Fuzzy Subsethood Measure Applied in Weights Setting of Decision Makers

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Abstract

In order to avoid deviations caused by autocratic and subjective attitudes in decision making process, bringing decision makers (DMs) together to obtain group consensus is one of the best choices for achieving accurate performance. Decision weighting of each decision maker (DM) is set equally in traditional performance evaluation model. Actually it is more suitable to be set by means of mutual interactions within decision group.

This paper summarized some methodologies, Similarity Aggregation Method (SAM), Optimal Aggregation Method (OAM), Least Squares Distance Method (LSDM), Defuzzification-based Least Squares Method (DLSM), to define the weights setting of DMs. Also a new developed method called Subsethood Aggregation Method (SbAM) is provided for the same purpose, which can be used for getting more objective group consensus.

Keywords: Group Decision Making, Fuzzy Subsethood Measure, Decision Weighs Setting, Group Data Aggregation.
1. Introduction

With reference to techniques of group consensus, Forman and Peniwati (1998) categorized them into two groups. One is Aggregation of Individual Judgments (AIJ), the other is Aggregation of Individual Priorities (AIP). AIJ related to qualitative techniques, while AIP related to quantitative techniques. The topics discussed below are just scopes of AIP.

The best advantage from AIP embedded is that AIP offers an objective result through mathematical calculation. AIP puts all measurements of DMs into an objective consideration, so that it is possible to keep off deviations caused by autocratic in decision making process. The calculation procedures of AIP are varied on methods applied, some methods with complicated algorithm usually need supporting software assisting to problem solved.

Decision weighting of each DM is set equally in traditional performance evaluation model, either by means of arithmetic average or by means of geometric average. Once bias has been caused by outliers, the aggregated final conclusion would be far apart from actual group consensus. Some existed methods for weights setting are discussed, which include SAM, OAM, LSDM and DLSM.

SAM provided by Hsu and Chen (1996) is based on similarity function with which to measure the agreement degree between each paired measurements. OAM provided by Lee (2002) is based on degree of similarity and distance of two fuzzy measurements. OAM solution provided by Lee is nonlinear problem orientation, so that it is hard to be solved. LSDM and DLSM provided by Wang and Parkan (2006) is easier than OAM.

SbAM, a new developed method based on fuzzy subsethood measure, is provided for getting more objective group consensus. If the measurement range of one DM is “wider”, then the degree of subsethood is “smaller”, and vise-versa. SbAM is prior to SAM while applying in group data aggregation.

2. Literature Review on Weights Setting of Decision Makers

1. Similarity Aggregation Method (SAM)

Hsu and Chen (1996) proposed SAM as the weighted average method to define corresponding weighting for each DM. The major considerations for SAM are based on both relative degree of importance role in decision group and agreement degree of each fuzzy measurement within decision group. Basic assumption of SAM is that the similarity degree of two fuzzy measurements is defined by fuzzy similarity measure. The more one DM to have degree of similarity to the others, the larger he/she will get corresponding decision weighting.

Degree of similarity can be explained by geometric meaning, the larger the overlapped area is, the more degree of similarity will be. For example the overlapped area illustrated in figure 1 is larger than that shown in figure 2, so that degree of similarity in figure 1 has larger
value than that listed in figure 2. If the measurements of two decision makers are overlapped completely, then degree of similarity is equal to 1. While there is no interaction between two measurements, then degree of similarity is equal to 0.

![Fig 1: Overlapped Area and Degree of Similarity (A)](image1)

![Fig 2: Overlapped Area and Degree of Similarity (B)](image2)

2. Optimal Aggregation Method (OAM):

Lee (2002) proposed an objective methodology to set up weighting of each DM. OAM defined decision weightings of DMs based on degree of similarity and distance of two fuzzy measurements. The optimal aggregation can be reached when get the minimum value for additions of non-similarity between aggregation of all measurements and each individual measurement. Figure 3 illustrate geometric demonstration diagram of three individual measurement came from expert E1, E2, E3, and results of OAM, the best final conclusion existed with the minimum “total distance” to individual measurement.

![Fig 3: OAM Geometric Demonstration Diagram](image3)

3. Least Squares Distance Method (LSDM)

Wang and Parkan (2006) bore the same conclusion as Lee(2002) had, both of them...
thought, group consensus should be made by means of aggregate individual fuzzy measurements suitably. Wang and Parkan thought that OAM methodology is nonlinear problem orientation, so that it is hard to be solved. LSDM is easier than OAM by means of least squares distance calculation. Basic assumption of LSDM is listed as followings:

Assumed two individual measurements can be expressed as $\tilde{R}_i = (r_{i1}, r_{i2}, r_{i3})$ and $\tilde{R}_j = (r_{j1}, r_{j2}, r_{j3})$, their corresponding weighting are $w_i$ and $w_j$. The weighted Euclidean distance can be defined as followings, $m = 3$ for T.F.N.:

$$d_{ij} = f(w_i, R_i, w_j, R_j) = \sqrt{\sum_{k=1}^{n}(w_i r_{ik} - w_j r_{jk})^2}$$

$$\text{Min } J = \sum_{i=1}^{s} \sum_{j=1}^{t} d_{ij}^2 = \sum_{i=1}^{s} \sum_{j=1}^{t} \left[ \sum_{k=1}^{n} (w_i r_{ik} - w_j r_{jk}) \right]^2$$

$$s, t, \sum_{i=1}^{n} w_i = 1, w_i \geq 0, i = 1, K, s, j = 1, K, n.$$

4. Defuzzification-based Least Squares Method (DLSM)

DLSM provided by Wang and Parkan (2006) can be assumed as the rough-cut method for LSDM. Let one of a list of fuzzy numbers as $\tilde{R}_i = (r_{i1}, r_{i2}, r_{i3})$ with its corresponding weights $w_i$, then the weighted defuzzification number, objective function, and constraints are defined as followings:

$$\bar{z}_i = \frac{1}{m} \sum_{k=1}^{m} w_i r_{ik} = w_i \left( \frac{1}{m} \sum_{k=1}^{m} r_{ik} \right) = w_i \bar{z}_i$$

$$\text{Min } J = \sum_{i=1}^{s} \sum_{j=1}^{t} (w_i z_i - w_j z_j)^2$$

$$s, t, \sum_{i=1}^{n} w_i = 1$$

$$w_i \geq 0, i = 1, ..., n.$$

3. Subsethood Aggregation Method (SbAM)

Contents of SbAM was derived from fuzzy subsethood measure defined by Kosko (1986), degree of likelihood measure created by Zwick et al (1987) and SAM methodology provided by Hsu and Chen (1996).

The basic concept of fuzzy subsethood measure is based on “area” of mutual interaction between two fuzzy numbers divided by “area” of individual fuzzy number. If the measurement of one DM is “wider”, then the degree of subsethood is “smaller”, and vise-versa. On the contrary, degree of similarity defined by Zwick et. al. ignored the difference between two separate measures. Procedures of SbAM are expressed as following:

1. Calculate degree of fuzzy subsethood measure for each paired measurements, where N
measurements can compose \((N - 1) \times N / 2\) pairs.

\[
S(A, B) = \deg \text{rec}(A \subseteq B) = \frac{\sum_{i \in A, j \in B} \mu_{A}(x) \cap \mu_{B}(x)}{\sum_{i \in A, j \in B} \mu_{A}(x)} = \frac{\mu(A \cap B)}{\mu(A)}
\]

2. Construct a \(n \times n\) agreement matrix (AM), each cell represents degree of fuzzy subsethood measure for each corresponding paired measurement \(S_{ij}\), where \(S_{ij} = S(I, J)\).

\[
AM = \begin{bmatrix}
S_{11} & S_{12} & \cdots & S_{1j} & \cdots & S_{1n} \\
S_{21} & S_{22} & \cdots & S_{2j} & \cdots & S_{2n} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
S_{i1} & S_{i2} & \cdots & S_{ij} & \cdots & S_{in} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
S_{n1} & S_{n2} & \cdots & S_{nj} & \cdots & S_{nn}
\end{bmatrix}
\]

3. Calculate average agreement \(A(E_{i})\) for each corresponding DM \(E_{i}, i = 1, 2, \ldots, n\).

\[
A(E_{i}) = \frac{1}{n-1} \sum_{j=1}^{n} S_{ij}, \quad j \neq i
\]

4. Calculate relative agreement degree \(RAD_{i}\) for each corresponding DM.

\[
RAD_{i} = \frac{A(E_{i})}{\sum_{i=1}^{n} A(E_{i})}, \quad i = 1, 2, \ldots, n
\]

5. Calculate consensus degree coefficient \(CDC_{i}\) for each corresponding DM, where \(\beta\) represents linear combination coefficient for \(W_{i}\) and \(CDC_{i}\). \(W_{i}\) represents importance factor in decision group, for example we set factor of manager as 0.3 and set that of engineer as 0.2. If all DM are equally important, then value of \(\beta\) would be set to zero.

\[
CDC_{i} = \beta \times w_{i} + (1 - \beta) \times RAD_{i}
\]

\[
0 \leq \beta \leq 1
\]

6. Aggregate all individual measurement product their corresponding \(CDC_{i}\), then objective group consensus can be got.

\[
\tilde{R} = \sum_{i=1}^{n} CDC_{i} \otimes \tilde{R}_{i}
\]

