Replacement services for planned maintenances of the infrastructure of a public transport system

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Abstract

This paper deals with planning the replacement services in case of a scheduled maintenance of parts of the infrastructure of a public transport system. The considered maintenances last over a long period, e.g., days or weeks, and involve the blockade of some segments of the affected line. As a consequence, replacement services have to be planned thoroughly. These services typically include the establishment of additional lines and augmented frequencies of some existing lines. For this purpose, a mixed integer programming model is developed. The model is then used to solve a case of the city of Vienna using CPLEX.

Introduction

For a public transport system maintenance measures are needed on a regular basis to keep the system functioning. Some tasks may be processed over night when the particular part of the system is not in use. However, occasionally large maintenance measures have to be conducted that involve the blockade of some segments of the infrastructure for several days or even weeks. In the latter case, appropriate replacement services have to be planned.

When looking at a city, there are typically several modes of transport available, e.g., bus, tramway, and metro. In case of a blockade of a metro line, for example, people may resort to another mode of transport. Hence, the frequencies of the existing lines have to be re-planned. Besides the amplification of existing lines, some additional lines might have to be implemented in order to guarantee a proper passenger flow through the system without a significant increase of the travel times compared to the original situation. The typical example for replacement services are extra bus lines, as buses can be operated on most roads and do not rely on the existence of tracks. The planning of these extra lines consists of setting their routes and their frequencies. Subsequent planning tasks include timetabling and scheduling aspects, which will not be addressed in this paper, though.

Particularly with regard to the case of replacing blocked metro lines by buses one has to note that the capacity of a bus is significantly less than the one of a metro train. Given the typical constraint of a minimum headway along bus lines, preserving the same passenger flow as in the original situation may still be a problem even with extra bus lines. The establishment of replacement services may therefore include additional tasks for the operator, such as informing the customers about the planned measures and suggesting different routes in order to partly re-direct the passenger flow. In this paper, we focus solely on the line planning aspect and assume that the passengers take the provided frequencies along the lines into account and adapt their routes accordingly.

Public transport planning is typically divided into several tasks, including network planning, frequency setting, timetabling, rolling stock scheduling and driver scheduling (Ceder & Wilson, 1986). These tasks are either processed in the mentioned order or some are solved simultaneously. We refer to the comprehensive survey by Guihaire and Hao (2008) for an overview of problems, objectives and approaches in the field of public transport planning. The planning tasks considered in this paper are closely related to standard line planning. For an overview of different models and approaches in line planning we refer to the survey by Schöbel (2012). In line planning, the transportation network is given, i.e., the stations and the streets or tracks connecting them are fixed. Line planning then consists of setting the number of lines, their routes and their respective frequencies. Thereby, constraints have to be taken into account to guarantee feasibility. Typical objectives are either passenger- or cost-oriented. Passenger-centered objectives include the minimization of travel times and the maximization of direct travelers, i.e., travelers that need no transfers. Cost-centered models may incorporate additional constraints in order to provide a minimum service quality, while fixed and/or variable costs for operating the lines are minimized.

The problem addressed in this paper deals with planning the replacement services in case of planned maintenances with a blockade of a line segment for several days or weeks. The tasks involve setting the frequencies of existing lines, potentially re-directing or extending them and establishing new lines for the replacement services. The passenger flow in the transport system is also part of the planning. The objective is to minimize the total travel times of the passengers including transfers. Costs are implicitly considered by limiting the number of additional vehicles for the replacement services. Therefore, a Mixed Integer Linear Programming (MILP) model is developed, which can be used to solve instances of a reasonable size by using a commercial solver like CPLEX. The model shares some features with our model for line planning in case of unforeseen disruptions (Kiefer, Kritzinger, & Doerner, 2016). In our previous work we considered only the demand along the arcs in the network and we were therefore unable to determine the passenger flow conversely to the paper at hand. Moreover, since we dealt with unforeseen disruptions, we had to model the allocation of vehicles from operating lines in order to install the replacement services more quickly. In contrast, when considering blockades as part of planned activities, the public transport provider is able to allocate vehicles from depots.

