A Possibilistic Programming Solution Method for a Bi-Objective Open Shop Scheduling Problem with Fuzzy Parameters

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Abstract

This paper presents a novel, bi-objective possibilistic mixed-integer linear programming model for an open shop scheduling problem. Machine-dependent setup times, fuzzy processing times and fuzzy due dates with triangular possibility distributions are the main constrains of this model. The objectives are to minimize the total weighted tardiness and total weighted completion times. An interactive fuzzy programming solution approach proposed by Torabi and Hassini (TH) is applied to convert the original model into an auxiliary single-objective crisp model. By the use of a classic approach given in the literature, some numerical instances in small sizes are generated at random and solved in order to obtain the Pareto-optimal solutions.

Keywords: Open shop scheduling problem; Total weighted tardiness; Total weighted completion times; Possibilistic programming; Fuzzy parameters.

1. Introduction

Scheduling is an act of defining priorities or arranging activities to meet certain requirements, constraints, or objectives (Sule, 1997). An open Shop scheduling problem (OSSP) is a kind of shop scheduling, in which the sequencing of operations on each machine and the processing route of each job are two important issues should be specified in this problem. The processing order of operations in the OSSP is immaterial. So, the solution space of the OSSP is much larger than job shop and flow shop problems. Matta and Elmaghraby (2009) described a proportionate multiprocessor open shop problem, in which the processing times are stage-dependent, not both job and stage-dependent and the objective is to minimize the makespan. They presented polynomial time algorithms for two special cases. Lin et al (2008) addressed a mixed-integer programming for a multi-processing-stage open shop problem with the characteristics of movable dedicated machines and no-wait restriction, also known as no intermediate queue. The objective is to minimize the total occupation time for all the processing stages. They proposed a two-phase heuristic algorithm for solving the given problem. Liaw (2005) designed a preemptive open shop problem to minimize the total tardiness. He developed an efficient constructive heuristic for solving large-sized problems and presented a branch-and-bound algorithm that incorporates a lower bound scheme based...
on the solution of an assignment problem as well as various dominance rules for solving medium-sized problems. Liaw (2008) addressed the problem of scheduling two-machine preemptive open shops to minimize the total completion time. He designed a non-linear mixed-integer programming to minimize the total weighted completion time, and then developed a dynamic programming algorithm to optimally solve small-sized problems. Finally, he proposed an efficient heuristic for solving large-sized problems.

The inherent uncertainty in the parameters of models takes into account in several fields. Torabi and Hassini (2008) designed a new multi-objective possibilistic mixed-integer linear programming model for supply chain master planning. After the use of appropriate strategies to convert this possibilistic model into an auxiliary crisp multi-objective linear model, they proposed a novel, interactive fuzzy programming approach to solve this model and obtain a preferred compromising solution. Konno and Ishii (2000) designed a preemptive open shop scheduling problem with the fuzzy resource and allowable time. This problem holds a bi-criteria term to be maximized, i.e., minimal satisfaction degree with respect to the processing intervals of jobs and, minimal satisfaction degree of resource amounts used in the processing intervals. They presented a solution procedure based on the network flow algorithm.

In scheduling problems, when setup tasks are considered separated from processing times, they can be started in advance when the machine is available before the job. Roshanaei et al. (2009) considered non-preemptive open shop scheduling problems with machine and sequence-dependent setup times in order to minimize the makespan. They proposed two new advanced meta-heuristics, namely multi-neighbourhood search simulated annealing and hybrid simulated annealing, to solve this problem. Low and Yeh (2008) addressed an open shop scheduling problem as a 0–1 integer programming model with the objective of minimizing the total job tardiness and some assumption, such as sequence-independent setup and sequence-dependent removal times. They proposed some hybrid genetic-based heuristics to solve the problem in an acceptable computation time. Mosheiov and Oron (2008) addressed batch scheduling problems on an m-machine open-shop with identical processing time jobs, machine and sequence-independent setup times assumptions. The objectives are to minimize the makespan and flow time. They proposed an $O(n)$ time algorithm for the flow time minimization problem.

