Optimization of Sales Promotion Activities for Customers under the Condition of Budget Constraint

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Abstract

The purpose of this study is to create a model of sales promotion activities to each customer where the summation of expected profits under budget constraint is maximized, as a multi-dimensional knapsack problem, where the purchase probability of a customer for various channels is already known. In addition, we have proposed a metaheuristic algorithm for that problem. This problem has been formulated using a defined expected cost, an expected profit, and a risk measure that is the square root of a semi-variance of risks. We could obtain the tool to decide “to whom and how (channel) what sort of products should be promoted” in order to maximize an expected total profit under a budget constraint and a risk tolerance. The relation among an expected total profit, a budget and a risk tolerance were also obtained.

Keywords: CRM, Semi-variance, Combinatorial optimization, Risk measure, Strategic oscillation

1. Introduction

In recent years a concept, “CRM (Customer Relationship Management)”, has spread in corporate management (S.Verma, R.Chaudhuri 2009, S.Doyle 2009, A.Krasnikov, et al. (2009), S.Maklan, et al. 2008). The concept of CRM is to strengthen the relationship with customers and to maintain a long and continuous trade relationship. To realize CRM, it is always necessary to maximize customer satisfaction by approaching each customer on the assumption that each customer is different from others instead of considering that a customer is merely one of the purchasers and to propose and provide products and services which match respective customers’ needs and tastes. To decide “to whom and how (channel) what sort of products and services should be promoted” will become an important issue for a corporation that will have an effect on sales and profits.

The purpose of this study is to create a model of a knapsack problem which is a problem of combinatorial optimization of sales promotion activities which a corporation carries out on each customer such as sending direct mails or e-mails to or telephoning existing customers concerning products, etc. and to further propose a metaheuristic algorithm as a
method of solution of the modeled problem. The algorithm in this study will make it possible to decide “to whom and how (channel) what sort of products and services should be promoted” in order to maximize an expected total profit under a budget constraint on the assumption that the purchase probability of each customer by product and channel is already known. Furthermore, it will enable decision making from the three viewpoints of profits, costs and risks by introducing risk measure when expanding the model. In this study, we will carry out a numerical experiment using virtual data for the modeled problem to indicate its validity.

2. Model

A corporation carries out a promotion of a product \( j \) to a customer \( i \) through a channel \( k \).

![Diagram]

- **Receive a promotion**
- **Purchase a product**
- **Purchase a product**
- **Place an order for a sample product**
- **Do not place an order for a sample product**
- **Purchase a product (B)**
- **Purchase a product (C)**

**Figure 1. Probability of shifting acts of customers \( i \)**

A certain corporation is considering a sales promotion plan to maximize a total profit within a limited budget and a predetermined risk tolerance. In this problem, the corporation will sell \( J \) kinds of products (\( j = 1, \ldots, J \)), and will promote a product to \( I \)
numbers of existing customers \((i = 1, \ldots, I)\) using any of \(K\) numbers of channels \((k = 1, \ldots, K)\) respectively. In addition, the corporation has been assumed to have prepared respective free sample products. Customers who have received a promotion decide whether they will purchase the product or not. Where a customer does not purchase any product, the customer will decide whether it will place an order for a sample product or not. If the customer places an order for a sample product, it will further decide whether it will purchase a product or not (Figure 1). The number of promotions to a customer is once at most. Gross profits (product price less cost) of products, selling and general administrative expenses, and cost of channels vary.

Here, the probability \(r_{ijk}\) of direct purchase, the probability \(s_{ijk}\) of placing an order for a sample product, the probability \(t_{ijk}\) of purchasing a product after placing a sample product at the time of carrying out a promotion of a product \(j\), to a customer \(i\) through a channel \(k\) are assumed to be already known respectively. Where acts of customers who have received a promotion move as shown in Figure 1, acts that customers can possibly carry out will eventually be classified into 4 (purchase directly, purchase after placing an order for a sample product, not to purchase despite having placed an order for a sample product, and do nothing), and the probability of shifting to respective acts will be shown as in Table 1. In addition, Table 2 shows predetermined costs and profits of a corporation where customers who have received a promotion have carried out respective acts.