Problem description

The considered problem consists of planning the replacement services for a large public transport network with different modes of transport. The network is defined on a directed graph, where the nodes correspond to the stations and two stations might be connected by a directed arc. The weight of the arcs indicates the respective travel time between the stations. Hence, each mode of transport has its own network. The direction of the arcs is specified by the infrastructure. In some cases, the physical station has to be split into several artificial nodes, as explained later.

Besides the blocked line under maintenance for which the replacement services are needed, there are several operating lines in the system. The term line refers to a directed closed path in the network. Hence, a line is characterized by the stations on the path and the order of their visits. Note that defining a line as an ordinary path is generally insufficient, as due to limitations of the infrastructure visiting the same stations on both directions may not always be possible. It is reasonable that only those parts of the network need to be considered for the planning that are close to the blocked line.

The aim is to find a proper replacement service by planning extra bus lines, adjusting the frequencies of all lines and potentially extending existing lines. Thereby, vehicles are generally allowed to turn short at a station, if the infrastructure provides this possibility. Passenger flows are incorporated and are also part of the planning. Therefore, an origin-destination-matrix (OD-matrix) of the passengers for each pair of stations is used. The objective is to minimize the travel time of the passengers including a penalty for each transfer, subject to restrictions regarding a minimum headway of the lines and a maximum utilization of the arcs. Moreover, the fleet size for each mode of transport is given. Note that the consideration of planned blockades allows the allocation of vehicles from depots, which might not be possible in case of unplanned disruptions.

The problem can be formulated as a MILP model. The notation is listed below. With regard to the set of stations one has to note that occasionally physical stations have to be split into several artificial ones, e.g., if two tramway tracks intersect at a station but the infrastructure does not allow a turn from one track to another. Hence, the set of all nodes I that includes the artificial stations differs from the set of physical stations G. A starting node σ_k has to be defined for each line k which is then used for the connectivity constraints of the lines.

I	Index set of the nodes/stations
N	Index set of the transport modes
G	Index set of the physical stations
$\mathcal{F}$	Set system; each set corresponds to a physical station with its artificial stations as elements
W_n	Index set of stations, where a vehicle of type n is able to turn, $W_n = \{i \in I : \tau_{in} < \infty\}$
A_{in}	Stations with an incoming connection from station i, $A_{in} = \{ j \in I : t_{ijn} < \infty\}$
E_{in}	Stations with an outgoing connection towards station i, $E_{in} = \{ j \in I : t_{jin} < \infty\}$
K	Index set of lines
K_n	Index set of lines of mode n (partition of K)
g_{ijn}	Maximal number of vehicles of type n on the arc (i, j) per time unit
h_{ijn}	Minimal number of vehicles of type n on the arc (i, j) per time unit
κ_n	Number of available vehicles of type n
f_n	Capacity of a vehicle of type n

t_{ijn} Travel time of a vehicle of type *n* on the arc (*i*, *j*)
τ_{in} Time needed of a vehicle of type *n* for turning at station *i*
σ_k Starting node of line *k*
d_{lm} Demand according to the OD-matrix from the physical station *l* to the physical station *m*
α_k Penalty of a transfer to line *k* per passenger
β Penalty of the slack variables

The following decision variables are used in the model. The variables a_{ijk} and u_{ik} are incorporated to model the connectivity constraints and will be explained together with the constraints.

$$x_{ijk} = \begin{cases} = 1 & \text{if line } k \text{ traverses arc } (i,j) \\ = 0 & \text{otherwise} \end{cases} \quad \forall\ n \in N,\ k \in K_n,\ i \in I,\ j \in A_{in}$$

y_{ijk} number of vehicles of k serving (i,j) per time unit $\forall\ n \in N,\ k \in K_n,\ i \in I,\ j \in A_{in}$
r_{ik} number of vehicles of k turning at i per time unit $\forall\ n \in N,\ k \in K_n,\ i \in W_n$
u_{ik} variables for connectivity constraints $\forall\ i \in I,\ k \in K$

$$a_{ijk} = \begin{cases} = 1 & \text{increase } u_{jk} \text{ with respect to } u_{ik} \\ = 0 & \text{no increase} \end{cases} \quad \forall\ n \in N,\ i \in I,\ j \in A_{in},\ k \in K_n$$

