Cash Flow Simulation Embedded Real Options

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Abstract

Cash flow simulation embedded option is an option whose value is based on choosing the optimal decision in each time step during a single cash flow calculation simulation run. Cash flow simulation embedded options are mostly operative options with continuous, gradual and nearly immediate exercise with well-known payoff or benefits. Typically these options are difficult to model with other methods than Monte Carlo simulation. However, cash flow simulation embedded options and the common, once exercisable options can be applied simultaneously in an investment valuation by using the simulated cash flow with embedded options as the underlying asset for the lattice, which is then used in valuing other lattice type options and their interactions.

Keywords: real options, cash flow simulation

1. Introduction

Real options analysis (ROA) is a fairly new approach for valuing managerial flexibility in investment decisions. These decisions can be related for example to deferring, extending or abandoning the project. According to real options thinking, investments are characterized by sequential and irreversible investments made under uncertainty (Dixit and Pindyck 1995). The expression real options stems from utilizing the mathematics commonly used in valuing financial options related to some uncertain real investments. According to Trigeorgis and Mason (1987) and Brealey and Myers (2003), real options analysis can be also illustrated as an improved, economically corrected version of decision-tree analysis that has adopted the market perspective allowing determination of expected values using risk-neutral probabilities and risk-free discounting rate.

Cash flow simulation embedded option is an option whose value is based on choosing the optimal decision in each time step during a single cash flow simulation run. Characteristic for these cash flow simulation embedded options is that their value is based on on-going activities where different states of operation change back and forth during their lifespan. These options are usually continuous with gradual and nearly immediate option exercise with sufficiently well known payoff. As such, their valuation with any analytical or lattice method is very difficult if not impossible. Therefore, cash flow simulation options differ from typical real options that are modeled either with analytical closed-form solution or with lattices as once exercisable, irreversible opportunities during a project. Also, these decisions often happen during certain stages of the project. Most typical of these are the decisions of go or no-go in a staged product development project or extending or abandoning the project.
The strength of the *cash flow simulation embedded options* and the ordinary lattice type options – after this these ‘ordinary’ options are called *lattice options* to make the distinction between the two - are that they can be applied simultaneously in a valuation case. The simulated cash flow with the embedded options is used as the underlying asset in the option valuation lattice construction. Therefore, the underlying asset value and its stochastic process parameters are derived from the simulated, consolidated cash flow calculation, and these process parameters, usually at least the underlying asset value and volatility, are used in lattice construction and in valuing lattice options.

After the introduction, different real option valuation approaches and methods are presented. Then, Section 3 discusses use of Monte Carlo simulation in option pricing. Section 4 presents cash flow simulation embedded options and compares them to ordinary lattice type options. Section 5 discusses how cash flow embedded options can be used together with lattice type options. Section 6 presents the conclusions.

2. Real options valuation methods

Real option valuation methods can be classified according to Amram & Kulatilaka (1999) into partial differential equation (PDE) methods, dynamic programming, and simulation. Lander and Pinches (1998) present a categorization of continuous-time models, finite-difference schemes, binomial models, and other lattice models. Pritsch (1999) categorizes the valuation methods firstly to analytical (closed-form and approximations of closed form solutions) and numerical methods. Numerical methods can be further divided to approximations of stochastic processes (simulations and lattice approaches) and approximations of differential equations with finite difference models.

Based on the above mentioned references, the following categorization of Figure 1 can be constructed. Firstly, methods can be classified into *analytical continuous time models* and *numerical discrete time models*. Another clear classification can be done between *partial difference equation models, stochastic process approximations, and general risk-neutral lattice and tree approaches*. Despite of some simulation and general risk-neutral pricing models, all the numerical methods apply *dynamic programming* in the solution procedure. The procedure suggested in this paper combines both the use of *Monte Carlo and other simulations* and *Numerical lattice approximation of stochastic process approaches* together.

Most academic models are based on having the tradeoff between realism and simplicity tilted in favor of simplifying assumptions, because this allows analytical closed-form solutions while preserving the major insight. Qualitative essence of the results may often remain even with more realistic assumptions leading to more complex models (Bjerksund & Ekern 1993). While these models may provide generalized scientific knowledge and managerial insights, they are often of no practical use in investment appraisal. As pointed out by Charnes (2007), many purists prefer to use only models that avail themselves to analytic solution. When confronted by a realistic complication that precludes analytic solution, the complication is simply assumed away, whereas practitioners wish to include realistic complications and are happy to accept the trade-off of getting an approximate solution with simulation.
Figure 1: Categorization of different real option valuation approaches.