According to the literature survey, there are a few researches regarding the fuzzy approach and its combination with multi-objective optimization in open shop scheduling problems. So, in this paper, a bi-objective possibilistic mixed-integer linear programming (BOPMILP) model is designed for the OSSP and then solved optimally.

The rest of this paper is organized as follows. The designed mathematical programming is presented in Section 2. The proposed interactive fuzzy programming solution approach is presented in Section 3. The computational results are shown in Section 4. Finally, the conclusion remark is provided in Section 5.

2. Mathematical Programming

The OSSP considered in this paper consists of $n$ jobs that should be processed on at most $m$ machines. Two performance measures, namely total weighted tardiness and total weighted completion times, are considered to be minimized.

2.1. Problem Assumptions

- Each job should be processed on at most $m$ machine.
- The processing route of each job is arbitrary.
- At any time, at most one job can be processed on each machine.
• The overlap between operations of a job is not allowed.
• All jobs are available at time 0.
• No preemption is allowed.
• Machine-dependent setup time is considered for each operation. This time is assumed to be separated from its processing times.
• Setup task can be started in advance when the certain machine is free.
• All the processing times and due dates are considered as fuzzy parameters with the triangular possibility distribution.

2.2. Notations

\( N \) Set of jobs, \( N = \{1, 2, \ldots, n\}; |N| = n \\
\( L \) Set of machines, \( L = \{1, 2, \ldots, m\}; |L| = m \\
\( i, k \) Job indices, \( (i, k = 1, 2, \ldots, n) \) \\
\( j, h \) Machine indices, \( (j, h = 1, 2, \ldots, m) \) \\
\( M \) A large positive number \\
\( w_i \) Tardiness penalty of job \( i \) \\
\( v_i \) Priority of job \( i \) \\
\( O_{ij} \) Operation of job \( i \) on machine \( j \); \( \forall i \in N; \forall j \in L \) \\
\( S_{ij} \) Set up time of job \( i \) on machine \( j \) \\
\( \tilde{p}_{ij} \) Fuzzy processing time of job \( i \) on machine \( j \) \\
\( \tilde{d}_i \) Fuzzy due date of job \( i \) \\
\( T_{Sij} \) Starting time of set up task for operation \( O_{ij} \) \\
\( T_i \) Tardiness of job \( i \) \\
\( C_i \) Completion time of job \( i \)

\[ Y_{ijh} = \begin{cases} 
1 & \text{if } O_{ij} \text{ precedes } O_{ih}, \text{ for job } i \\
0 & \text{Otherwise} \\
\end{cases} \quad \forall i \in N; \forall j, h \in L, j \neq h \]

\[ X_{ikj} = \begin{cases} 
1 & \text{if } O_{ij} \text{ precedes } O_{kj}, \text{ on machine } j \\
0 & \text{Otherwise} \\
\end{cases} \quad \forall i, k \in N, i \neq k; \forall j, h \in L \]

2.3. Mathematical Model

A possibility distribution can be stated as the degree of occurrence of an event with imprecise data (Torabi and Hassini, 2008). The processing times and due dates are considered as fuzzy parameters with THE triangular possibility distribution as follows:

\[ \tilde{p}_{ij} = (p_{ij}^m, p_{ij}^f, p_{ij}^o) \]  \hspace{2cm} (1)

\[ \tilde{d}_i = (d_i^m, d_i^f, d_i^o) \]  \hspace{2cm} (2)

\((p_{ij}^m, d_i^m), (p_{ij}^f, d_i^f), (p_{ij}^o, d_i^o)\) are most pessimistic, most possible, and most optimistic values, respectively, in which can be determined by the decision maker. The presented BOPMILP model is formulated bellow.
Min $Z_1 = \sum_{i=1}^{n} w_i T_i$ \hspace{1cm} (3)

