Rough Set Analysis for Solid Waste Management and Evaluation

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Abstract. The paper introduces a simple exemplification application concerning four different typical alternatives as regards to the problem of solid urban waste disposal (discharge, incinerator, balanced use of incinerator and composting plant, recycling), concerning the sub-region West Sicily, estimated by means of economic, environmental and technical criteria; with reference to those alternatives, we even know arrangements (global evaluations) expressed by ten different groups (decision makers). We use a useful methodology of decisional support, the Rough Set Analysis; this approach allows to take into consideration quantitative and qualitative data, to stress which are the most important attributes of the performed analysis, and to express results in terms of decisional rules, easily comprehensible and implementable. We speak about some obtained results with underscrement approach, by stressing methodology and application advantages brought by this approach, useful instruments of success for management of waste and in general for all problems of support to decision in environmental matters. In: Sheibani K (ed). Proceedings of the 1st International Conference on Applied Operational Research – ICAOR (2008), pp 275–285. Lecture Notes in Management Science Vol. 1.
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1 Introduction

We introduce a simple exemplification application concerning four different typical alternatives as regards to the problem of solid urban waste disposal, estimated by means of twelve criteria; with reference to those alternatives, we even know arrangements (global evaluations) expressed by ten different groups (decision
makers), consisting in the assignment of each alternative to one decision class, taking into account different attributes (sorting). All data deal with a case of management of solid urban waste concerning Sicily, and more precisely the sub-region including provinces of Palermo and Trapani, with 27 municipalities, for the extension of about 215,000, a population of about a million inhabitants and with a production of solid urban waste of about 500,000 tons a year.

Four typical considered alternatives, though analyzed with reference to specific territorial localization, have characteristics that are somehow generalizable, that is to say standard in spite of the peculiar context in which they are inserted.

In this application, above-mentioned data will be used, by applying classical rough set approach and with different purposes in the field of a decisional problem. The four alternatives (objects) that have been dealt with are considered as a sample of reference actions, well known by the decision maker (DM) that expresses his global preferences, from which the approach extracts the most important information on preferences in the problem of waste management. This information can then be used for supporting the same DM in solving analogue decisional problems, with reference to a wider group of admissible alternatives, described by the same criteria hereby used, the so-called conditional criteria. In particular, application of analysis by means of the rough sets is aimed at obtaining some peculiar and interesting results typical of that approach: decision rules and computation of reducts, of the subsets of the most “important”, criteria in the considered problem.

More precisely, the reducts are the subset of conditional attributes (or criteria, that is to say attributes with a values set ordered according to preferences) that alone give the same information quality on preferences obtainable by considering all the attributes. Therefore, it would be sufficient to know evaluation of alternatives will respect to criteria contained even in one reducts to “explain” preferences expressed by the DM, thus neglecting all other attributes, therefore considered as redundant. The distribution of different attributes into obtained reducts supplies, therefore, even very useful information on the importance of each one of them, distinctly considered and in synergy with the other ones, in the analysis taken into consideration. This “importance”, therefore, is not supplied by the DM, as it happens in all other approaches, but it is the result of an input information performed study.

The decisional rules synthesize preferences of the DM in the form of logic sentences of the type “if..., then...”, expressed in natural language, easily comprehensible and applicable at their turn to solve multicriteria decisional problems (of choice, of sorting and of ranking) concerning a new group of alternatives described by the same attributes. They can be certain, if conclusions are univocal, uncertain or possible on the contrary case. By comparing, as a matter of fact, the “premise” of a rule (that is to say the description “if...” in terms of values of some criteria) with the profile describing a new alternative, if this profile satisfies the conditions described in the premise, to this alternative we apply the “conclusion” (“then...”) of the same considered decisional rules. In the hereby presented study, in particular, this approach is applied to a PCT (Pairwise Comparisons Table), where instead of single alternatives, each time we consider pairs of them; therefore, descriptions and conclusions of each rule concern pairs of alternatives, thus
respecting useful results to solve even problems of choice and ranking, and not only of classification, as it happens by applying the traditional analysis of rough sets.