3. Monte Carlo simulation in option valuation

Until present, when the choice between a conceptually superior but numerically less tractable model and one with easy analytical evaluation had to be made, very often the easier model was chosen even at the expense of oversimplifying the matter at hand (Jäckel 2002). This was mostly due to limited computing power and less advanced algorithms available. When compared with a purely mathematical analysis, simulation allows models that are closer to the real world with more complexity and fewer restricting assumptions of reality. With the aid of ever faster computers and improved Monte Carlo techniques and numerical methods, we are nowadays in the position to use conceptually superior and more realistic modelling approach. Techniques that were previously considered to be prohibitively computationally expensive and not feasible are nowadays often the method of choice. (Jäckel 2002). An important advantage of simulation techniques is that they lend themselves well to parallel computing architecture (Longstaff & Schwartz, 2001).

Original use of Monte Carlo method for option valuation has been a Monte Carlo integration, where instead of attempting to carry out directly high-dimensional integrations involving the transition probabilities of many possible intermediate events and states by the use of lattice methods, single chains of events are sampled. Although Monte Carlo simulation can be used for numerical integration in valuing options, more common ways to use the methodology is to use it either for stochastic path sampling or for combining different uncertainties related to the option pricing together, e.g. in valuing exchange options, basket options, or for consolidating cash flow simulation uncertainties.
Direct Monte Carlo simulation method (also called trace driven simulation) suggested by Boyle (1977) is based on the idea of approximating the stochastic process numerically. The valuation procedure consists of the following steps:

1) Random path for the underlying asset is sampled in a risk-neutral world and the payoff of the option at the maturity is calculated.
2) This is repeated at least several thousand times
3) The mean of the sample payoffs is calculated to get an estimate of the expected payoff in a risk-neutral world
4) Payoff is discounted at risk-free rate to get the present option value.

Direct Monte Carlo simulation has several advantages. It can accommodate complex payoffs and complex stochastic processes and take into account situations where payoff depends on some function of the whole path followed by a variable, not just its terminal value. Payoffs may occur at several times during the life of the derivative (Hull, 2005). The approach has become more attractive relative to other numerical techniques because it is flexible as well as easy to implement and modify (Boyle et al. 1997). The terminal value distribution may also be of any arbitrary shape or form and it does not need to be related to any stochastic process.

The direct Monte Carlo method does not allow valuation of American options because it does not take into account the early exercise capability. The problem is that it is difficult to perform backward optimization with forward simulation, which in general is necessary for American-type options (Dias 2000). Also, changing some aspects of the model may force to change the whole optimization procedure. Longstaff & Schwartz (2001) use least-squares analysis to determine the best-fit relationship between the value of continuing and values of relevant variables at each time an early exercise decision has to be made. At each exercise time, the payoff from immediate exercise is compared with the expected payoff from continuation. The optimal exercise strategy is determined by the conditional expectation function that is estimated from the cross-sectional information in the simulation by using least squares. This is found by regressing the ex post realized payoffs from continuation on functions of the values of the state variables, and this fitted value provides a direct estimate of the conditional expectation function. A complete specification of the optimal exercise strategy along each path is obtained by estimating the conditional expectation function for each state. Dias (2000) suggests using genetic algorithms to determine optimal early exercise.

However, one significant strength of Monte Carlo simulation is capability to value path-dependent options whose payoff depends on the path followed by the price of the underlying asset, and not just on its final value. Examples of such options are Asian options whose value depends on the average price of the underlying asset. The price of such option is mean of the sample values, which are calculated by sampling a random path for the underlying asset in a risk-neutral world, calculating the path-dependent payoff, and discounting the payoff at the risk-free interest rate (Hull 2006). The only difference is that in a single simulation run the payoff is the sum of the discretized path values instead of being only dependent of the final payoff value.
The approach is efficient for real options purposes, because it allows use of any kind of stochastic processes including jumps and mean-reversion for the parameters. Because some of these options are rather operational by nature, their stochastic processes may also differ from those typical for financial derivatives modeling. While financial models often assume geometric Brownian motion and perhaps mean-reversion and jumps, industrial operational processes may also have trends, seasonality, cyclicality, S-shape curves and even extreme value like events.

