Real asset appraisal based on a multi-expert approach using the pay-off method for real option valuation

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Abstract

The pay-off method is a novel method that is designed for the valuation and analysis of real assets. The method is based on cash-flow scenarios that are used to create a pay-off distribution for a project, from which the real option value is calculated. The reliability of the used cash-flow scenarios influences the reliability of the analysis results. Evaluation of future cash-flows is a complicated issue, as it is often impossible to identify or specify in sufficient detail any relevant processes that underlie the real asset cash-flows. This means that often the best information available on the future of the cash-flows that the asset generates is in the heads of project managers / experts. Managers’ opinions about the future may not be in concert thus calling for consensus building. In this paper we show how consensus on project dynamics can be modelled and how the resulting cash-flows can be used as a basis for real an analysis of project profitability with the pay-off method.

Keywords: consensus in group decisions, fuzzy preference relations, pay-off method, real option valuation, fuzzy logic

1. Introduction

In (Collan, et al., 2009b) a new “fuzzy pay-off method” for real option valuation is introduced that uses a fuzzy project pay-off distribution for the calculation of the real option value for the project. The method relies on information / knowledge provided by (a team of) managers, who are responsible for the construction of cash-flow scenarios that are used as basis for the fuzzy pay-off distribution for the analysed project. The project cash-flow scenarios are designed by trusting the
involved managers’ past experience and they are constructed, based on the values of the variables that most contribute to the cash-flows and the future market outlook (of these variables). The effectiveness of the real option analysis and the reliability of the results depend on the way in which the information / knowledge of the managers is represented and combined during the process of construction of the cash-flow scenarios.

Even though it’s realistic to assume that time series of data related to the variables involved are available, nevertheless it’s a matter of fact that the complexity of the relationships between the variables, the difficulty of providing numerical forecasts in the long period, and the influence of “black swans” (Taleb, 2007) generate a very large uncertainty that cannot be managed using traditional mathematical modelling techniques. Linguistic variables, word-based computing algorithms and approximate reasoning models are more suitable (Zadeh, 1975) to address the problem of the representation and manipulation of managers’ knowledge, and the modelling of consensual dynamics aiming to reach the agreement on linguistically expressed opinions (opinions on cash-flow components numerically represented by fuzzy numbers). Finding a consensus between experts on how the future cash-flow scenarios look is important, because if the scenarios are not accepted by (all) the involved managers the credibility of any obtained results may suffer.

The process of finding commonly accepted cash-flow scenarios is often facilitated by a moderator who supervises work sessions and most of all the interactions between the managers, who via exchange of information, rational arguments, bargaining, etc. tries to persuade the individuals to change their opinions. The moderator works in favour of a common acceptance of the reached result. One of the key issues the moderator has to address (maybe in co-operation with the managers) is to settle the initial set of linguistic terms and the granularity of the corresponding set of fuzzy numbers (triangular). After having agreed on these, the cash-flow of the first scenario (pessimistic) is designed, and the managers present their opinions (fuzzy preferences). When the opinions have been definitely shaped, the consensus reaching process is activated. The same process is carried out for the other, in the triangular case, two scenarios; optimistic and “most likely”.

In this paper, we present a blueprint for a decision support system (DSS) for supporting the work of managers involved in the estimation of profitability of investments based on cash-flow scenarios and performing real-options- and profitability analysis based on the scenarios. The decision support
system supports both the finding of a common “vision” for the created cash-flow scenarios and the actual profitability analysis.

The first core component of the DSS is the soft consensus model which was proposed in (Kacprzyk and Fedrizzi, 1988; 1989) in the standard framework of numerical fuzzy preferences and expanded in (Fedrizzi, et al., 1988; Kacprzyk, et al., 1992; Kacprzyk, et al., 1997). Then, the model has been extended to a dynamical context (Fedrizzi, et al., 1999; 2007) that has been adopted in this paper involving linguistically expressed opinions according to the approach developed in (Herrera-Viedma, et al., 2005).

