An Approximate Reasoning Approach to Rank the Results of Fuzzy Queries

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Abstract
In this paper we suggest the use of a context-dependent fuzzy aggregation method to rank the results of fuzzy queries over fuzzy ontologies. In our approach the fuzzy aggregation rules are provided by the experts, the coefficients of the consequence part of the rules are derived from the linguistic values used in the conditional part of the rules and the rank of a search result is determined by the Takagi-Sugeno fuzzy reasoning scheme.

1 Introduction
The emerging concept of "knowledge mobilisation" may be defined as putting available knowledge into active service to benefit society. It may be considered as an enhancement of knowledge management methods and technologies and it can represent the next step in implementing new forms of modern ICT in management processes.
Gruber (1993) defined ontology as 'an explicit specification of a conceptualization'. Parry (2004, 2006) introduced a definition for fuzzy ontology that proposes to represent it as a fuzzy relation. A fuzzy ontology structure can
be defined as consisting of concepts, of fuzzy relations among concepts, of a concept hierarchy or taxonomy, of non-hierarchical associative relationships and of a set of ontology axioms, expressed in an appropriate logical language (for a reference see Sanchez and Yamano, 2006). Fuzzy ontologies have shown their usefulness in many fields as, for example, information retrieval (Parry, 2006) and summarization (Lee et al, 2005). Carlsson et al (2010) developed a fuzzy ontology as the core of knowledge mobilisation. They have showed how a fuzzy keyword ontology can be applied for imprecise queries by using fuzzy information granulation.

In this work we present a fuzzy case-based reasoning approach to refine the results of fuzzy queries over fuzzy ontologies. It consists of two stages: The first stage is to retrieve the results from a fuzzy ontology by using a fuzzy (keyword) search. It can be done, for example, by using the FQUERY for Access system designed by Kacprzyk and Zadrozny (2001); Zadrozny and Kacprzyk (2009). In the second stage we rank the results of fuzzy queries by using fuzzy aggregation rules and a Takagi-Sugeno fuzzy reasoning scheme.

2 Ranking the results of fuzzy queries by fuzzy aggregation rules

Without loss of generality we will suppose that we have (fuzzy) three keywords and a fuzzy query gives us an output with degrees of similarity (or match) \( x_1 \), \( x_2 \) and \( x_3 \), usually in percentage terms. We should combine these degrees of similarities into an overall degree of similarity. Then we can rank the results of a fuzzy query according to their overall degree of match. One can use a fixed aggregation operator to find the overall match to the query. In many situations, however, this simple aggregation operator may not bring us the correct answer. More often than not we should use a context-dependent aggregation process instead. This context-dependent aggregation can be implemented by using a set of fuzzy if-then rules supplied by experts. Suppose the experts can provide us with some fuzzy aggregation rules of type,

\[
\text{if } x_1 \text{ is } A_1 \text{ and } x_2 \text{ is } B_1 \text{ and } x_3 \text{ is } C_1 \text{ then } o_1 = a_1 x_1 + b_1 x_2 + c_1 x_3
\]

where \( A_1 \), \( B_1 \) and \( C_1 \) are fuzzy numbers, and \( a_1 \), \( b_1 \) and \( c_1 \) are crisp numbers depending on \( A_1 \), \( B_1 \) and \( C_1 \). For example,

\[
\text{if } x_1 \text{ is SMALL and } x_2 \text{ is SMALL and } x_3 \text{ is MEDIUM then } o_1 = 1/5 x_1 + 1/5 x_2 + 3/5 x_3
\]

which should be interpreted as, if the degree of match to the first keyword, \( x_1 \), is small and the degree of match to the second keyword \( x_2 \) is small and
the degree of match to the third keyword $x_3$ is medium then the aggregated value should be $o_1 = 1/5x_1 + 1/5x_2 + 3/5x_3$.

Suppose we have the following fuzzy aggregation rules,

if $x_1$ is $A_i$ and $x_2$ is $B_i$ and $x_3$ is $C_i$ then $o_i = a_i x_1 + b_i x_2 + c_i x_3$

\[ \cdots \]

if $x_1$ is $A_m$ and $x_2$ is $B_m$ and $x_3$ is $C_m$ then $o_m = a_m x_1 + b_m x_2 + c_m x_3$

where $A_i$, $B_i$ and $C_i$ are fuzzy numbers, $i = 1, \ldots, m$. The procedure for obtaining the fuzzy output of such a knowledge base consists of the following three steps:

- Find the firing level of each of the rules.
- Find the output of each of the rules.
- Aggregate the individual rule outputs to obtain the overall system output.

Sugeno and Takagi use the following architecture Takagi and Sugeno (1985). The firing levels of the rules are computed by

$$ \alpha_i = t\text{-norm}(A_i(s_1), B_i(s_2), C_i(s_3)) $$

where $t\text{-norm}$ is a triangular norm, and $s_1$, $s_2$ and $s_3$ are the crisp observations. The individual rule outputs are computed by,

$$ o_i = a_i s_1 + b_i s_2 + c_i s_3, \ i = 1, \ldots, m, $$

and then the crisp output is obtained by,

$$ o = \frac{\alpha_1 o_1 + \cdots + \alpha_m o_m}{\alpha_1 + \cdots + \alpha_m} \quad (1) $$

Suppose further that we can assign some context-dependent weights to the fuzzy numbers in the conditional part of the rules, denoted by $w(A_i)$, $w(B_i)$ and $w(C_i)$ and let us derive the coefficients of the consequence part of the rules from the linguistic values of the conditional part as

$$ o_i = \frac{w(A_i) \times s_1 + w(B_i) \times s_2 + w(C_i) \times s_3}{w(A_i) + w(B_i) + w(C_i)} $$

It is clear that $0 \leq o_i \leq 1$ for all $i = 1, \ldots, m$.

