Measuring Equity Similarity by Market Price
Sensitivity of Financial Instruments

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Abstract. The paper addresses the problem of classifying a company’s financial instruments into equity and debt. We take the perspective of an investor to the company and derive a measure of equity similarity which reflects the risk characteristics associated with financial liability. This approach will allow to assign an equity percentage to every financial instrument by standard valuation methods of finance theory. In: Shebani K (ed). Proceedings of the 1st International Conference on Applied Operational Research – ICAOR (2008), pp 87–99. Lecture Notes in Management Science Vol. 1. ISSN 2005-0050.

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1 Introduction

Financial theory as well as accounting theory distinguish between equity and debt when regarding financial instruments of companies. However, both apply this distinction for different reasons. While accounting aims to establish an unequivocal basis of disclosure principles, the financial perspective primarily is a desire for a flexible and multi-faceted characterisation of possible instruments, helping to explain and support finance and investment decisions. The decision to buy or issue a certain financial instrument rests on the nexus of all features that come along with the instrument. These attributes have to match the needs and circumstances of the investor and the issuing company. Nevertheless, the reflection of the instrument in the financial statements influences decisions of investors and managers. The equity-to-assets ratio, for instance, is regarded an important indicator for a company’s financial solidity and therefore affects financing conditions and business prospects.
Furthermore, the classification determines the profit figure and the amount of taxes a company must pay. But the dichotomous distinction of financial instruments increasingly raises questions when applied to real companies. Many modern financial instruments combine typical features of both categories positioning them in between the traditional notions of equity and debt. The classification in two clear-cut categories therefore might have become arbitrary.

The prevailing opinion regarding the classification issue seems to be that a dichotomous distinction will never cope with the whole and ever-growing diversity of financial instruments. Therefore, some authors suggested that the information traditionally given in the balance sheet should be supplemented by ratios that quantify specific aspects of an instrument relevant for certain addressees [6, pp. 13–18]. Such a ratio can be interpreted as a measure of equity similarity. Tailoring such measures to different characteristics of equity could result in a whole range of ratios that highlight different perspectives and therefore address different informational needs.

This paper proposes a ratio for equity similarity capturing an investor’s point of view. The measure will be derived from a specific definition of financial liability. Being a sensitivity to price movements, it belongs to a class of risk measures well established and applied in financial risk management. As a further advantage, the estimation of the ratio requires standard finance methods only, in particular asset pricing and contingent claims analysis. Application within the context of capital structure analysis and financial reporting allows to provide relevant information in a single but meaningful figure. We also show that equity similarity expressed as a sensitivity delivers new and valuable insight into the relative risk positions held by different capital providers.

The paper is structured as follows: In Sect. 2 we introduce the current discussion about the classification of equity and debt. Section 3 derives and defines price sensitivity as an appropriate measure for equity similarity, and later explains its content on the basis of an example company with different financial instruments. The last section summarises and concludes.

2 Approaches to Classifying Equity and Debt

Current Research and Hybrid Finance

Frequent inconsistencies in the reporting for mezzanine instruments have put a question into the spotlight of research and practice: How do these instruments blend with traditional debt and equity in the financial statements? Accountants generally argue that an instrument’s characteristics should determine its classification, e. g. [1, p. 2149]. But in reality it sometimes works the other way round [18]: Many companies seek to report a high fraction of equity finance in the balance sheet. Thus, their financial instruments are designed to meet this specific requirement
while avoiding other equity characteristics. Typical examples for this are financing programmes for medium-sized enterprises by which the client company issues a hybrid instrument – also called mezzanine capital – to the bank. Most such products are specifically designed to be treated as equity in the balance sheet, but as debt for tax reasons so that the company can deduct the interest payments from its tax base. For example, to address different accounting standards the product Equi-Notes [14, 7] actually offers two different variants: type A is considered equity under the Commercial Code of Germany (HGB), but only type B will meet the more restrictive equity definition of the International Financial Reporting Standards (IFRS).

The range of problems that accompany the accounting for hybrid capital has been addressed by a series of contributions in the literature. Many accountants entertain doubts whether the established distinguishing criteria still comply with the objectives of financial reporting [20, p. 41]. As a result the debt and equity distinction based on their characteristics has been re-discussed recently. Several accounting researchers came forward with proposals on how to more consistently classify financial instruments [1, 3]. The International Accounting Standards Board (IASB) and the United States standard-setter Financial Accounting Standards Board (FASB) are running a joint research project regarding the classification issue [13]. Different discussion papers are circulating, issued by the FASB [9], the IASB [12] and the ‘Proactive Accounting Activities in Europe’ initiative (PAAiM) [20].