In other terms, we obtain some simplified and easily comprehensible descriptions of the reasons that explain the global preferences expressed by single DM with respect to a sample of reference of alternatives for the management of solid urban waste, of their main behavioural models, in order to be able to use them in a wider context, that is to say to support them to solve complex real decisional problems, in which concrete alternative feasible solutions for the examined problem are considered, estimated and compared by different points of view (environmental, technical and economical).

From global preference expressed by each DM in terms of total ranking of alternatives, it is possible to immediately obtain relations of outranking relation S with reference to considered pairs of alternatives. In particular, if a doesn’t follow b in this ranking, we will say that a outranks b, in the sense that “a is at least as good as b”, globally considered.

In the hereby proposed application, we will use, for calculation of approximations of outclassing relations, before the relation of indiscernibility (concerning alternatives having the same description) and then that of dominance, both applied to a PCT, in order to compare the results, above all with reference to their potential value. In particular, the considered PCT will contain as objects all possible ordered pairs of the four considered alternatives, as conditional attributes the same attributes (criteria) taken into consideration and as decisional attributes outranking relations built on the base of different (10) rankings of the alternatives supplied by different DM. The use of dominance relation allows to stress ordinal properties of criteria; if, in fact, the alternative a is better evaluated than the alternative b on all considered criteria, by using indiscernibility, it is only possible to affirm that a and b are discernible; by using dominance relation, on the contrary, it is possible to highlight that a dominates b (and therefore a is even preferred by b), thus being also able to detect eventual cases of inconsistency of the DM with respect to the principle of dominance.

2 Alternatives and Criteria

The four considered alternatives represent different ways to manage the problem of solid urban waste; they can be considered in abstract, that is to say without taking into consideration peculiar localisation, but only peculiar characteristics that each one shows. They are: 1) discharge, that substantially represents the current state, in which almost all waste are transported in proper discharges; 2) incinerator, which provides for incineration with energy recycling together with a differentiated collection of waste; 3) balanced use of incinerator and compositing plant, with a selection of the organic component of waste, to be transformed into a compound, and energy production from recycling of waste with the incinerator; 4) recycling, which is aimed at maximising resource retrieval from waste and it is fundamentally based onto a sufficient differentiation of waste.
We have considered a consistent family of criteria (exhaustive and not redundant), that should describe the most important economic, environmental and technical aspect of the problem.

**Economic criteria:**
- **Investment cost,** which includes construction and purchasing costs of technical equipment; it is measured on cardinal scale (million of lira/tons/days) and it must be minimised;
- **Exploitation cost,** which considers all management and maintenance costs of the plant; this too is measured on cardinal scale (million of lira/year) and it must be minimised;
- **earnings,** which includes possible earnings coming from retrieved resources and costs saves for their final discharge; they are measured on cardinal scale (million of lira/year), they must be maximised;
- **Local employment,** which considers new potential work places created into the area by different alternatives; it is measured on cardinal scale and it must be maximised.

**Environmental criteria:**
- at local level:
  - **impact on air quality,** which indicates risks for health by air pollution; it is measured on cardinal scale and it must be minimised;
  - **impact on water quality,** which reflects contamination danger of superficial waters and of layers; it is measured on cardinal scale and it must be minimised;
  - **impact on landscape,** which synthesise loss of amenity and aesthetic beauty as a consequence of realised alternatives; it is measured on ordinal scale and it must be minimised;
- **global effect,** which represents climatic change from the greenhouse effect consequent to each alternative; it is measured on ordinal scale and it must be minimised;
- **Impact on ecosystem,** which reflects risks on flora and fauna from the exploitation of different alternatives; it is measured on ordinal scale and it must be minimised.