The second core component of the DSS is the pay-off method for real option valuation that is able to use the assessed cash-flow scenarios directly in creating a pay-off distribution that is actually a fuzzy net present value (NPV) for the project. From the pay-off distribution the project real option value and single number (crisp) NPV can be easily calculated for analysis purposes. The pay-off method is usable with different kinds of real investments (Collan, 2010 ; 2010 (submitted); Collan, et al., 2009a; Collan and Heikillä, 2010 (accepted)) and is an intuitively understandable method for profitability analysis and real option valuation.

This is the first paper to propose a combination of a consensus model with the pay-off method; similar issues discussed together with real option valuation (other models) can be found, e.g., in (Driouchi, et al., 2009).

In the next section we present the modelling of the consensual dynamics that result in cash-flow scenarios for a project under analysis, followed by a presentation of the pay-off method for real option valuation that is used to perform real options and profitability analysis based on the created cash-flow scenarios.

2. Modelling the consensual dynamics
One of the first issues to address when modelling the consensus dynamics is how to settle the linguistic term set $L$, i.e. the set of the linguistic values used by the managers to estimate the cash flow values. As pointed out in (Mata, et al., 2009) the granularity of $L$ “should be small enough so as not to impose useless precision levels on the users, but large enough to allow a discrimination of the assessments in a limited number of degrees”. The semantics of $L$ in our paper is given by triangular fuzzy numbers and we assume that all the managers agree on the use of the same linguistic term set, even though they may have different levels of knowledge.

A consensus reaching process is an iterative process in which managers have to express their opinions on the set of linguistic terms and modify them according to the advises given by a moderator who is in charge of the driving of the agreement towards a consensus threshold fixed in advance. We assume that each manager is represented by a pair of connected nodes, a primary node (dynamic) and a secondary node (static). The $n$ primary nodes form a fully connected sub network and each of them encodes the individual opinion (fuzzy preference relation) of a single manager. The $n$ secondary nodes, on the other hand, encode the individual opinions originally declared by the managers, denoted $s_i = [s_i \in [0, 1]]$, and each of them is connected only with the associated primary node.

The consensual dynamics could be represented according to the following steps (Fedrizzi, et al., 2007). The iterative process of opinion transformation corresponds to the gradient dynamics of a cost function $W$, depending on both the present and the original network configurations. The value of $W$ combines a measure $V$ of the overall disagreement in the present network configuration and a measure $U$ of the overall change from the original network configuration.

The various interactions involving node $i$ are mediated by interaction coefficients whose role is to quantify the strength of the interaction. The diffusive interaction between primary nodes $i$ and $j$ is mediated by the interaction coefficient $v_{ij} \in (0, 1)$, whereas the inertial interaction between primary node $i$ and the associated secondary node is mediated by the interaction coefficient $v_{si} \in (0, 1)$. It turns out that the values of these interaction coefficients are given by the derivative $f'$ of the scaling function.

The diffusive component of the network dynamics results from the consensual interaction between each node $x_i$ and the remaining $n-1$ nodes $x_{j\neq i}$ in the network. The aggregated effect of
these $n-1$ interactions can be represented as a single consensual interaction between node $x_i$ and a virtual node $\bar{x}_i$ containing a particular weighted average of the remaining opinion values.

The interaction coefficient $v_i \in (0,1)$ of this aggregated consensual interaction controls the extent to which manager $i$ is influenced by the remaining managers in the group. In our soft consensus model the value $v_i$, as well as the weighting coefficients $v_j \in (0,1)$ in the definition of $\bar{x}_i$ as given below, depend non-linearly on the standard euclidean distance between the opinions $x_i$ and $x_j$,

$$v_j = f''((x_i - x_j)^2)$$  \hspace{2cm} (1)

$$v_i = \sum_{j \neq i} v_j / (n-1)$$  \hspace{2cm} (2)

and the average preference $\bar{x}_i$ is given by

$$\bar{x}_i = \frac{\sum_{j \neq i} v_j x_j}{\sum_{j \neq i} v_j}$$  \hspace{2cm} (3)

In formula (1), $f''(x) \in (0,1)$ is the decreasing sigmoid function which plays a crucial role in the network dynamics and is defined as $f''(x) = 1 / \left(1 + e^{\beta(x-\alpha)}\right) = \sigma(x)$, where $f(x)$ is a scaling function defined as $f(x) = -\frac{1}{\beta} \ln \left(1 + e^{-\beta(x-\alpha)}\right)$.