Min $Z_2 = \sum_{i=1}^{n} v_i C_i$ \hspace{1cm} (4)

s.t.

\begin{align*}
T_{S_{ij}} + S_{ij} + \tilde{p}_{ij} &\leq C_i & \forall i \in N; \forall j \in L \\
T_{S_{ij}} + S_{ij} + \tilde{p}_{ij} - M(1 - Y_{ijh}) &\leq T_{S_{ih}} + S_{ih} & \forall i \in N; \forall j, h \in L, j \neq h \\
T_{S_{ih}} + S_{ih} + \tilde{p}_{ih} - M X_{ijh} &\leq T_{S_{ij}} + S_{ij} & \forall i \in N; \forall j, h \in L, j \neq h \\
T_{S_{ij}} + S_{ij} + \tilde{p}_{ij} - M(1 - X_{ikj}) &\leq T_{S_{kj}} & \forall i, k \in N, i \neq k; \forall j \in L \\
T_{S_{kj}} + S_{kj} + \tilde{p}_{kj} - M X_{ikj} &\leq T_{S_{ij}} & \forall i, k \in N, i \neq k; \forall j \in L \\
C_i - \tilde{a}_i &\leq T_i & \forall i \in N \\
Y_{ijh} + Y_{ihj} & = 1 & \forall i \in N; \forall j, h \in L, j \neq h \\
X_{ikj} + X_{klj} & = 1 & \forall i, k \in N, i \neq k; \forall j \in L \\
T_{S_{ij}} &\geq 0 & \forall i \in N; \forall j \in L \\
T_i, C_i &\geq 0 & \forall i \in N \\
X_{ikj} &\in \{0,1\} & \forall i, k \in N, i \neq k; \forall j, h \in L, j \neq h 
\end{align*}

Two objective functions (i.e., total weighted tardiness and total weighted completion times) are shown by equalities (3) and (4). Constraint (5) describes the completion time of job $i$. Constraints (6) and (7) express the relationship between two operations of the job $i$ that are not required to be consecutive. The starting time for processing operation $O_{ih}$ is greater than or equal to the completion time of operation $O_{ij}$. Constraints (8) and (9) characterize the constraint of the operational sequence of the operations which are processed on the same machine. The setup task of operation $O_{ij}$ cannot be started until machine $j$ has finished the processing task of operation $O_{ij}$. Constraint (10) describes the tardiness for each job $i$, which is defined by:

$$T_i = \max \left(0, C_i - \tilde{a}_i\right) \hspace{1cm} \forall i \in N$$

Constraint (11) expresses the order of any two operations of a job, if $Y_{ijh} = 1$ then $Y_{ihj} = 0$; otherwise, $Y_{ijh} = 0$ and $Y_{ihj} = 1$. Constraint (12) expresses the order of any operation pairs $(O_{ij}, O_{ik})$ on the same machine $j$, if $X_{ikj} = 1$ then $X_{kij} = 0$; otherwise, $X_{ikj} = 0$ and $X_{kij} = 1$. Constraint (13) expresses that all jobs should be available for scheduling at time 0. Constraints (14) and (15) define the continuous and binary decision variables.

### 2.4. An Equivalent Auxiliary Crisp Model

The weighted average method (Lai and Hwang, 1992a) is used for the defuzzification process and transforming two fuzzy parameters in the left-hand sides of Constrains (5) to (10) into crisp numbers. The equivalent auxiliary crisp constraints are represented as follows:

\begin{align*}
T_{S_{ij}} + S_{ij} + w_1 p^P_{ij,\beta} + w_2 p^m_{ij,\beta} + w_3 p^0_{ij,\beta} &\leq C_i & \forall i \in N; \forall j \in L \\
T_{S_{ij}} + S_{ij} + w_1 p^P_{ij,\beta} + w_2 p^m_{ij,\beta} + w_3 p^0_{ij,\beta} - M(1 - Y_{ijh}) &\leq T_{S_{ih}} + S_{ih} & \forall i \in N \\
\end{align*}
\[ T_{slh} + S_{lhi} + w_1 p_{lih}^p + w_2 p_{lih}^m + w_3 p_{lih}^o - M \times Y_{ijh} \leq T_{sjl} + S_{jhl} \] 
\[ \forall i \in N; \forall j, h \in L, j \neq h \] (19)