**Technical criteria:**
- **implementation time,** which indicates necessary time to make each alternative operate; it is measured on ordinal scale (months) and it must be minimised;
- **residual waste,** which represents the quantity of waste, for each alternative, which cannot be furtherly treated; they are measured on cardinal scale (percentage of waste sent to the discharge) and they must be minimised;
• Risk of serious accidents, estimated on the base of experience for each alternative; measured on cardinal scale, it must be minimised.

3 Matrix for Estimation and Warning

Once you have defined the alternatives (four) and chosen estimation criteria (twelve), it is possible to build an impact matrix (Table 1). It corresponds to the table of information of the approach of rough sets, by containing only conditional criteria and evaluation of each alternative (descriptions):

The value set of all ordinal criteria is the following one (codification used have been indicated between brackets): <very low (mb), low (b), more or less low (pmb), medium (me), more or less high (pma), high (a), very high (ma)>. These seven levels represent increasing or decreasing preferences respectively according to the fact that the corresponding criterion is maximised or minimised. As it is possible to easily observe, in the considered matrix, quantity (cardinal scales) and quality information (ordinal scales) coexists.

Table 1. Estimation matrix

<table>
<thead>
<tr>
<th>Altern.</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
<th>C12</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>0.6</td>
<td>0.8</td>
<td>ma</td>
<td>a</td>
<td>a</td>
<td>95</td>
<td>0.18</td>
<td>38.6</td>
<td>957</td>
<td>0</td>
<td>mb</td>
<td>12</td>
</tr>
<tr>
<td>I</td>
<td>0.8</td>
<td>0.4</td>
<td>a</td>
<td>pma</td>
<td>ma</td>
<td>25</td>
<td>1</td>
<td>102</td>
<td>2734</td>
<td>6736</td>
<td>pma</td>
<td>24</td>
</tr>
<tr>
<td>I+C</td>
<td>0.6</td>
<td>0.35</td>
<td>a</td>
<td>pma</td>
<td>a</td>
<td>14.8</td>
<td>0.65</td>
<td>93</td>
<td>2991</td>
<td>5037</td>
<td>a</td>
<td>30</td>
</tr>
<tr>
<td>R</td>
<td>0.4</td>
<td>0.3</td>
<td>me</td>
<td>pmb</td>
<td>mb</td>
<td>25.9</td>
<td>0.1</td>
<td>44</td>
<td>2551</td>
<td>2252</td>
<td>ma</td>
<td>30</td>
</tr>
</tbody>
</table>

Legend

Alternatives: D: Discharge; I: Incinerator; I+C: Incinerator and composing plant; R: Recycling

Conditional criteria: c1: impact on air quality; c2: impact on water quality; c3: impact on landscape; c4: impact on ecosystem; c5: global effect; c6: residual waste; c7: risk of serious accident; c8: investment cost; c9: exploitation cost; c10: earnings; c11: local employment; c12: implementation time.

4 Analysis of Rough Sets

We remind that from the analysis of the table of decisions by means of the rough sets, it is possible to obtain information on the hereby-contained preferences that result to be better at the increasing of the number of considered alternatives and criteria.
Into the mentioned work, however, we have quoted even an “equity matrix”, in which considered alternatives are estimated with linguistic variables from ten different local interest groups (DM), precisely:

the so-called DM, that is to say groups directly involved in the decisional process: Ministry of Environment, Sicilian regional Government, Local administrations, Citizens' Administrations, Local agricultural Association, Municipalised bodies, Tourism association; pressure groups, that is to say subjects that try to have an influence on the progress according to their own interests: Local groups of environmental safety (NGOs), Lobbies, Future generations.

Estimation of these “experts” are independent from data quoted in the table of decisions and have been independently expressed from each DM, that is to say without knowing judgement of other groups; they have been directly detected by means of conversations, interviews, discussions with each one of the considered groups and/or by means of consulting of proper material and they have been quoted into Table 2.