<table>
<thead>
<tr>
<th>Acts</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(r_{ijk})</td>
</tr>
<tr>
<td>B</td>
<td>((1 - r_{ijk})s_{ijk}t_{ijk})</td>
</tr>
<tr>
<td>C</td>
<td>((1 - r_{ijk})s_{ijk}(1 - t_{ijk}))</td>
</tr>
<tr>
<td>D</td>
<td>((1 - r_{ijk})(1 - s_{ijk}))</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Acts</th>
<th>Costs</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(C_j + L_k)</td>
<td>(P_j - (C_j + L_k))</td>
</tr>
<tr>
<td>B</td>
<td>(C_j + L_k + F_j)</td>
<td>(P_j - (C_j + L_k + F_j))</td>
</tr>
<tr>
<td>C</td>
<td>(L_k + F_j)</td>
<td>(-(L_k + F_j))</td>
</tr>
<tr>
<td>D</td>
<td>(L_k)</td>
<td>(-L_k)</td>
</tr>
</tbody>
</table>

Acts of A to D shown in Table 1 and Table 2 are decided in the order as shown below.

A : \(1 \rightarrow 2\)
B : \(1 \rightarrow 3 \rightarrow 4 \rightarrow 6\)
C : \(1 \rightarrow 3 \rightarrow 4 \rightarrow 7\)
D : \(1 \rightarrow 3 \rightarrow 5\)

Where,
\(C_j\): Selling and administrative cost of a product \(j\)
\(F_j\): Cost of a sample of a product \(j\)
$p_j$: Gross profit of a product $j$

$L_k$: Promotion cost of a channel $k$

From the above, where a product $j$ is promoted to customers $i$ through a channel $k$, an expected cost $c_{ijk}$ and an expected profit $p_{ijk}$ are defined as follows:

$$
\begin{align*}
  c_{ijk} &= r_{ijk} (C_j + L_k) + (1 - r_{ijk}) s_{ijk} t_{ijk} (C_j + L_k + F_j) \\
  &+ (1 - r_{ijk}) s_{ijk} (1 - t_{ijk}) (L_k + F_j) + (1 - r_{ijk})(1 - s_{ijk})L_k \\
  p_{ijk} &= r_{ijk} \{p_j - (C_j + L_k)\} + (1 - r_{ijk}) s_{ijk} t_{ijk} \{p_j - (C_j + L_k + F_j)\} \\
  &+ (1 - r_{ijk}) s_{ijk} (1 - t_{ijk}) \{- (L_k + F_j)\} + (1 - r_{ijk})(1 - s_{ijk})(-L_k)
\end{align*}
$$

(1)

Semi-variance (W. Yan, et al. 2007) $v_{ijk}$ will be introduced using an expected cost $c_{ijk}$ defined above.

Essentially it is not a problem that an actual cost incurred becomes smaller than an expected value. However, it is a problem that a cost incurred becomes larger than an expected cost, and it is not so desirable that the summation of expected costs exceeds a budget. $v_{ijk}$ is let to be made an expected value of a square of a larger value between 0 and a value obtained by deducting an expected cost $c_{ijk}$ from a cost of each act (A to D) that can be carried out by customers $i$ when a product $j$ is promoted to customers $i$ through a channel $k$ (Equation (3)). By so doing, when a cost incurred is greater than an expected cost can be taken as a problem. In this study, the square root of $v_{ijk}$ is let to be made a risk measure.

$$
\begin{align*}
  v_{ijk} &= r_{ijk} \left[ \max \left\{ (C_j + L_k - c_{ijk}, 0) \right\} \right]^2 + (1 - r_{ijk}) s_{ijk} t_{ijk} \left[ \max \left\{ (C_j + L_k + F_j - c_{ijk}, 0) \right\} \right]^2 \\
  &+ (1 - r_{ijk}) s_{ijk} (1 - t_{ijk}) \left[ \max (L_k + F_j - c_{ijk}, 0) \right]^2 + (1 - r_{ijk})(1 - s_{ijk}) \left[ \max (L_k - c_{ijk}, 0) \right]^2
\end{align*}
$$

(3)

This problem will be formulated using a defined expected cost $c_{ijk}$, an expected profit $p_{ijk}$, and a semi-variance $v_{ijk}$, as a multi-dimensional knapsack problem which is a combinatorial optimization problem as follows:

$$
\max \quad g = \sum_{i=1}^{J} \sum_{j=1}^{J} \sum_{k=1}^{K} p_{ijk} x_{ijk}
$$

(4)
\[ s.t. \quad c = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} c_{ijk} x_{ijk} \leq b \]  

(5)

\[ r = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \sqrt{v_{ijk}} x_{ijk} \leq a \]  

(6)

\[ \sum_{j=1}^{J} \sum_{k=1}^{K} x_{ijk} \leq 1, \quad i = 1, \ldots, I \]  