\[ T_{slj} + S_{lij} + w_1 p_{lij}^p + w_2 p_{lij}^m + w_3 p_{lij}^o - M(1 - X_{ikj}) \leq T_{skj} \] 
\[ \forall i, k \in N, i \neq k; \forall j \in L \] (20)

\[ T_{skj} + S_{kjl} + w_1 p_{kjl}^p + w_2 p_{kjl}^m + w_3 p_{kjl}^o - M \times X_{ikj} \leq T_{slj} \] 
\[ \forall i, k \in N, i \neq k; \forall j \in L \] (21)

\[ C_i - w_4 d_{li}^p + w_2 d_{li}^m + w_3 d_{li}^o \leq T_i \] 
\[ \forall i \in N \] (22)

where, \( \beta \) is the minimal acceptable possibility and usually determined by the decision maker. It is worth noting that the events with the possibility more than or equal \( \beta \) are the acceptable events (Lai and Hwang, 1992b). Also, \( w_1 + w_2 + w_3 = 1 \), and \( w_1, w_2 \) and \( w_3 \) denote the weights of the most pessimistic, most possible and most optimistic values of fuzzy parameters, respectively. By the use of the most likely solution concept proposed by Lai and Hwang (1992b), these parameters can be set as: \( \beta = 0.5 \), \( w_1 = 1/6 \), \( w_2 = 4/6 \), \( w_3 = 1/6 \). It should be noted that \( (p_{ijl}^p, d_{ijl}^p), (p_{ijl}^m, d_{ijl}^m), \) and \( (p_{ijl}^o, d_{ijl}^o) \) denote the most pessimistic, most possible and most optimistic values of the acceptable events (i.e., acceptable processing times and due dates) respectively, based on the specified value of \( \beta \). Thus, the auxiliary crisp bi-objective mixed-integer linear programming (BOMILP) model is as follows:

\[ \text{Min } Z = [Z_1, Z_2] \]  
\[ \text{s.t. } \]  
\[ v \in F_{(u)} \] (23)

where, \( v \) denotes a feasible solution vector consists of all of the continuous and binary variables in the original model, and \( F_{(u)} \) denotes the feasible area involving the crisp constraints given in (11) to (15) and (17) to (22).

3. Interactive Fuzzy Programming Solution Approach

In this paper, an efficient interactive fuzzy programming solution approach, which belongs to the priori optimization methods group for solving multi-objective decision making (MODM) problems, is used. This approach is applied to achieve the Pareto-optimal solutions of the obtained bi-objective crisp model. This method is named as TH method proposed by Torabi and Hassini (2008). The steps of the TH method are presented below:

**Step 1)** Determine appropriate triangular possibility distributions for the fuzzy parameters and develop the original BOPMILP model for the OSSP.

**Step 2)** Given the minimal acceptable possibility level for fuzzy parameters, \( \beta \), convert the fuzzy constraints into the corresponding crisp ones, and formulate the auxiliary crisp BOMILP model.

**Step 3)** Determine the positive ideal solution (PIS) and negative ideal solution (NIS) for each objective function by solving the corresponding MILP model as follows:

\[ Z_1^{PIS} = \text{Min } \sum_{i=1}^{n} w_i T_i \]  
\[ \text{s.t. } v \in F_{(u)} \] (24)

\[ Z_1^{NIS} = \text{Max } \sum_{i=1}^{n} w_i T_i \]  
\[ \text{s.t. } v \in F_{(u)} \] (25)

\[ Z_2^{PIS} = \text{Min } \sum_{i=1}^{n} v_i C_i \]  
\[ \text{s.t. } v \in F_{(u)} \] (26)
\[ Z_{2}^{NIS} = \text{Max} \sum_{i=1}^{n} v_i c_i \quad \text{s.t.} \quad v \in F(v) \quad (27) \]