<table>
<thead>
<tr>
<th>Altern.</th>
<th>d1</th>
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<th>d3</th>
<th>d4</th>
<th>d5</th>
<th>d6</th>
<th>d7</th>
<th>d8</th>
<th>d9</th>
<th>d10</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>mc</td>
<td>mc</td>
<td>c</td>
<td>pmc</td>
<td>pmc</td>
<td>pmc</td>
<td>mc</td>
<td>pmc</td>
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<td>b</td>
<td>pmc</td>
<td>mc</td>
<td>c</td>
<td>pmc</td>
<td>me</td>
<td>c</td>
<td>me</td>
<td>pmc</td>
</tr>
<tr>
<td>I+C</td>
<td>mb</td>
<td>b</td>
<td>b</td>
<td>pmc</td>
<td>pmc</td>
<td>pmc</td>
<td>b</td>
<td>pmc</td>
<td>pmc</td>
<td>b</td>
</tr>
<tr>
<td>R</td>
<td>mb</td>
<td>mb</td>
<td>pmc</td>
<td>mb</td>
<td>mb</td>
<td>mb</td>
<td>mb</td>
<td>pmc</td>
<td>pmc</td>
<td>pmc</td>
</tr>
</tbody>
</table>

Legend

Alternatives: D: Discharge; I: Incinerator; I+C: Incinerator and composting plant; R: Recycling

Deciders: d1: Ministry of environment; d2: Sicilian regional government; d3: Local Administrations; d4: Citizens' association; d5: Local agricultural association; d6: Municipalised bodies; d7: Local groups of environmental safeguard (NGOs); d8: Tourism association; d9: Future generations; d10: Lobbies

Estimation: b: good; c: bad; ec: extremely bad; mb: very good; mc: very bad; me: medium; pmc: more or less good; pmc: more or less bad.

5 Pet and Indiscernibility

Therefore, by considering, as distinct criteria the above mentioned evaluation of the experts, it is possible to build ten tables of decisions having each one as conditional criteria, the twelve ones already considered and as decisional attribute respectively each one of the ten above mentioned groups.

This way to operate, however, does not solve the problem of the limited number of alternatives (always four) and it only allows a traditional analysis of rough sets (multiattribute classification based on relation of indiscernibility). Two alter-
natives are said *indiscernible* if they show the same description in terms of considered conditional criteria. It is then preferable to build ten PCT. In each one of these, the lines contain all the possible (12) ordered pairs of the four alternatives, the first twelve columns conditional criteria, the thirteenth column the decisional attribute.

To make a model of graduated preferences, which constitute the values of PCT, we have proceeded as follows. With reference to conditional criteria, we have built the rankings of alternatives with respect to each one of them, by assigning whole values from 1 to 4, and precisely the value 1 to the best alternative (first place in the ranking) and the value 4 to the worst one (fourth place in the ranking); the degree of preference of each relation is then given by the opposite of the difference of values of ranges of two examined alternatives in the ranking, compared to the considered criterion. We have proceeded in this way even with reference to quantity criteria (cardinal scales); as a matter of fact, if on one hand this operation makes you lose information on “preference intensity” (calculated, for example, as difference between cardinal values of the two alternatives), on the other hand you avoid the problem of introduction of thresholds for model building of preferences and you avoid that small imprecision on data detecting can have important effects on results.

As regards to the decisional attribute, you build the ranking of the alternatives on the base of quality evaluation (ordinal) given by each expert (10 total orders) and you associate to each pair of (x,y) alternatives an outranking relation S or its negation $S'$, on the base to what has been previously observed. The partial rankings with respect to the 12 conditional criteria and to global arrangements assigned by each expert (decisional criteria of, i=1,2,...,10) are quoted in Table 3.

It is possible to observe some correspondences between these rankings, precisely between deciders of local agricultural Association (d5) and Tourism association (d8) and among deciders NGOs (d7) and Future generations (d9).