There are three ways to define a distribution in use simulation modeling: 1) trace-driven simulation with direct use of data values in modeling, 2) use of empirical distribution based on data values, and 3) use of theoretical distribution were data is used to fit the data values into certain predefined distributions (Law & Kelton, 2000). These different approaches are introduced briefly as they have different qualities and may be used with different approaches for valuing different real options together in the same project.

In trace-driven simulation data values are used directly in modeling. Traditional Monte Carlo simulation approach described earlier (Boyle 1977) is an example of this approach. Each simulation trial as such is used to define the value of a single option. The advantage of the approach is that it is completely independent of the assumptions of the stochastic process and the distribution form of the terminal values. However, this method is not that efficient in comparison with other approaches and requires more computing time. It does not either suit for valuing (financial) American put options without significant modifications, because the method does not handle the possible early exercise. It is not generally simple to determine the optimal exercise of the real option if the option payoff is dependent of the forthcoming expected future cash flows. Especially cases with several interacting sequential and parallel options make this method impractical if not impossible without use of some other methods and advanced optimization algorithms. However, the approach is flexible for such cases were the real option value is dependent of the value of the underlying asset at each point during the option life and the exercise is determined purely based on the existing value of the underlying asset value at each moment during the option life. Asian options are examples of this. Also the cash flow simulation embedded options are calculated according to this approach.

Another use of simulation is to determine empirical distribution based on data values. In this approach, data is grouped and then used for valuation purposes. This approach does not limit the form of the distribution, but it requires more computational time. Therefore the method is suggested only for the cases were no theoretical distribution can provide sufficient accuracy. Statistical analysis is also more difficult and the use of the resulting distribution for a practical valuation purpose is limited. However, several option valuation approaches (e.g. Barraquand & Martineau 1995) are based on determining the empirical “bins” of underlying asset states and values and then computing the transition probabilities between these states.

In use of theoretical distribution data is used to “fit” the data values into certain predefined distributions. Theoretical distributions are very compact way of representing a set of data values. The method also allows statistical analysis. Theoretical distribution also smoothes out the data and may provide information on the overall underlying
distribution. Theoretical distributions are also easier to change. Present knowledge of different distributions and the possibility to shift or truncate them also allows them to be used nearly always instead of empirical distributions.

Use of theoretical distributions is by far the most popular valuation approach for options. Most common approach in practice is to assume that the process follows geometric Brownian motion with normally distributed rate of return having then lognormal underlying asset value distribution. The approach is also used for valuing basket option where the payoff is dependent on the value of a portfolio (or basket) of assets. A European basket option can be valued with direct Monte Carlo simulation, by assuming that the assets follow correlated geometric Brownian motion processes. However, a much faster approach is to calculate the first two moments of the basket at the maturity of the option in a risk-neutral world and then assume that value of the basket is lognormally distributed at that time (Hull 2006). This approach can be further improved with a 3-parameter lognormal distribution by shifting the distribution, which provides more accuracy yet allows similar flexible calculation with closed form solutions (Brigo et al. 2004). Also American option valuation is possible with a lattice approach, assuming that stochastic process and the inner node values in the tree can be deducted from the terminal distribution end node values. This approach has also been applied in valuing cash flow simulation based lattice options (Haahtela 2006, 2008 2010).

Some of the real option approaches, even without using simulation, apply one of the previous distribution parameterization approaches in valuation. Both the trace-driven and theoretical distribution approaches often use real simulated data, but they may also be based on managerial assumptions. However, majority of the subjective estimate models assume use of a theoretical distribution, most of them – unnecessarily - use of lognormal distribution, while trace-driven approach usually assumes use of historical or simulated data. Angelis (2002) approach, similarly to Datar & Matthews (2007), may either use trace-driven approach or theoretical distribution approach. Mun (2003, 2006) suggests using of theoretical distributions based on managerial assumptions for volatility estimation. This may be done for example by determining the percentiles or the minimum, most likely and maximum value for the distribution. Also nearly all the Monte Carlo simulation based volatility estimation procedures use the assumption of a theoretical (lognormal) distribution. The approach suggested in this paper combines trace-driven approach for cash flow simulation embedded options in the consolidated underlying asset modeling, which is then fitted into theoretical (displaced lognormal) distribution(s) used in the lattice parameterization similarly to Haahtela (2010).