Obtaining the above ideal solutions requires solving four mixed-integer linear programs. To reduce the computational time, the negative ideal solutions can be determined by the use of following heuristic rule.

\[ Z_{h}^{NIS} = \text{Max} (Z_h(v_h^*)) ; \quad h = 1,2 \quad , k = 1,2 \quad (28) \]

It is worth noting that \( v_h^* \) and \( Z_h(v_h^*) \) denote the decision vector associated with the PIS of the \( h \)-th objective function and the corresponding value of the \( h \)-th objective function, respectively (Torabi and Hassini, 2008). The results are shown in Table 1.

<table>
<thead>
<tr>
<th>Table 1. Payoff table</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z_1 )</td>
</tr>
<tr>
<td>( v_1^* )</td>
</tr>
<tr>
<td>( v_2^* )</td>
</tr>
</tbody>
</table>

**Step 4)** Specify a linear membership function for each objective function computed below. Figure 1 represents the graph of these membership functions.

\[ \mu_{Z_1}(v) = \begin{cases} 1 & Z_1 < Z_1^{PIS} \\ \frac{Z_1^{NIS} - Z_1}{Z_1^{NIS} - Z_1^{PIS}} & Z_1^{PIS} \leq Z_1 \leq Z_1^{NIS} \\ 0 & Z_1 > Z_1^{NIS} \end{cases} \quad (29) \]

\[ \mu_{Z_2}(v) = \begin{cases} 1 & Z_2 < Z_2^{PIS} \\ \frac{Z_2^{NIS} - Z_2}{Z_2^{NIS} - Z_2^{PIS}} & Z_2^{PIS} \leq Z_2 \leq Z_2^{NIS} \\ 0 & Z_2 > Z_2^{NIS} \end{cases} \quad (30) \]

**Figure 1.** Linear membership function for \( Z_1(Z_2) \)
Step 5) Convert the auxiliary BOMILP model into an equivalent single-objective MILP by the use of the following auxiliary crisp formulation:

\[
\begin{align*}
\text{Max} & \quad \lambda(v) = \gamma \lambda_0 + (1 - \gamma) \sum_h \theta_h \mu_{Z_h}(v) \\
\text{s.t.} & \quad \lambda_0 \leq \mu_{Z_h}(v), \quad h = 1, 2 \\
& \quad v \in F(v) \\
& \quad \lambda_0, \gamma \in [0,1] 
\end{align*}
\] (31)

According to two objective functions of this problem, Constraint (32) is written by:

\[
\begin{align*}
Z_1^{NIS} - \sum_{i=1}^{n} w_i T_i & \geq \lambda_0 (Z_1^{NIS} - Z_1^{PIS}) \\
Z_2^{NIS} - \sum_{i=1}^{n} v_i C_i & \geq \lambda_0 (Z_2^{NIS} - Z_2^{PIS})
\end{align*}
\] (35) (36)

where, \( \mu_{Z_h}(v) \) and \( \lambda_0 = \min_h \{ \mu_{Z_h}(v) \} \) denote the satisfaction degree of the \( h \)-th objective function and the minimum satisfaction degree of objectives, respectively. Moreover, \( \theta_h \) and \( \gamma \) indicate the relative importance of the \( h \)-th objective function and the coefficient of compensation, respectively. Parameter \( \theta_h \) is determined by the decision maker based on her/his preferences such that \( \sum_h \theta_h = 1, \theta_h > 0 \). Also, \( \gamma \) controls the minimum satisfaction level of objectives as well as the compromised degree among the objectives implicitly. That is, the proposed formulation is capable of yielding both unbalanced and balanced compromised solutions for a given problem based on the decision maker’s preferences through adjusting the value of parameter \( \gamma \) (Torabi and Hassini, 2008).