(7)

\[ x_{ijk} \in \{0,1\}, \quad i = 1, \ldots, I, \quad j = 1, \ldots, J, \quad k = 1, \ldots, K \]  

(8)

Where

\[ x_{ijk} = \begin{cases} 
0, \text{ where a product } j \text{ is not promoted to customers } i \text{ through channel } k \\
1, \text{ where a product } j \text{ is promoted to customers } i \text{ through channel } k 
\end{cases} \]

\[ b : \text{budget} \]

\[ a : \text{maximum risk tolerance} \]

Equation (4) will become an objective function, and the purpose will be maximization of the summation \( g \) of expected profits. Equation (5) is a restricted equation where the summation \( c \) of expected costs must become below a budget \( b \). Equation (6) will become constraint that the summation \( r \) of a square root of a semi-variance must be less than the maximum risk tolerance \( a \). In addition, Equation (7) is a restricted equation where the number of times of promotion to a customer is once at most. Equation (8) is constraint of a determining variable \( x_{ijk} \), and \( x_{ijk} \) will be 1 where promotion of a product \( j \) is carried out to customers \( i \) through channel \( k \), and will be 0 where it is not carried out.

This problem can be said to be a multi-dimensional knapsack problem as it consists of Equation (4), an objective function, Equation (5), a budget constraint, and Equation (6), a risk constraint. In addition, the constraint condition (7) where the number of times of promotion to a customer is once at most can be deemed to be exclusive constraint. Accordingly, this problem is a multi-dimensional knapsack problem with an exclusive constraint.
3. Algorithm

3-1 Basic Strategy

It is necessary to apply the metaheuristic method as the scale of the problem is large. Firstly, initial solution will be made using a greedy method created for this problem. Procedures for the improvement of a solution will be repeated with the solution as the starting point. Good solutions of restricted problems such as knapsack problems exist near the boundary between the feasible solution region and the infeasible solution region. A method to search inside and outside the boundary alternatively to intensively search near the boundary is called strategic oscillation (K. A. Dowsland 1998, S.Hanafi, A.Freville 1998, F.Glover, et al. 1998).

By strategic oscillation, search is made using different types of neighborhood operation. In this study, neighborhood operation to increase the summation $g$ of expected profits which are the value of an objective function and neighborhood operation to reduce the summation costs $c$ or the summation risks $r$ will be used depending on the situation.

3-2 Creation of Initial Solution

The following greedy method will be adopted to create initial solution. $d_{ijk}$ in step 2 is a value of dividing $p_{ijk}$ by the summation of the rate of $c_{ijk}$ to budget $b$ and the rate of $\sqrt{v_{ijk}}$ to the maximum risk tolerance $a$. The value of $d_{ijk}$ means efficiency when a product $j$ is promoted to customers $i$ through a channel $k$, and the higher the value, the greater priority should be given. Even if promotion is carried out in the order of a higher $d_{ijk}$, there is no guarantee that an optimal solution is obtained, but a quality solution can be obtained relatively easily.

Step 1:
$x_{ijk} := 0$ as to all $i,j,k$.

Step 2: Calculate

$$d_{ijk} := \frac{p_{ijk}}{c_{ijk} + b + \sqrt{v_{ijk}} }$$

(9)

as to all $i, j$ and $k$, and arrange in the order of a greater value of $d_{ijk}$:

$$(d_{ijk})_1 \geq (d_{ijk})_2 \geq \cdots \geq (d_{ijk})_{i,j,k}$$

(10)

and $n := 1$
Step 3: If 
\[ c_{ijk} \leq b, \sqrt{v_{ijk}} \leq a \quad \text{and} \quad \sum_{j=1}^{J} \sum_{k=1}^{K} x_{ijk} = 0, \quad x_{ijk} := 1 \quad \text{and} \quad b := b - c_{ijk}, a := a - \sqrt{v_{ijk}} \]
as to all \( i, j, k \) for \( \{d_{ijk}\}^n \).

Step 4:
\[ n := n + 1 \]

Step 5:
If \( n > I \cdot J \cdot K \), it will terminate. If \( n \leq I \cdot J \cdot K \), it will return to Step 3.

3-3 Improvement of Solutions

Algorithm to improve the initial solution will be shown below. Processing (a) and processing (b) in Step 2 are neighborhood operations to carry out strategic oscillation. This enables the area near the boundary to be searched by carrying out processing (a), where the present solution is feasible and by carrying out processing (b) where it is infeasible.