4. Cash flow simulation options

The idea of cash flow simulation embedded options is based on choosing the optimal forward-looking decision in each time step during a single cash flow simulation run. Characteristic for these cash flow simulation embedded options is that their value is based on on-going activities where different states of operation change back and forth during their lifespan. These options are usually continuous with gradual and nearly immediate option exercise with sufficiently well known payoff. Typical modeling cases for these kinds of options are 1) switching options between raw materials and end
products, 2) capacity subcontracting for peak demand, and 3) contracts having clauses with prices and quantities that are related to each other and have certain maximum and minimum levels for each time period.

For example, dual-fuel boiler may change operational model between fuels according to their market prices. In mining industry unprofitable mines may be closed down when commodity price is low. In charter shipping, older and less profitable ships travel only during high charter prices (Howell et al. 2001). Similarly cheap-to-build, inefficient energy generator peaker plants are only to be used when electricity prices are high and to be shut down when price goes down (Mun 2002). Oil and gas refineries change their outputs according to the market demand and cyclacity (Mun 2002). Large international manufacturing companies may change their products and production strategy according to the changes in customer taste, production and logistics and raw material costs, exchange rates among different global market areas (Trigeorgis 1996).

As such, cash flow simulation options are not by their use so different from traditional real options. The difference between these cash flow simulation embedded options and traditional options is rather in the difference of ease and convenience of modeling the flexibility. While other methods are rather good at valuing once exercisable methods (especially the binomial lattices and trees), they are not that good at valuing options whose value is accumulated during certain time period. As such, all types of different real options and their combinations with interdependencies – e.g. expanding, switching, abandoning, contracting, scaling etc. can be modeled either with a lattice method or by cash flow simulation. However, in some cases modeling some of these as cash flow simulation embedded options is easier. Even better, both approaches can be applied simultaneously in a project valuation.

There are three different considerations on how (and whether) the option should be rather modeled as a cash flow simulation embedded option or a lattice (binomial tree) option. The first question is related to the switching costs between different alternatives. Second question is if there is time lag between the decision and the actual operational change. The third question is if the change is irreversible once it is done. According to these aspects, the complexity and modeling of these cash flow simulation embedded options can be categorized into three different types according to their properties and dependence of the forward-looking nature in relation to their exercise. Cases with immediate decision and payoff without significant exchange costs and once exercisable exchange options are sufficiently straightforward to model, while options with exercise costs and several exchanges between different alternative operational stages are significantly more complex.

These options are typically based on such decisions that are done continuously during the project. If the decisions and operational changes can be done immediately without time lag or changing costs, such decision can be modeled in each time period as an option to choose between different alternatives max(x1, ..., xn). In simple cases, the selection criteria are straightforward, and the most optimal alternative is selected. This is typical if there are no changing costs and the change can be done without time lag. In terms of simulation modeling, during each simulation trial, the value of the option is defined as

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\( \Sigma \max(x_1, x_2) \). After several thousand simulation trials, the results approximate the option value sufficiently precisely. Value of the option is the difference between the value of the case without this simulated option and a

Figure 2 shows a single sample path for one cash flow simulation parameter value with and without an option. The option guarantees to its holder a minimum unit price of 22 regardless of the market value. As a result, the value of this option to the holder is the difference between the value with options less the value without options over the time of investment. Options similar to this example are extremely difficult to value without the approach of using Monte Carlo cash flow simulation. There are no easy – if any - analytical closed form solutions or binomial approximations available for these kinds of options, especially if the parameter value does not follow any common stochastic process. On the other hand, cash flow simulation approach is immune to the pattern of the stochastic process. Another advantage is that the cash flow simulation parameters may be both auto and cross- correlated, and the approach is still capable for modeling their value. The payoff may also be other than a simple \( \max(x_1, \ldots, x_n) \) and even of a complex functional form.