Step 6) Given the coefficients (\( \theta_h, \gamma \)). Solve the equivalent single-objective MILP. If the decision maker is satisfied with the obtained efficient compromise solution, stop. Otherwise, change the value of some controllable parameters (say \( \beta \) and \( \gamma \)), and then go back to Step 2.

The validity and efficiency of the proposed OSSP model is checked and shown by the use of some numerical instances in Section 4.

It should be noted that changing the values of controllable parameters of equivalent single-objective MILP from (31) to (36), results to achieve a set of non-dominated solutions as the Pareto-optimal solutions for the BOMILP model given in (23).

4. Computational Results

To test and analyze the validity and efficiency of the mathematical model presented in Section 2 and the proposed MODM method called TH method presented in Section 3, some numerical examples in small sizes are generated randomly by the use of a classic approach of the literature. These problems are solved exactly by the use of the Lingo 8 software in a few minutes and the results of the TH method are analyzed.

4.1. Generating Numerical Instances

The processing times and due dates are considered as symmetric triangular possibilistic distributions given bellow:
\[ \bar{p}_{ij} = \left( p_{ij} - u_{ij}, p_{ij}, p_{ij} + u_{ij} \right) \]
\[ \bar{d}_i = \left( d_i - u_i, d_i, d_i + u_i \right) \]

where, \( p_{ij} \) and \( d_i \) are the most possible values, and \( u_{ij} \) and \( u_i \) represent the extension values of these fuzzy numbers. In this paper, a classic approach in the literature proposed by Loukil et al. (2005) to generate numerical instances for scheduling problems is used as follows. The most possible values of processing times and due dates are uniformly distributed in the intervals \([0,100]\) and \([1 - T - \frac{R}{2}, 1 - T + \frac{R}{2}]\), respectively.

where, \( P = (m + n - 1) \bar{p} \), in which \( \bar{p} = \sum_{i=1}^{n} \sum_{j=1}^{m} p_{ij} / (n \times m) \) is the mean of most possible values of the processing times. Two parameters \( R \) and \( T \) take their values in the sets \( \{0.2, 0.6, 1\} \) and \( \{0.4, 0.6, 0.8\} \), respectively. The values of \( u_{ij} \) and \( u_i \) are uniformly distributed in the interval \([1,12]\). Also, the setup times are random variables between 10 and 50. Finally, the values of priority and tardiness penalty, \( v_i \) and \( w_i \) are selected randomly between 1 and 5.

### 4.2. Parameter Setting

Several experiments are implemented in different small sizes and different sets of controllable parameters. Then, the effective values in terms of solution quality are chosen. The parameters of the TH method and those of the generating instances approach are as follows. For the final MILP model given in Section 3, parameter \( \gamma \) takes its values in the set \( \{0, 0.1, \ldots, 1\} \). Also, it is assumed that the preference information corresponding to the relative importance of the objective functions are specified linguistically by the decision maker as: \( \theta_1 = \theta_2 \) and \( \theta_1 > \theta_2 \). Thus, the values of these parameters in the first case are \( \theta_1 = \theta_2 = 0.5 \), and in the second case are \( \theta_1 = 0.8 \) and \( \theta_2 = 0.2 \). The values of controllable parameters \( R \) and \( T \) are presented in the next subsection.

### 4.3. Computational Results

Two kinds of sample examples in small sizes with different combinations of jobs and machines are considered. For each kind of these examples, three numerical instances are generated randomly. The characteristic of sample examples and the values of controllable parameters for each numerical instance are presented in Tables 2 and 3. In Table 4, the values of positive ideal solutions \( (Z^PIS_i) \) and negative ideal solutions \( (Z^NIS_i) \) for each numerical instances are illustrated. Using the obtained values of PIS and NIS of each objective function, the final MILP model for each numerical instances are exactly solved by the Lingo 8 software. The results of example 4×4-b is presented in Table 5.