Step 1:
Create initial solution, which will be made a temporary solution, and \( n := 0 \).

Step 2:
Select customers \( i \) at random, and if \( c \leq b, \quad r \leq a \), processing (a) will be carried out, and, otherwise, processing (b) will be carried out.

(a): Calculate the following equation (11) as to all \( j, k \) (where promotion is not carried out to a customer \( i \) in the present solution, \( p_{ijk} = 0, \ c_{ijk} = 0, \) and \( \sqrt{v_{ijk}} = 0 \)). Find out the product \( j' \) and channel \( k' \), where the value of the following equation is the largest, and makes \( x_{ij'} := 1 \). If it is impossible to find out any, do nothing.

\[
\frac{p_{ijk'}}{c_{ijk'}} + \frac{\sqrt{v_{ijk'}}}{b - c + c_{ijk}} + \frac{\sqrt{v_{ijk}}}{a - r + \sqrt{v_{ijk}}}
\]

(b): Select channels and products (or promotion is not carried out) at random, and find out a product \( j' \) and a channel \( k' \) which satisfy \( c_{ijk'} < c_{ijk} \) (Where promotion is not carried out to customers \( i, \ c_{ijk} = 0 \)). Or find out a product \( j' \) and a channel \( k' \) which satisfy \( \sqrt{v_{ijk'}} < \sqrt{v_{ijk}} \) (Where promotion is not carried out to customers \( i, \sqrt{v_{ijk}} = 0 \)). If
any is found out, \( x_{ik} := 1 \), and \( x_{ik} := 0 \). If nothing is found out, do nothing.

Step 3:
Where a solution is changed, calculate a value of an objective function \( g \) and the summation of expected costs \( c \) and the summation of risks \( r \), and if \( g < g^* \) (where \( g^* \) is the value of an objective function of a temporary solution) and \( c > b \) and \( r > a \), let the present solution a temporary solution, and go to Step 5. Otherwise, go to Step 4.

Step 4:
If \( g > g^* \) and \( c \leq b \) and \( r \leq a \), let the temporary solution be the present solution. Otherwise, the solution will not be changed.

Step 5:
\( n := n + 1 \)

Step 6:
If the number of \( n \) has reached an iteration value, output a temporary solution and end it. Otherwise, go to Step 2.

4. Numerical Experiment

<table>
<thead>
<tr>
<th>( i )</th>
<th>( r_{i1} )</th>
<th>( s_{i1} )</th>
<th>( t_{i1} )</th>
<th>( r_{i21} )</th>
<th>( s_{i21} )</th>
<th>( t_{i21} )</th>
<th>( r_{i31} )</th>
<th>( s_{i31} )</th>
<th>( t_{i31} )</th>
<th>( r_{i41} )</th>
<th>...</th>
<th>( t_{i43} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.001</td>
<td>0.132</td>
<td>0.159</td>
<td>0.012</td>
<td>0.179</td>
<td>0.024</td>
<td>0.036</td>
<td>0.133</td>
<td>0.048</td>
<td>0.196</td>
<td>...</td>
<td>0.092</td>
</tr>
<tr>
<td>2</td>
<td>0.045</td>
<td>0.125</td>
<td>0.085</td>
<td>0.282</td>
<td>0.202</td>
<td>0.205</td>
<td>0.071</td>
<td>0.035</td>
<td>0.145</td>
<td>0.103</td>
<td>...</td>
<td>0.173</td>
</tr>
<tr>
<td>3</td>
<td>0.021</td>
<td>0.234</td>
<td>0.322</td>
<td>0.028</td>
<td>0.191</td>
<td>0.195</td>
<td>0.046</td>
<td>0.241</td>
<td>0.217</td>
<td>0.137</td>
<td>...</td>
<td>0.108</td>
</tr>
</tbody>
</table>
| ...   | ...  | ...  | ...  | ...  | ...  | ...  | ...  | ...  | ...  | ...  | ...| ...
| 50000 | 0.248| 0.184| 0.282| 0.107| 0.126| 0.184| 0.099| 0.206| 0.003| 0.207| ...| 0.078|

A numerical experiment has been carried out using the number of customers \( I = 50,000 \), the number of products \( J = 4 \), and the number of channels \( K = 3 \). Immediately after a product \( j \) is promoted to customers \( i \) through a channel \( k \), the probability \( r_{jk} \) in which a product \( j \) is purchased, the probability \( s_{jk} \) in which an order for a sample of a product \( j \) is placed, and the probability \( t_{jk} \) in which a product \( j \) is purchased after placing an order for its sample are prepared at random respectively. Table 3 shows some
of $r_{gk}$, $s_{gk}$ and $t_{gk}$. Gross profit $P_j$, selling and general administrative expenses $C_j$ and costs of a sample $F_j$ with respect of a product $j$, and promotion cost $L_k$ of a channel $k$ were set as per Table 4 and Table 5, respectively.