![Diagram](image)

*Figure 2: Illustration of the value of the cash flow simulation embedded real option.*

In case of switching costs, the problem becomes more complex, because the analysis requires forward-looking approach. In such cases it may not be justified to make the change immediately when the price of another raw material goes above the price of another material but only after it has increased enough above certain threshold level. A hysteresis band is a range of values that the state variable may take for which mode switching is passed up, even when short-term conditions make switching appear profitable. In such cases, the threshold values for switching operational mode may have to be modeled separately. This can be done in practice with simulation software with numerical optimizing capabilities, e.g. with least-squares regression (Longstaff & Schwartz, 2001) or with genetic algorithms (Dias 2000). The resulting optimal state dependent switching boundaries or their approximations can be inserted into the cash flow simulation model in a functional form. In many cases, even a good approximation of the optimal threshold level and timing may cause only a minimal ex-ante loss in comparison with the perfect optimal solution.
However, in most real life cases, if operational changes are possible, the operational switching cost and time is very small. As a result, the option cost is not likely to be a switching cost during the running project but rather the option cost is related to the higher equipment purchase price, because more flexible manufacturing technology is often more expensive. Therefore, these switching options do not necessarily require localized analyses for optimal or nearly optimal switching thresholds. Also existence of flexible technology, for example in energy technology with dual-fuel engines, may reduce the volatility of relative price changes between different energy sources. This may potentially reduce the future value of flexible equipment. In modeling perspective, this means that most of the decisions become more like cases with immediate decisions and payoffs.

As a result, we can compare the typical properties of the cash flow simulation embedded options and the lattice type options. Table 1 illustrates the differences between the two approaches.

<table>
<thead>
<tr>
<th>Cash flow simulation embedded real option</th>
<th>Lattice valuation type option</th>
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<tbody>
<tr>
<td>• Continuous, gradual and nearly immediate option exercise with well-known payoff or benefit</td>
<td>• Option exercised once, with not well-known and delayed anticipated option payoff</td>
</tr>
<tr>
<td>• Based on on-going activities where different states of operation change back and forth during their lifespan</td>
<td>• Typically one-time, completely or nearly completely irreversible actions</td>
</tr>
<tr>
<td>• Use of peak energy plants</td>
<td>• Abandoning the project</td>
</tr>
<tr>
<td>• Switching options between raw materials and end products, e.g. use of dual-fuel boilers</td>
<td>• Continuation of an R&amp;D project (go or no-go decision)</td>
</tr>
<tr>
<td>• Use of capacity subcontracting</td>
<td>• Expansion options (new market or technology)</td>
</tr>
<tr>
<td>• contracts having clauses with prices and quantities that are related to each other and have certain maximum and minimum levels</td>
<td></td>
</tr>
<tr>
<td>• Variables influencing the cash flows have a wide variety of correlations and patterns (mean-reversion, jumps, S-curves, trends, cyclicity, seasonality, extreme values events) of which some are not typical for the financial option underlying asset movements</td>
<td>• Process of the underlying asset value (present value of the future cash flows) is likely to have a more continuous random stochastic process regardless of how irregular the pattern of future cash flows during each time period.</td>
</tr>
<tr>
<td>• Mostly operative options by nature</td>
<td>• More often strategic than operative options</td>
</tr>
</tbody>
</table>

Table 1: Comparison of cash flow simulation embedded options and lattice type options

5. **Lattice method valuation with cash flow simulation embedded options**

Similarly to Monte Carlo simulation, lattice approach changes the mathematical valuation problem into a discretized numerical approach. Lattice models are accurate, robust, and intuitively appealing tools for valuing financial and real options (Hahn 2005, p. 6). Lattices are much more easily explained to and accepted by management because the methodology is much simpler to understand (Mun 2006). This is valuable especially with sequential and parallel compound options, which is often the case in real applications (Trigeorgis 1996, Copeland & Antikarov 2001). They allow valuation of American options as well as barrier options. Typically, lattice methods are of binomial (two states) or trinomial (three states) type, but there are also quadranimal lattices, e.g. for jump-
diffusion process, and pentanomial lattices for rainbow options with two combined and correlated underlying assets (Mun 2006).

Lattice valuation models are based on a simple representation of the evolution of the underlying asset value. The two main ideas with lattice approaches are 1) the modeling of the continuous process with a discrete random walk and 2) the assumption of risk-neutral pricing. In the continuous limit, a lattice with an infinite number of time steps to expiration represents a continuous risk-neutral evolution of the asset value. In a lattice method, a tree of possible values of underlying asset prices and their probabilities, given an initial asset price, is built. This tree determines the possible asset prices and the associated probabilities of these asset prices being realized. In other words, a lattice determines the asset prices and probabilities in a state space during each time period over the life-time and at the expiry of the security. The possible values of the security and therefore also the payoff of the option at expiry can then be calculated, and finally, by working back down the tree, the security can be valued. (Wilmott et al., 1995, 180-182).