A different line of research follows the idea to supplement the information given about financial instruments with additional ratios. A German expert group has recently published a proposal about how to calculate a measure of equity similarity based on a scoring model [6]. The method quantifies several classical equity characteristics and combines them by a weighting scheme. While being very operational such a method inevitably suffers from arbitrariness regarding the choice of the weights. The partly recognition of hybrid financial instruments as equity by rating agencies – seeking to avoid distortions that follow arbitrary classification rules – can also be interpreted as a measurement of equity similarity. However, the equity percentage attributed to instruments has for long been the result of a rough guess, which was additionally restricted to a maximum of 50%. Since 2005 the major rating agencies have been using different assessment frameworks, allowing for higher and adaptive percentages to assess the equity component of a financial instrument [10]. Our work directly contributes to this line of research by proposing a measure of equity similarity that is consistently derived from the notion of financial liability. By using valuation methodology from finance theory for accounting purposes our work is also related to the literature that explores the relations between accounting and capital markets [2]. Those relations have been studied extensively, most often with the objective to assess the informational content of accounting figures [17]. So called association studies measure the correlation of equity price changes and disclosure of accounting information whereas event studies try to capture price reactions on the release of specific types of information. An overview for studies within an international context can be found in [19]. A different line of literature explores the effect of accounting on the cost of capital [8]. In both areas of literature, however, the results of existing accounting rules on capital
markets are studied. By contrast, we use pricing approaches from finance theory to derive accounting figures with a high informational content.

**Dichotomous Distinctions Based on Attributes**

Standard finance textbooks like Brealey/Myers/Allen [5, p. 361] characterise equity and debt with regard to two types of rights that those instruments provide for the investors: cash flow rights and control rights. Debt holders, they state, have the first claim on cash flow and are promised definite cash payments. In contrast, equity holders can exercise control of the firm. Other characterisations additionally consider the legal form of a binding contract and the duration of the investment.

From an economic point of view the priority of the claims is decisive for the amount of return a capital provider can expect. In this context, subordination of equity means that if the company is liquidated, the owner’s claims strictly rank behind debt, the former being served only if all debt claims are completely satisfied. On the other hand, holders of equity are entitled to the rights of control in the company. This compensates for the risk which is inherent in their residual claim. Generally, it can be established that all financial instruments tend to compensate higher risk by more control rights. For this important relation the traditional catalogue of attributes used to characterise debt and equity – Table 1 gives an example – just marks ends of a wide range. The margins could be called pure equity and pure debt [4, 20].

<table>
<thead>
<tr>
<th>attribute</th>
<th>pure equity</th>
<th>pure debt</th>
</tr>
</thead>
<tbody>
<tr>
<td>returns</td>
<td>variable, depending on profits</td>
<td>fixed</td>
</tr>
<tr>
<td>taxation of returns paid</td>
<td>no deduction</td>
<td>deducted from tax base</td>
</tr>
<tr>
<td>control rights</td>
<td>all rights of control</td>
<td>no rights of control</td>
</tr>
<tr>
<td>duration of investment</td>
<td>undefined</td>
<td>fixed</td>
</tr>
<tr>
<td>priority in liquidation</td>
<td>subordinate</td>
<td>first</td>
</tr>
</tbody>
</table>

Although this catalogue proves highly useful in describing typical equity and debt, a multiple list of criteria inevitably is ambiguous when used for a dichotomous classification. Therefore, in case of doubt, it is common to concentrate on the priority in liquidation and disregard all other attributes. This guarantees clearness at the cost of diminishing information.
Continuous Characterisation Based on Risk

Let us assume that we have identified the decisive attribute and apply it as the criterion for classification. If we further accept that pure equity and pure debt are elusive we are lead to model real financial instruments on a continuum [20]. This can be shown as a line from pure equity to pure debt, see Fig. 1. Each instrument is then represented by a point on that equity-debt line; the grouping into two classes is replaced by a smooth transition. In order to identify a financial instrument with a point, the line must be scaled. That means measuring the equity similarity of instruments, e.g. by a percentage of how well the instrument resembles pure equity.