<table>
<thead>
<tr>
<th>$j$</th>
<th>$P_j$ (yen)</th>
<th>$C_j$ (yen)</th>
<th>$F_j$ (yen)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3,000</td>
<td>500</td>
<td>300</td>
</tr>
<tr>
<td>2</td>
<td>3,500</td>
<td>700</td>
<td>400</td>
</tr>
<tr>
<td>3</td>
<td>4,000</td>
<td>600</td>
<td>500</td>
</tr>
<tr>
<td>4</td>
<td>4,500</td>
<td>900</td>
<td>600</td>
</tr>
</tbody>
</table>

Table 5 Promotion costs $L_k$ of a channel $k$

<table>
<thead>
<tr>
<th>$k$</th>
<th>$L_k$ (yen)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>300</td>
</tr>
</tbody>
</table>

5. Results

![Figure 2 Total profit vs. number of iteration ($b = 10^5$, $a = 10^6$)](image)

Figure 2 shows that the value of the objective function at the feasible solution is increased as the iteration $n$ increases. In this case, the maximum value of the iteration $n$ is set to be $n = 10^9$. The value of the objective function is increasing slightly at $n = 10^9$ and the solution is not the global optimal solution. But the average value by the ten trials at $n = 10^7$ is 4,422,748 and 1469.53 respectively. The standard deviation is small and it is considered that the solution close to the global optimal solution was obtained by strategy oscillation method while escaping from local optimal solutions.
Next, we fixed the maximum risk tolerance \( a = 10^6 \) and checked the values of an objective function by solving problems by the method by changing a budget \( b \) by \( 10^5 \) in ascending order from \( 50 \times 10^4 \) to \( 150 \times 10^4 \). The results are shown in Figure 3. If \( b \) exceeds \( b = 110 \times 10^4 \), the objective function will scarcely increase. The reason is considered to be that when the value of \( b \) increases, budget constraint will be loosened, but risk constraint will not change. Risk constraint will become rigid in relative terms, and the number of feasible solutions will not increase. In addition, a similar tendency has been seen when problems are solved by a budget being fixed at \( b = 10^6 \), and the maximum risk tolerance \( a \) being changed by \( 10^4 \) from \( 50 \times 10^4 \) to \( 150 \times 10^4 \) (Figure 4).

![Figure 3 Values of objective function when budget b is changed](image1)

![Figure 4 Values of objective function when maximum risk tolerance a is changed](image2)
6. Conclusions and Future Issues

In this study, we have created a model of sales promotion activities to each customer where the summation of expected profits under budget constraint and risk tolerance is maximized, where the purchase probability of a customer for various channels is already known. In addition, we have proposed a metaheuristic algorithm for that problem.

This problem can be said to be a multi-dimensional knapsack problem as it consists of an objective function, a budget constraint, and a risk constraint. In addition, the constraint where the number of times of promotion to a customer is once at most can be deemed to be exclusive constraint. This problem has been formulated using an expected profit, a defined cost and a risk measure that is the square root of a semi-variance of risks. As the results, we could obtain the tool to decide “to whom and how (channel) what sort of products should be promoted” in order to maximize an expected total profit under a budget constraint and a risk tolerance. We also obtained the relation between an expected total profit and a budget under a fixed maximum risk tolerance and the relation between an expected total profit and a maximum risk tolerance under a fixed budget.

Nowadays, tools to obtain purchase probability of a customer such as neural network, CHAID and etc. are established. Therefore the method obtained in this study will be useful for a sales promotion to customers and a promotion budget planning.

Firstly, further improvement of the algorithm should be taken up as a future issue. In addition, this time, there was an additional constraint condition that the number of promotions to a customer was once at most. However, it is possible to consider to make the number of this constraint, the maximum number of promotions, twice or more (or no ceiling is set), and it is necessary to consider the algorithm in that case. Furthermore, there can be another problem that the summation of risks, which is the value of an objective function, should be minimized, while making the objective function the summation of risks and maintaining the expected profits within a budget constraint at a certain defined value or more.

7. References


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