**Table 3.** Ranking with respect to conditional criteria and to decisional groups.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>c1</th>
<th>c2</th>
<th>c3</th>
<th>c4</th>
<th>c5</th>
<th>c6</th>
<th>c7</th>
<th>c8</th>
<th>c9</th>
<th>c10</th>
<th>c11</th>
<th>c12</th>
<th>d1</th>
<th>d2</th>
<th>d3</th>
<th>d4</th>
<th>d5</th>
<th>d6</th>
<th>d7</th>
<th>d8</th>
<th>d9</th>
<th>d10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discharge</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>1</td>
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<td>4</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Incinerator</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>4</td>
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<td>3</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td></td>
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<tr>
<td>Inc. + Comp.</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>1</td>
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<tr>
<td>Recycling</td>
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<td>1</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

The corresponding 10 PCT are represented into a unique Table 4, where the 12 lines are referred to ordered pairs of distinct alternatives, the first 12 columns represent preference degrees compared to each one of the conditional criteria, calculated as above mentioned, and the remaining 10 columns collect decisional attributes concerning global assignment (codified by “outranks” and “does not outrank”) of each one of the ten experts, respectively indicated with di, i=1,2,...,10. The analysis of the 10 PCT (one for each expert) thus built have been carried out by considering only a decisional criterion $d_i$ each time, that is to say the opinion of one group per time.
Table 4. PCT with ordinal data

<table>
<thead>
<tr>
<th>(x,y)</th>
<th>c1</th>
<th>c2</th>
<th>c3</th>
<th>c4</th>
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<th>c6</th>
<th>c7</th>
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<th>d9</th>
<th>d10</th>
</tr>
</thead>
<tbody>
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<td>D,1</td>
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<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>2</td>
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<td>C</td>
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<td>C</td>
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Upper and lower approximations of the two decisional classes \((S,C)\) have been calculated by using relation of indiscernibility (classic approach). Obtained results show a quality of classification equal to one per each DM (that is to say that all the alternatives are always correctly classified). This result, the best obtainable one, however does not surprise if you take into consideration the reduced number of objects to be classified. As a consequence, even the decisional rules are all certain. We calculated a group of minimal decisional rules for each expert, which (almost all) have the premise constituted by only one describer, as a consequence of what above observed. Hereby we supply some examples of these rules among the most significant ones; the number in parenthesis indicates the force of each rule, that is to say the number of object that supports this same rule.

\[
d_1: \text{Ministry of the environment}
\begin{align*}
\text{if (impact on landscape = -1)} & \rightarrow (\text{Dec} = C) & (3) \\
\text{if (impact on landscape = 1)} & \rightarrow (\text{Dec} = S) & (3) \\
\text{if (impact on water quality = -2)} & \rightarrow (\text{Dec} = C) & (2) \\
\text{if (impact on air quality = 0)} & \rightarrow (\text{Dec} = S) & (4)
\end{align*}
\]

\[
d_3: \text{Local administrations}
\begin{align*}
\text{if (residual waste = 1)} & \rightarrow (\text{Dec} = S) & (3) \\
\text{if (residual waste = -1)} & \rightarrow (\text{Dec} = C) & (3) \\
\text{if (impact on air quality = 0)} & \rightarrow (\text{Dec} = S) & (4)
\end{align*}
\]

\[
d_{10}: \text{Lobbies}
\begin{align*}
\text{if (impact on air quality = -2) and} \\
\text{if (impact on water quality = -2)} & \rightarrow (\text{Dec} = C) & (1) \\
\text{if (impact on air quality = 0)} & \rightarrow (\text{Dec} = S) & (4) \\
\text{if (global effect = -2)} & \rightarrow (\text{Dec} = S) & (2) \\
\text{if (global effect = 2)} & \rightarrow (\text{Dec} = C) & (2) \\
\text{if (residual waste = 3)} & \rightarrow (\text{Dec} = S) & (1) \\
\text{if (investment cost = 2)} & \rightarrow (\text{Dec} = C) & (2)
\end{align*}
\]
As title of example, we supply the interpretation in common language respectively of the first two rules of decider d1:

if the degree of preference of the alternative a on the alternative b compared to the impact on landscape is of degree −1 (therefore a is not preferred to b), then the alternative a does not outrank (that is to say is not as much as good as it) the alternative b; if the degree of preference of the alternative a on the alternative b compared to the impact on landscape is of the degree of 1 (therefore a is preferred to b), then the alternative a outranks (that is to say is at least as much as good as it) the alternative b.