![Equity-debt line](image)

**Fig. 1.** Equity-debt line

With respect to the fact that equity similarity incorporates different aspects, many methods of quantification exist. In order to end up with a result that is meaningful for the addressee, why to classify should determine how to classify. Therefore, before defining a method of quantification, the purpose of the classification must be identified. Different purposes will bring and should bring different results concerning the capital character of an instrument. For example, the fact that Equi-Notes type A are considered equity under HGB but liabilities under IFRS can be traced back to the different purposes the different accounting frameworks follow. The same applies to the catalogue of attributes in Table 1: Depending on the purpose, the one or the other attribute will be appropriate as the final criterion for distinction.

Nevertheless, most of the attributes in the traditional catalogue can be regarded as different components of a common purpose. They are related to the risk the payments to the investor are subject to, but each place emphasis on a different aspect. Equity, uniformly, is the side which is associated as being risky, whereas debt represents the secure side. Equity similarity, hence, can be interpreted as some kind of risk measure. The use of risk measures for the classification of financial instruments has long been proposed by Swoboda [24, 25, pp. 55–59]. The standard deviation of returns \( \sigma \) he suggested captures a specific aspect of investment risk – the variability of return – that does not come close to the notion of risk primarily assigned to equity. This might be one reason why the use of this number has not been accepted within the context of financial reporting. In a more traditional understanding the risk an equityholder faces is named financial liability, what is closely related to the obligation to bear losses. This notion of risk therefore forms the starting point for the derivation of our measure of equity similarity.
3 Market Price Sensitivity as a Continuous Measure of Equity Similarity

The Notion of Financial Liability

Following the last section, a ratio shall be developed which is tied to the character of a financial instrument being more equity- or debt-like. The purpose of the ratio is to express financial liability. It should be noted that the word “liability” is not used synonymously for debt. It is rather understood in a sense of “to be liable” and relates to the meaning of limited liability companies. In common speech as specified in a standard dictionary [23, p. 820], there are three meanings of the word “liable”:

1. “responsible by law, . . . , subject by law”
2. “likely to do something,” and
3. “likely to experience (something undesirable).”

The second meaning can be excluded here. One might be tempted to accept the first meaning, and indeed this might come close to the initial intention, when limited liability was invented. But if we consider the position of a typical investor, the following situation appears: The investor has put money or other assets in the company, and in return the company issued a financial instrument to them. In all but the rarest cases no further obligation of the investor will arise from the instrument. How, then, can they be liable in the sense of being legally responsible? Hence, the first meaning as stated above does not conform to reality. Rather it is the third meaning that is appropriate here: Liability means to be exposed to negative effects.

The risk the investors assume by buying a financial instrument materialises in the compensation they actually get. They might have some expectation or a legal claim on returns. But it is uncertain what wealth, normally cash, they will obtain when the instrument becomes due. The company, eventually, has to meet all claims with its assets. Thus, the value of the assets is the final source of any payments and defines the total amount available. According to this, we define losses as an immediate decrease in the company’s asset value. Note that this loss definition differs from the accounting notion of losses (or expenses), which encompasses changes in the obligations of the company as well. The company’s assets will be distributed with respect to the different ranks of financial instruments. The distribution of payments within the different classes of investors is defined by the seniority of the claims as specified in the financing contracts. Based on this, we define liability as the participation of a financial instrument in losses incurred by the company. From the investor’s perspective, losses of the company will be reflected in a lower value of their investment. Therefore, liability is the reduction of the value of an financial instrument when the company’s asset value decreases.
As the actual economic situation of the instruments’ holders is to be captured here we use market values, i.e. prices, for both the asset value and the value of the financial instrument. Nevertheless, in order to derive the measure for equity similarity it is not necessary to observe market prices of the company’s assets and its financial instruments. For financial liability is just the relation between those prices. The value – meaning a hypothetical price – of a financial instrument can be expressed by a valuation formula that in general is derived by some contingent claims valuation (mark to model). Our purpose only requires that the company’s asset price together with its capital structure work as input to the valuation. For example, with respect to credit risk, this means that we need a so called asset value model. The basic version, often referred to as the Merton model, considers shares as a call option on the company’s asset value, with the strike price being equivalent to the claims of all debtholders [11, p. 587]. Financial liability is then expressed by the variation of the price estimated by this model when the input asset value changes, both in negative direction.