The tricky part here is the determination of the weights from the logical combination of the keywords. Suppose we have three fuzzy keywords, $K_1$, $K_2$ and $K_3$, and we search for $f(K_1, K_2, K_3)$, where $f$ denotes any fuzzy logic-based function that may connect the keywords. For example, if $f(K_1, K_2, K_3) = K_1 \cap K_2 \cap K_3$ then the weights of the linguistic values can be obtained from their natural ordering.
3 An example

Suppose we have three fuzzy keywords,

\[ K_1 = \{\text{Very High Brightness}\} \]
\[ K_2 = \{\text{High Consistency}\} \]
\[ K_3 = \{\text{Low Air content}\} \]

of equal importance and we search for

\{Very High Brightness\} \textbf{and} \{High Consistency\} \textbf{and} \{Low Air content\}

that is, we use a simple fuzzy connective ’and’. Suppose further that we have five linguistic values \{VERY SMALL, SMALL, MEDIUM, BIG, VERY BIG\} for fuzzy numbers in the conditional part of the rules. From the definition of logical ’and’ operation it follows that we can assign the following weights to linguistic values

\[
w(\text{VERY SMALL}) = 1, \\
w(\text{SMALL}) = 2, \\
w(\text{MEDIUM}) = 3, \\
w(\text{BIG}) = 4, \\
w(\text{VERY BIG}) = 5,
\]

and suppose the experts can provide us with the following three fuzzy aggregation rules,

\[
\begin{align*}
\text{if} & \quad x_1 \text{ is SMALL and } x_2 \text{ is SMALL and } x_3 \text{ is MEDIUM} \\
\text{then} & \quad \alpha_1 = 1/5x_1 + 1/5x_2 + 3/5x_3 \\
\text{if} & \quad x_1 \text{ is BIG and } x_2 \text{ is SMALL and } x_3 \text{ is MEDIUM} \\
\text{then} & \quad \alpha_2 = 4/9x_1 + 2/9x_2 + 3/9x_3 \\
\text{if} & \quad x_1 \text{ is SMALL and } x_2 \text{ is MEDIUM and } x_3 \text{ is MEDIUM} \\
\text{then} & \quad \alpha_3 = 2/8x_1 + 3/8x_2 + 3/8x_3
\end{align*}
\]

Then a search result with similarity degrees \((s_1, s_2, s_3)\) is propagated through this fuzzy inference system and its overall degree of similarity is computed by (1). For example, let the universe of discourse be the unit interval and let we define the following semantics (membership functions) for linguistic labels,

\[
\begin{align*}
\text{VERY SMALL}(u) &= (1 - u)^2, \\
\text{SMALL}(u) &= 1 - u, \\
\text{MEDIUM}(u) &= \max\{0, (1 - 2|1/2 - u|)\}, \\
\text{BIG}(u) &= u \\
\text{VERY BIG}(u) &= u^2
\end{align*}
\]
and let us have a result with degrees of similarity

\[(s_1, s_2, s_3) = (0.3, 0.4, 0.8).\]

Then the firing levels of the rules are computed by

\[
\alpha_1 = \min\{\text{SMALL}(0.3), \text{SMALL}(0.4), \text{MEDIUM}(0.8)\} \\
= \min\{0.7, 0.6, 0.4\} = 0.4
\]

\[
\alpha_2 = \min\{\text{BIG}(0.3), \text{SMALL}(0.4), \text{MEDIUM}(0.8)\} \\
= \min\{0.3, 0.6, 0.4\} = 0.3
\]

\[
\alpha_3 = \min\{\text{SMALL}(0.3), \text{MEDIUM}(0.4), \text{MEDIUM}(0.8)\} \\
= \min\{0.7, 0.8, 0.4\} = 0.4
\]

and the individual rule outputs are,

\[
\sigma_1 = \frac{1}{5} \times 0.3 + \frac{1}{5} \times 0.4 + \frac{3}{5} \times 0.8 = 0.420,
\]

\[
\sigma_2 = \frac{4}{9} \times 0.3 + \frac{2}{9} \times 0.4 + \frac{3}{9} \times 0.8 = 0.488,
\]

\[
\sigma_3 = \frac{2}{8} \times 0.3 + \frac{3}{8} \times 0.4 + \frac{3}{8} \times 0.8 = 0.525,
\]

Then the overall score (degree of match) is

\[
o = \frac{\alpha_1 \sigma_1 + \alpha_2 \sigma_2 + \alpha_3 \sigma_3}{\alpha_1 + \alpha_2 + \alpha_3} = \frac{0.4 \times 0.420 + 0.3 \times 0.488 + 0.4 \times 0.525}{1.1} = \frac{0.1680 + 0.3144 + 0.21}{1.1} = 0.629.
\]

The principal reason for this relatively big overall score, 0.629, (from individual degrees of similarity, 0.3, 0.4 and 0.8) is that the weight of 0.8 is relatively big in the conditional part of the fuzzy aggregation rules.

4 Summary

We have suggested the use of fuzzy aggregation rules and a Takagi-Sugeno fuzzy reasoning scheme to rank the results of fuzzy queries over fuzzy ontologies. The fuzzy aggregation rules have been provided by the experts. The coefficients of the consequence part of the rules have been derived from the linguistic values used in the conditional part of the rules.
References