It is interesting too to carry out an elementary “meta-analysis” on quality and quantity of criteria that each decider has effectively used (that is to say sufficient to build approximation) into describers of their own rules. Some have used only a criterion (Local agricultural association (d5), Tourism association (d8) and citizens’ association (d4), which have taken into consideration only air quality); other ones, two criteria (Ministry of environment (d1), Sicilian regional government (d2), NGOs (d7), Future generations (d9) and Municipalised bodies (d6), which have used only air quality and the impact on landscape; local Administrations (d3) have, on the contrary, considered only air quality and residual waste). Only a decider (Lobby, d10) has used 5 criteria and a rule, which the predecessor composed of two describers.

It is particularly meaningful to observe that the rule “if (impact on air quality = 0) → (Dec = S)” has been adopted by all DsM; this rule can be well defined, therefore, the “rule of the rules”. In any case, the air quality clearly reveals the most used and most important decisional criterion (the “core”), present in rules of three deciders that have used it alone with all the levels of preference (from −3 to 3).

In conclusion, remark that Lobby Dm (d10) is the only one that apparently shows inconsistency into preferences. As a matter of fact, it adopts, among the others, the following rules:

if (global effect = −2) → (Dec = S),
if (global effect = 2) → (Dec = C).

It should be expected that, on the base of the principle of the preference monotony, if estimation of an alternative compared to a criterion improves, comparative judgement of the considered alternative should not get worse. It is, on the contrary, what happens in the hereby mentioned rules: the degree of preference of a on b compared to the global effect ranges from −2 to 2 (strong improvement in favour of a), however preference is inverted (from “a outclasses b” and “a does not outclass b”). A deeper analysis of corresponding profiles into the PCT reveals that, really, the dominance principle has not been violated, as the preferences concerning examined alternatives are inverted compared to criteria of costs and implementation time (that is to say it there not dominance). This more simply means that the Lobby Dm implicitly assigns a great importance to these criteria. The presence of this particular inconsistency, as previously remarked, cannot be stressed by the approach of the rough sets based on indiscernibility relation; only the use of dominance relation for approximation of classes can, as a matter of fact, detect the eventual presence of this type of great inconsistency into Dm’s preferences.
7 Conclusions

We have briefly shown some applications of rough sets to particular decisional problems in the field of management of solid urban waste. Despite the limited number of alternatives present in the sample of the study case, we have obtained results of a certain application interest, which have stressed methodological features of this instrument of data analysis and some different aspects of preferences of considered DMs, which could have been difficultly stressed with other techniques. This information, easily comprehensible as expressed in natural language, explain the main and simplified descriptions of reasons of taken decisions and can be then to applied decisional problems concerning new alternatives on the base of knowledge of preference of DMs extracted from used decisions' table. In the whole approach, we have not used technical parameters or information on preferences explicitly required to DMs and, then, difficult to obtain from them and to directly insert them as input of the model. All this information is, on the contrary, obtained as results (output) of the carried out analysis, on the base of preferences effectively expressed by DM on a sample of cases, as knowledge extracted from the table of decisions of the studied problem and which can be usefully applied to help DM to efficiently solve similar decisional problems, concerning new alternatives.

In other terms, when we will pass to the operational phase and face the analysis of a real problem concerning solid waste disposal, all information obtained from the performed analysis (reducts, rules of decisions) can be immediately applied, both to take into consideration only the attributes (criteria) that are considered more relevant (that is to say the reducts), with great saving of time and detecting costs, both to build ranking or to make a choice with reference to the new set of alternatives taken into consideration. The DM, in fact, after having discussed and acknowledged the decisional rules previously obtained from the above mentioned analysis, will have no difficulty to accept the same rules, which explicit its own preferences, as guide for its future action, as concrete and powerful support to decisional activity.

References