**Price Sensitivity as a Ratio for Financial Liability**

Put formally we propose a ratio called “equity similarity”, $e$. The ratio is the change of the financial instrument’s price $P$ by a marginal change of the asset value $V$. If left-hand limit and right-hand limit differ, $e$ is the one-sided limit for a marginal decrease of $V$. Put into formula:

$$ e = \frac{\partial P(V,...)}{\partial V} = -\frac{\Delta P}{\text{marginal loss}} \quad (1) $$

By consequently formalizing what we regard an appropriate understanding of liability we thus end up with a definition of equity similarity that is nothing else but the price sensitivity measured as the first derivative of the instrument’s price with respect to the asset value.

We would expect $e = 1$ for pure equity and $e = 0$ for pure debt, i.e., losses are reflected in an equivalent drop in equity prices, while the price of debt remains unchanged. This relates to the attribute of pure debt that all its returns are fixed, and consequently the price of debt should not be affected by changes in the asset value. As a first check we can consider a default-free bond. The value of such a bond is the present value of its cash flows, which are assumed to be paid with certainty. Therefore, this present value is independent of the company’s asset value and the first derivative with respect to it is indeed zero.

However, real financial instruments will rarely show ideal numbers of zero or one. When the asset value decreases, ceteris paribus the repayment of debt might become less certain. So we expect $e < 1$ for all but perfect secured claims to the company. If we consider shares as the instrument, this might for example be modeled by the Merton model mentioned above that models equity as a call option. Equity
similarity as defined here will then coincide with the well known option delta. But in general a company’s capital will consist of several types of financial instruments with different rank. The higher the rank the less probable the default on the promised payments. For a company having \( n \) different classes of financial instruments, let \( P_1, P_2, \ldots, P_n \) be their respective prices. The following two conditions hold:

\[
\sum_{i=1}^{n} P_i = V \quad \text{(2)}
\]

\[
\sum_{i=1}^{n} e_i = 1 \quad \text{(3)}
\]

Equation (2) states that the asset value divides into all classes of financial instruments. It reflects the fact that all assets are eventually distributed to the holders of financial instruments. This argument can be extended to explain (3): Losses can be seen as negative assets and thus have to be apportioned between the instruments. Consequently, all instruments in sum carry 100% of the losses. If we restate (1) as

\[
e_i = \lim_{h \to 0} \frac{P_i(V) - P_i(V-h)}{h} \quad \text{(4)}
\]

it is easy to prove (3) with the help of (2):

\[
\sum_{i=1}^{n} e_i = \lim_{h \to 0} \frac{\sum_{i=1}^{n} P_i(V) - \sum_{i=1}^{n} P_i(V-h)}{h} = \lim_{h \to 0} \frac{V - (V-h)}{h} = 1 \quad \square
\]

With the help of contingent claims valuation the equity similarity ratio can be derived analytically. For example, we again refer to the Merton model that considers the shares as a call option on the company’s asset value. As mentioned before, the equity similarity then turns out be the option delta and can be calculated as a partial derivative of the Black/Scholes formula. However, the Black/Scholes model rests on some restrictive assumptions, that often are not satisfied. In particular, following Black/Scholes insolvency is only allowed at the maturity date and not before [16, pp. 177–179]. In our context, however, it is most important to consider insolvency at all relevant times, because financial liability materialises in these special cases. Therefore, the following section shall assess the informational content of our measure with the help of a reduced two period example, which allows a detailed analysis of all possible future states of the world.

**The Informational Content of Price Sensitivity**

Suppose today, in \( t = 0 \), a company issues three financial instruments (in monetary units): 60 regular bonds (1), 30 subordinate bonds (2) and 10 shares (3), resulting in 100 assets (V). There are two points of time in the future, \( t = 1 \) and \( t = 2 \), and
the bonds become due at the latter. Promised interest is 10% for the regular bond and 40% for the subordinate bond. In \( t = 1 \) there are two scenarios with equal probability: In the superior case (sc. 1) total assets rise to 132, in the inferior case (sc. 2) total assets fall to 92, causing insolvency. Only for the superior case two similar scenarios exist in \( t = 2 \): In the superior case (sc. 1.1) total assets rise to 189, in the inferior case (sc. 1.2) total assets fall to 106.7, causing insolvency. Figure 2 draws a time-tree of the asset value and Fig. 3 shows the balance sheets using the values transferred.