\(\tilde{R}\) : Aggregated fuzzy measurement

\(\tilde{R}_{i}\) : Fuzzy measurement of \(i^{th}\) DM, \(i = 1, 2, \ldots, n\)

4. Empirical Example for SbAM
Application procedures of SbAM are demonstrated with a simple empirical example, assumed there are 3 DM with their fuzzy measurement listed below.

\[ \tilde{R}_1 = (1,2,3) ; \quad \tilde{R}_2 = (1,3,5) ; \quad \tilde{R}_3 = (3,4,5) \]

1. Calculate degree of fuzzy subsethood measure for 6 paired measurements.

\[ s(\tilde{R}_1, \tilde{R}_2) = \frac{|\tilde{R}_1 \cap \tilde{R}_2|}{|\tilde{R}_1|} = 0.667 \quad s(\tilde{R}_1, \tilde{R}_3) = \frac{|\tilde{R}_1 \cap \tilde{R}_3|}{|\tilde{R}_1|} = 0.333 \]

\[ s(\tilde{R}_2, \tilde{R}_1) = \frac{|\tilde{R}_2 \cap \tilde{R}_1|}{|\tilde{R}_2|} = 0 \quad s(\tilde{R}_2, \tilde{R}_3) = \frac{|\tilde{R}_2 \cap \tilde{R}_3|}{|\tilde{R}_2|} = 0 \]

\[ s(\tilde{R}_3, \tilde{R}_1) = \frac{|\tilde{R}_3 \cap \tilde{R}_1|}{|\tilde{R}_3|} = 0.333 \quad s(\tilde{R}_3, \tilde{R}_2) = \frac{|\tilde{R}_3 \cap \tilde{R}_2|}{|\tilde{R}_3|} = 0.667 \]

2. Construct a 3x3 agreement matrix (AM).

\[ AM = \begin{bmatrix} 1 & 0.667 & 0 \\ 0.333 & 1 & 0.333 \\ 0 & 0.667 & 1 \end{bmatrix} \]

3. Calculate average agreement \( A(E_i) \) for each corresponding DM (skip \( S_{ii} = 1 \)).

\[ A(E_1) = \frac{0.667 + 0}{3 - 1} = \frac{0.333}{0.333} = 1 \]

\[ A(E_2) = 0.333 \]

\[ A(E_3) = 0.333 \]

4. Calculate relative agreement degree RAD, for each corresponding DM.

\[ RAD_1 = 0.333/(0.3 + 0.333 + 0.333) = 0.333 \]

\[ RAD_2 = 0.333/(0.3 + 0.333 + 0.333) = 0.333 \]

\[ RAD_3 = 0.333/(0.3 + 0.333 + 0.333) = 0.333 \]

5. Assumed \( \beta = 0 \) (all DM are equally important), CDC\(_i\) is equal to RAD\(_i\).

\[ CDC_1 = RAD_1, CDC_2 = RAD_2, CDC_3 = RAD_3 \]

6. Aggregate all individual measurement product their corresponding CDC\(_i\), (1.67,3,4.33) would be the value of group consensus.

\[ \tilde{R} = 0.333(\tilde{R}_1) + 0.333(\tilde{R}_2) + 0.333(\tilde{R}_3) = (1.67,3,4.33) \]

5. Comparisons of SAM vs. SbAM Approach

Methodologies for weights setting of DM under group decision making have been
discussed in previous sections. We provide three sets of test data, each one contains three measurements in forms of T.F.N. arranged by three experts in respective. Table 1 lists these test data, weights setting via various approaches, and aggregated fuzzy measurement. \( \tilde{R}_i \) represents individual fuzzy measurement, \( \tilde{s} \) represents aggregated fuzzy measurement, data in the same row corresponding to \( \tilde{R}_i \) represent Weighs Setting via assigned approach. Each decision maker’s weighting is set equally and all measurements are aggregated via arithmetic average in traditional performance evaluation model. Approach of arithmetic average neglects mutual interactions among DM, but the others include SAM, SbAM, LSDM, DLSM and OAM are all approaches of mutual interactions.

Both SAM and SbAM are based on degree of “superposition” for each paired measurements. If “span” of all measurements are equal, then same conclusions will be got for both SAM and SbAM. Test data set 1 listed in table 1 demonstrates the conclusion. According to test data set 2, if “span” of a certain measurements is varied, e.g. DM 2 revised its measurement from (2, 3, 4) to (1, 3, 5), and assumed the others were left unchanged. Based on test data set 2, we observed that results referred to SAM kept unchanged, while referred to SbAM, weighting of DM 2 has been reduced from 0.5 to 0.333 due to its measurement with wide “span” (less precise). It’s just fit to the practical conclusion “the wider the decision range is, the smaller the decision effect should be”.