<table>
<thead>
<tr>
<th>Sample example</th>
<th>representation</th>
<th>No. of jobs</th>
<th>No. of machines</th>
<th>No. of decision variables</th>
<th>No. of constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4×3</td>
<td>4</td>
<td>3</td>
<td>104</td>
<td>197</td>
</tr>
<tr>
<td>2</td>
<td>4×4</td>
<td>4</td>
<td>4</td>
<td>152</td>
<td>309</td>
</tr>
</tbody>
</table>
Table 3. Values of controllable parameters for each numerical instance.

<table>
<thead>
<tr>
<th>Numerical instances</th>
<th>R</th>
<th>T</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.2</td>
<td>0.8</td>
<td>0.5</td>
</tr>
<tr>
<td>b</td>
<td>0.6</td>
<td>0.6</td>
<td>0.5</td>
</tr>
<tr>
<td>c</td>
<td>1</td>
<td>0.4</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Considering the computational results, the following results are obtained.

- In some cases with different relative importance of objective functions, the TH method performs well and efficiently. It means according to the relative importance, the solutions found by this method are unbalanced compromised solutions. But in other cases, this method does not perform well and the satisfaction degree of the objective function with lower importance is more than that of the objective function with higher importance.
- When the relative importance is equal, in some cases, the satisfaction degrees of objective functions are very close to each other. It means the solutions found by the TH method are approximately balanced compromised solutions.
- As mentioned in Section 3, each optimal solution of the final MILP model is an efficient solution to the BOMILP model. Thus, in all cases, by changing the values of controllable parameters (i.e., \( \gamma \) and \( \theta \)), two Pareto-optimal solutions are found by the TH method. Figure 2 indicates the Pareto-optimal frontier of example 4×4-b.

Table 4. Values of positive and negative ideal solutions

<table>
<thead>
<tr>
<th>Sample example</th>
<th>( Z_1 )</th>
<th>Z₂</th>
<th>( \mu_\text{PIS} )</th>
<th>( \mu_\text{NIS} )</th>
<th>( \mu_\text{PIS} )</th>
<th>( \mu_\text{NIS} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4×3-a</td>
<td>478</td>
<td>529</td>
<td>620</td>
<td>707</td>
<td>459</td>
<td>462</td>
</tr>
<tr>
<td>4×3-b</td>
<td>293</td>
<td>376</td>
<td>459</td>
<td>462</td>
<td>660</td>
<td>688</td>
</tr>
<tr>
<td>4×3-c</td>
<td>107</td>
<td>117</td>
<td>660</td>
<td>688</td>
<td>577</td>
<td>706</td>
</tr>
<tr>
<td>4×4-a</td>
<td>621</td>
<td>707</td>
<td>577</td>
<td>706</td>
<td>850</td>
<td>1040</td>
</tr>
<tr>
<td>4×4-b</td>
<td>476</td>
<td>508</td>
<td>850</td>
<td>1040</td>
<td>566</td>
<td>583</td>
</tr>
<tr>
<td>4×4-c</td>
<td>471</td>
<td>484</td>
<td>566</td>
<td>583</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5. Computational results of example 4×4-b

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( \theta_1 = 0.5 ), ( \theta_2 = 0.5 )</th>
<th>( \theta_1 = 0.8 ), ( \theta_2 = 0.2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z_1 )</td>
<td>( Z_2 )</td>
<td>( \mu_\text{PIS} )</td>
</tr>
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\( \beta = 0.5 \), \( R = 0.6 \), \( T = 0.6 \)
5. Conclusion

In this paper, a novel, bi-objective possibilistic mixed-integer linear programming model was presented for an open shop scheduling problem. An interactive fuzzy programming solution approach, named TH, proposed by Torabi and Hassini (2008) was applied to convert the original model into an auxiliary single-objective crisp model in order to obtain the Pareto-optimal solutions for small-sized problems. Considering the related results, in more cases, the TH method performs well according to the relative importance of objective functions and two Pareto-optimal solutions are found.

6. References