\[ \text{Fig. 2. Time-tree of the asset value} \]

\[ \text{Fig. 3. Balance sheets for the scenarios} \]
Note on how the asset values were calculated: Expected return on capital is always 12% per time step, expected return on equity is always 20% per time step. At each time step, one of the two scenarios includes the insolvency of the company, i.e., subordinated debt is not fully repaid. The risk neutral [21, p. 11] probabilities $q$ were calculated using the formulae [11, p. 303]

$$q_+ = \frac{i - h_+}{h_+ - h_-} \quad \text{and} \quad q_- = \frac{i - h_-}{h_- - h_+}$$

assuming a risk-free interest rate $i$ of 10%.

![Table](image)

**Fig. 4.** Mark-to-model prices of financial instruments

To calculate equity similarity, we have to consider a marginal change of the asset value $V = 100$ at $t = 0$. Say, we reduce the value to 99. The prices of all financial instruments can be calculated as expectation values using risk neutral probabilities. The results are shown in Fig. 4. From these numbers the ratios of equity similarity can be estimated:

$$e_1 = \frac{\Delta P}{\Delta V} = \frac{9.69 - 10.02}{-1} = 10.02 - 9.69 = 33\%$$

$$e_2 = 29.98 - 29.31 = 67\%$$

$$e_3 = 0$$

Surprisingly, from this point of view the subordinate debt is closer to pure equity than the shares. Its ratio of 67% suggests that, should the company incur a small loss, the subordinate debt will participate in this loss by two thirds. This is due to the specification of the example, where the company ends up insolvent at $t = 2$ with a probability of 75%. Insolvency is detrimental to the subordinate debt which then will be repaid partly. Losses reduce the fraction of the subordinate claim that is repaid, whereas the shareholders’ payoff has already reached ground in the insolvent scenarios. The 33%-ratio of the shares is due to the best-case scenario 1.1, where only shares are affected by an immediate small loss.
Varying today’s asset value not only shows how equity similarity changes with different starting positions of a company, but can also be used as sensitivity analysis for the calculated ratios. Figure 5 shows the three ratios as a function of today’s asset value. The vertical line at \( V = 100 \) indicates the example which was considered above. Around this asset value all three function graphs run horizontally. That means for \( V \in [83;117] \) the equity similarity ratios take the values estimated above. It can be further derived that what we called a “small loss” is a loss not exceeding \( 100 - 83 = 17 \). In general, the functions might not have horizontal sections so that it makes sense to define the ratio as a limit value as we did. For example, the ratio curve for the shares in Fig. 5 resembles the typical curve of \( \Delta \) in option pricing, which would be the standard normal cumulative density function [11, p. 423], [15, p. 222], [22, p. 89]. In our opinion, extending the analysis done here to more general pricing formulas with relaxed assumptions is an interesting field for further research.

4 Conclusion

The paper addressed the problem of classifying a company’s financial instruments into equity and debt. Existing accounting rules stipulate a dichotomous distinction in two classes. For this purpose traditional approaches utilise a catalogue of attributes, which has proven useful to describe the characteristics of pure equity and pure debt. However, when applied to modern financial instruments like hybrid
capital, the catalogue provides ambiguous results: Attributes of equity intermingle with attributes of debt.

In the current discussion leading accountants have repeatedly proposed to replace the binary classification by continuous measures. We followed this principle but pointed out that such an approach also needs an appropriate measure which can be used for the classification of instruments. Thus, we suggested a ratio of equity similarity reflecting the risk notion of financial liability. By referring to liability we employed the attribute regarded most characteristic for equity from an economic point of view.

The ratio is measured as the price sensitivity of financial instruments to losses in the company’s assets. For calculation, the measure requires a valuation of the instrument that uses the asset value as an input variable. Such valuations are known from standard contingent claims valuation. In our illustration of the measures informational content we used an example which disclosed all possible future asset values. However, it is in principle not necessary to do so. The ratio of equity similarity only requires to model the functional relation between the financial instrument’s price and the company’s asset value.

For companies with a high probability of default, debt instruments might suffer higher losses than equity and thus carry higher equity similarity ratios than the latter. This result adequately reflects the risk notion of financial liability. Therefore, we consider equity similarity more flexible and significant than traditional approaches to classification.

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