The interaction coefficient $u_i \in (0,1)$ of this inertial interaction controls the extent to which the manager $i$ resists to opinion changes due to the collective consensual trend. In analogy with the diffusion coefficients, the value $u_i$ in our soft consensus model depends non-linearly on the standard Euclidean distance between the opinions $x_i$ and $s_i$,

$$u_i = f'((x_i - s_i)^2)$$  \hspace{2cm} (4)

where $f'(x)$ is the sigmoid function mentioned earlier.

Given the characteristics of $f'$, the inertial component of the network dynamics enforces a strong memory of the original opinions while the opinion changes are small, but essentially turns-off the inertial memory, when the opinion changes become large. In this way the dynamics is able to endogenously discard opinion outliers.

The individual disagreement cost $V(i)$ is given by
\[ V(i, j) = f((x_i - x_j)^2) \]  \hspace{1cm} (5) \\
\[ V(i) = \sum_{j \neq i} V(i, j) / (n-1) \]  \hspace{1cm} (6)

and the individual opinion changing cost \( U(i) \) is

\[ U(i) = f((x_i - s_i)^2) \]  \hspace{1cm} (7)

Summing over the various managers we obtain the collective disagreement cost \( V \) and inertial cost \( U \),

\[ V = \frac{1}{4} \sum_i V(i) \]  \hspace{1cm} (8) \\
\[ U = \frac{1}{2} \sum_i U(i) \]  \hspace{1cm} (9)

with conventional multiplicative factors of \( 1/4 \) and \( 1/2 \).

The full cost function \( W \) is then \( W = (1 - \lambda)V + \lambda U \) with \( 0 \leq \lambda \leq 1 \).

The consensual network dynamics, which can be regarded as an unsupervised learning algorithm, acts on the individual opinion variables \( x_i \) through the iterative process

\[ x_i \rightarrow x_i' = x_i - \varepsilon \frac{\partial W}{\partial x_i} \]  \hspace{1cm} (10)

We can analyse the effect of the two dynamical components \( V \) and \( U \) separately. The disagreement cost \( V \) induces a non-linear process of diffusion based on the gradient term

\[ \frac{\partial V}{\partial x_i} = v_i (x_i - \bar{x}_i) \]  \hspace{1cm} (11)

where the diffusion coefficients \( v_{ij} \) and \( v_i \) were defined in (1), (2) and the average preference \( \bar{x}_i \) was defined in (3).

As a result, the iterative step of the non-linear diffusion mechanism corresponds to a convex combination (with sufficiently small \( \varepsilon \) ) between the opinion value \( x_i \) and the weighted average \( \bar{x}_i \) of the remaining preference values \( x_j \),

\[ x_i' = (1 - \varepsilon v_i) x_i + \varepsilon v_i \bar{x}_i \]  \hspace{1cm} (12)
The inertial cost $U$, on the other hand, leads to a non-linear mechanism which opposes changes from the original opinions $x_i$, by means of the gradient term

$$\frac{\partial U}{\partial x_i} = u_i (x_i - s_i)$$ (13)

where the inertial coefficient $u_i$ was defined in (4).

The non-linearity of the opinion changing aversion mechanism is encoded in the inertial coefficient $u_i$ which modulates the convex combination between the opinion value $x_i$ and the original opinion value $s_i$.

$$x_i' = (1 - \varepsilon u_i) x_i + \varepsilon u_i s_i$$ (14)

The full dynamics associated with the cost function $W = (V + U) / 2$ acts iteratively on each expert $i$ through convex combinations of the opinion value $x_i$, the average opinion value $\bar{x}_i$, and the original opinion value $s_i$.

$$x_i' = (1 - \varepsilon (v_i + u_i)) x_i + \varepsilon v_i \bar{x}_i + \varepsilon u_i s_i$$ (15)

Accordingly, the manager $i$ is in dynamical equilibrium, in the sense that $x_i' = x_i$, if the following stability equation holds,

$$x_i = (v_i \bar{x}_i + u_i s_i) / (v_i + u_i)$$ (16)

that is, if the present opinion value $x_i$ coincides with an appropriate weighted average of the original opinion $s_i$ and the average opinion value $\bar{x}_i$.