Several authors have suggested different variations of applying Monte Carlo simulation on cash flow calculation to estimate the underlying asset value and volatility. The existing cash flow simulation based volatility estimation methods are the logarithmic present value approach of Copeland & Antikarov (2001), conditional logarithmic present value approach of Brandão, Dyer & Hahn (2005), two-level simulation and least-squares regression methods of Godinho (2006). All these methods are based on the same basic idea. Monte Carlo simulation on cash flows consolidates a high-dimensional stochastic process of several correlated variables into a low-dimension (univariate) gBm summary process. The volatility parameter $\sigma$ of the underlying asset is estimated by calculating the standard deviation of the simulated probability distribution for the rate of return.

The estimations used in the cash flow simulation may have highly subjective estimates. As such, common theoretical justification of the real options valuation according to the 1) replicating portfolio with comparables or 2) risk-neutral valuation does not hold anymore. Therefore, the results should not be considered as precise answers but rather as the best available estimations of the investment value. Also, the difference between ambiguity and volatility, e.g. first order and second order uncertainty in terms of Knight (1921), should be recognized. Use of a static univariate description of the stochastic process is also questionable as the volatility usually tends to decrease over the time.

One alternative to mitigate the previously mentioned problems is to use a recombining trinomial tree for valuing real options with changing volatility. The trinomial tree can be constructed by simultaneously choosing a parameterization that sets a judicious state space while having sensible transition probabilities between the nodes. The volatility changes are modeled with the changing transition probabilities while the state space of the trinomial tree is regular and has a fixed number of time and underlying asset price levels. Such trinomial lattice can be extended to follow a displaced diffusion process with changing volatility, allowing taking into account the level of the underlying asset price and also negative underlying asset values. The parameterization of such trinomial lattice based on cash flow simulation is not more difficult than using Copeland & Antikarov (2001) approach. (Haahtela 2010)
Similarly to Godinho (2006), Haahetla (2010) suggests estimating the continuation value with ordinary least squares regression. However, in case of cash flow simulation embedded options, the response in the forward-looking estimator as a result of a change in an explaining variable may not be anymore linear, and therefore also 1) higher order terms of cash flow calculation components $X_{k,t}$, 2) non-linear responses and 3) need for piecewise regression should be considered to guarantee the correct response. Therefore, a possible forward-looking estimator may be a combination of earlier years’ cash flows $CF_{1}...CF_{t-1}$, and cash flow calculation components $X_{k,t}$ of the present moment $t$, which are then used as explanatory variables according to Equation (1) as follows:

$$\hat{S}_t = a_t + CF_t + \cdots CF_{t-1} + b_{t,t}X_{t,t} + \cdots + b_{k,t}X_{k,t}$$

+ possible higher order terms of $X_{k,t}$
+ non-linear responses and piecewise regression

When the present value and its standard deviation according to the previous estimator is calculated at certain time points, it is possible to compute the average volatility (or displaced diffusion volatility) for each time period. Starting from the beginning of the process, volatility $\sigma_i$ for each time period can be calculated according to Equation (2):

$$\sigma_i = \sqrt{\frac{\ln \left( \frac{\text{std}(S)_i}{S_0 e^{rT}} \right)^2 + 1 - \sum_{t=0}^{t-1} \sigma^2 t_i}{t_i}}$$

Where $S_0$ is the underlying asset value in the beginning, std($S$)$_i$ is the standard deviation in the given moment, $r$ is risk-free interest rate, $t$ is time from the beginning, and $t_i$ is the length of the time period.

As a result, cash flow simulation with embedded options is calculated according to the trace-driven approach. Then, the resulting distribution (or distributions for different time periods) is used as an empirical displaced (shifted) log-normal distribution, from which the time-dependent (and displaced) volatilities are calculated. Finally, this parameterization is used to construct a recombining trinomial tree, which is used similarly to any other lattice method for valuing common “lattice options”. The advantage is that such options that are easier to model in a cash flow simulation are also calculated there, and those options that are suitable for lattice valuation are valued with a lattice method.

6. Conclusions

This paper presented a method for valuing certain types of real options as cash flow simulation embedded options, whose value is based on choosing the optimal decision in each time step during a single cash flow calculation simulation run. This paper also showed how to use cash flow simulation with embedded options as the underlying asset in constructing a recombining trinomial lattice with changing volatility, which can be used for valuing common lattice type options and their interactions. Thus, advantages of cash flow simulation embedded and lattice type options are achieved simultaneously.
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