<table>
<thead>
<tr>
<th>Methods Test Data</th>
<th>Arithmetic Average</th>
<th>SAM</th>
<th>SbAM</th>
<th>LSDM</th>
<th>DLSM</th>
<th>OAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{R}_i(1,2,3) )</td>
<td>0.333</td>
<td>0.250</td>
<td>0.250</td>
<td>0.450</td>
<td>0.461</td>
<td>0.323</td>
</tr>
<tr>
<td>( \tilde{R}_i(2,3,4) )</td>
<td>0.333</td>
<td>0.500</td>
<td>0.500</td>
<td>0.313</td>
<td>0.308</td>
<td>0.354</td>
</tr>
<tr>
<td>( \tilde{R}_i(3,4,5) )</td>
<td>0.333</td>
<td>0.250</td>
<td>0.250</td>
<td>0.237</td>
<td>0.231</td>
<td>0.323</td>
</tr>
<tr>
<td>( \tilde{s}(2,3,4) )</td>
<td>(2,3,4)</td>
<td>(2,3,4)</td>
<td>(1.8,2.8,3.8)</td>
<td>(1.8,2.8,3.8)</td>
<td>(2,3,4)</td>
<td></td>
</tr>
<tr>
<td>( \tilde{R}_i(1,2,3) )</td>
<td>0.333</td>
<td>0.250</td>
<td>0.333</td>
<td>0.468</td>
<td>0.461</td>
<td>0.325</td>
</tr>
<tr>
<td>( \tilde{R}_i(1,3,5) )</td>
<td>0.333</td>
<td>0.500</td>
<td>0.333</td>
<td>0.290</td>
<td>0.308</td>
<td>0.350</td>
</tr>
<tr>
<td>( \tilde{R}_i(3,4,5) )</td>
<td>0.333</td>
<td>0.250</td>
<td>0.333</td>
<td>0.242</td>
<td>0.231</td>
<td>0.325</td>
</tr>
<tr>
<td>( \tilde{s}(1.67,3,4.3) )</td>
<td>(1.5,2.5,3,5,3)</td>
<td>(1.67,3,4.3)</td>
<td>(1.5,2.8,4.1)</td>
<td>(1.5,2.8,4.1)</td>
<td>(1.6,3,4.4)</td>
<td></td>
</tr>
<tr>
<td>( \tilde{R}_i(1,2,3) )</td>
<td>0.333</td>
<td>0.50</td>
<td>0.600</td>
<td>0.537</td>
<td>0.541</td>
<td>0.342</td>
</tr>
<tr>
<td>( \tilde{R}_i(2,3,5) )</td>
<td>0.333</td>
<td>0.50</td>
<td>0.400</td>
<td>0.323</td>
<td>0.324</td>
<td>0.373</td>
</tr>
<tr>
<td>( \tilde{R}_i(7,8,9) )</td>
<td>0.333</td>
<td>0.0</td>
<td>0.0</td>
<td>0.140</td>
<td>0.135</td>
<td>0.285</td>
</tr>
<tr>
<td>( \tilde{s}(3,3,4,3,5,7) )</td>
<td>(1.5,2.5,4)</td>
<td>(1.4,2.4,3.8)</td>
<td>(2.2,3,2,4,5)</td>
<td>(2.1,3,1,4,5)</td>
<td>(2.9,3,9,5,3)</td>
<td></td>
</tr>
</tbody>
</table>

Continue to review on test data set 3, measurement of DM 3 has no interaction to both DM1 and DM2. Based on degree of “superposition”, there is no doubt about the fact that weighting of DM3 is set to zero. If weighting of a DM is set to zero, there is no effect on final
conclusion, which can be looked as the filter to outliers. Both SAM and SbAM support functionality to sift normal data from outliers. Let’s review test data set 3 in deep, the measurement range of DM1 is less than that of DM2, in other words, the judgment from DM1 is more precise than that of DM2. It is obvious that weighting of DM1 would be larger than that of DM2. SbAM made it sense, but SAM seemed to have a bias. The bias existed in the instance that the one with more precise judgment has same effect to that with loose.

6. Conclusions

The decision weighting of each DM is set equally in traditional performance evaluation model, and arithmetic average is applied as the method for group data aggregation. Actually it is not fair and not reasonable, especially in case of outliers are existed. Once bias has been caused by outliers, it is far apart from the actual group consensus. Even then the calculation results will be varied from the various methods applied, but all methods discussed are all dependent on group interaction. One word to say, decision weighting of each DM is impacted by total decision group.

The solution procedures of SbAM are similar to that of SAM provided by Hsu and Chen (1996). Both of them are based on degree of “superposition”, the only difference existed is that SbAM is derived from fuzzy subsedthood measure, but SAM is derived from fuzzy similarity measure. Through the discussion in previous section, it is obvious that SbAM is prior to SAM while applying in group data aggregation.

Reference