Result from the “consensus engine” is a desired number of cash-flow scenarios that will account for the “optimistic” and the “pessimistic” views of the future performance of the investment under analysis and will hence include a managerial assessment of the investment risk, or more precisely the estimation uncertainty involved in the cash-flows. These cash-flow scenarios will then be used in the profitability and real options analysis performed with the pay-off method.

3. The pay-off method for real option valuation

The pay-off method (POM) is a real options-thinking based profitability analysis method that is suitable for cases, where input information is in the form of cash-flow scenarios and when cash-flow estimation accuracy is not necessarily very high. The method is presented in detail in (Collan,
et al., 2009b). The POM is usable also in the valuation of compound real options (Collan, et al., 2009a) and other compound real assets that for which cash-flow scenarios can be built. These characteristics make the pay-off method a good fit with the profitability analysis of many different types of investments.

The pay-off method calculates a real option pay-off value for a project from the project pay-off distribution (i.e. project NPV distribution) that can be constructed from the project cash-flow scenarios. The created NPV distribution is treated as a fuzzy number. To calculate the real option value by using the pay-off method the area weighted mean of the fuzzy number (distribution) is calculated, such that distribution values below zero are counted as zero and multiplied by the ratio of area over the positive values over the whole area of the distribution.

Figure 2. An example triangular NPV distribution from where the real option value can be calculated. Values between a-α and 0 are counted as zero.

Definition 1.1. The pay-off method calculates the real option value from the project NPV distribution as follows:

\[ ROV = \frac{\int_0^\infty A(x) \, dx}{\int_{-\infty}^\infty A(x) \, dx} \times E(A_+) \]

Where A stands for the (fuzzy) NPV, E(A+) denotes the (fuzzy) mean value of the positive side of the NPV and \(\int_{-\infty}^\infty A(x) \, dx\) computes the area below the whole NPV distribution A, and \(\int_0^\infty A(x) \, dx\) computes the area below the positive part of A.

Definition 1.2. The possibilistic mean for different “cases” of the triangular distribution
For the purposes of profitability analysis with the pay-off method, we suggest that triangular or trapezoidal pay-off distributions are used, because of their simplicity. Derivation of the possibilistic mean for the positive side of different types of distributions in this context is presented in (Collan, et al., 2009b) the definition of the possibilistic mean is presented in (Carlsson and Fullér, 2001). Figure 2 shows an example of a triangular pay-off (NPV) distribution from where the real option value can be calculated with the pay-off method.

The pay-off method can be used in calculating the real option value for any project for which a pay-off distribution can be explicitly defined; most of the time good enough results can be reached with triangular and trapezoidal distributions.

4. Summary and conclusions

Combining a “consensus engine” by introducing consensual dynamics for the creation of cash-flows for profitability and real-option valuation is a new approach. To the best of our knowledge such combination has not been previously presented, although the use of cash-flow scenarios together with real option valuation is common practice in many industrial companies. This may be the case, because the traditional real option methods are not based on cash-flow scenarios, but only simulation based methods and the novel pay-off method for real option valuation are able to use them. Building a consensus on the cash-flow scenarios that depict the best available managers’ opinions on the future of the project under analysis is a step forward in obtaining a reliable presentation of the future as consensus on this background means a “picture of the future” that is commonly understood and accepted by the participating managers.

Such a common understanding is likely to provide a higher value from the obtained real option valuation and profitability analysis (NPV) results from the analysis, as there is no conflict about the input into the method; none of the involved expert managers is claiming “garbage in – garbage out” which is actually a common problem in any forward looking analysis. When working under consensus the results will also gain the agreement aura and therefore decision-making based on the
results, perhaps even by the same managers, is facilitated. Claiming definitely better analysis results would be premature at this time however, it is likely that combining consensus creation and the pay-off method together may indeed lead to better outcomes from the analysis.

Our aim for future research is to address the following issues: (i) provide a methodological framework for making the linguistic information uniform, assuming that the managers have different expertise and thus their linguistic term sets may exhibit different granularity; (ii) explore the conditions under which the membership functions of fuzzy numbers represent effectively the knowledge of the managers according to the canonical measurement paradigms; (iii) carrying out extensive simulations mostly based on the experimental results obtained in (Fedrizzi, et al., 2010)

5. References