z_{ijklm} passenger flow on arc (i,j) and line k of passengers traveling from l to m
$\forall\ n \in N,\ k \in K_n,\ i \in I,\ j \in A_{in},\ l \in G,\ m \in G,\ l \neq m$

v_{gklm} number of passengers traveling from l to m that transfer at station g to line k
$\forall\ n \in N,\ k \in K_n,\ l \in G,\ m \in G,\ l \neq m,\ g \in G,\ g \neq m$

s_{ijk} slack for capacity on (i,j) of k $\forall\ n \in N,\ k \in K_n,\ l \in G,\ m \in G,\ i \in I,\ j \in A_{in}$

The following objective function minimizes the sum of the travel times of the passengers and the penalty for transfers. A reasonable weight for the transfers is a usual transfer time and eventually a measure for the inconvenience that might depend on the involved line. Finally, the slack variables representing insufficient capacity are also added to the objective function with a high weight.

$$\min \sum_{\substack{n\in N, k\in K_n, i\in I, j\in A_{in} \\ l,m\in G, l\neq m}} t_{ijn} \cdot z_{ijklm} + \sum_{\substack{n\in N, k\in K_n \\ g,l,m\in G, l\neq m, g\neq m}} \alpha_k \cdot v_{gklm} + \sum_{\substack{n\in N, k\in K_n \\ i\in I, j\in A_{in}}} \beta \cdot s_{ijk}$$

s.t.

$$y_{ijk} \leq g_{ijn} x_{ijk} \quad \forall\ n \in N,\ k \in K_n,\ i \in I,\ j \in A_{in} \tag{1}$$

$$y_{ijk} \geq h_{ijn} x_{ijk} \quad \forall\ n \in N,\ k \in K_n,\ i \in I,\ j \in A_{in} \tag{2}$$

$$\sum_{j\in A_{in}} x_{ijk} \leq 2 \quad \forall\ n \in N,\ k \in K_n,\ i \in I \tag{3}$$

$$\sum_{j\in A_{in}} x_{ijk} = \sum_{j\in E_{in}} x_{jik} \quad \forall\ n \in N,\ k \in K_n,\ i \in I \tag{4}$$

$$\sum_{j\in A_{in}} y_{ijk} = \sum_{j\in E_{in}} y_{jik} \quad \forall\ n \in N,\ k \in K_n,\ i \in I \tag{5}$$

$$(|I|-1) \cdot \sum_{j\in A_{in}} a_{ijk} \geq \sum_{j\in A_{in}} x_{ijk} \quad \forall\ n \in N,\ k \in K_n,\ i \in I \tag{6}$$

$$a_{ijk} \leq x_{ijk} \quad \forall\ n \in N,\ k \in K_n,\ i \in I,\ j \in A_{in} \tag{7}$$

$$u_{ik} - u_{jk} + 2 \cdot a_{ijk} - 1 - (1 - x_{jik}) \cdot \left(1 - \frac{2}{|I|-1}\right) \leq (|I|-2) \cdot (1 - x_{ijk})$$
$$\forall\ n \in N,\ k \in K_n,\ i \in I,\ j \in A_{in} \setminus \{\sigma_k\} \tag{8}$$

$$\sum_{i\in I, j\in A_{in}, k\in K_n} t_{ijn} y_{ijk} + \sum_{i\in W_n, k\in K_n} \tau_{in} r_{ik} \leq \kappa_n \quad \forall\ n \in N \tag{9}$$

$$y_{lik} - \sum_{j\in A_{in}\setminus\{l\}} y_{ijk} \leq \begin{cases} r_{ik} & \text{if } i \in W_n \\ 0 & \text{otherwise} \end{cases} \quad \forall\ n \in N,\ k \in K_n,\ l \in I,\ i \in A_{ln} \tag{10}$$

$$\sum_{k \in K_n} y_{ijk} \leq g_{ijn} \quad \forall\, n \in N,\ i \in I,\ j \in A_{in} \tag{11}$$

$$\sum_{l,m \in G} z_{ijlmk} \leq f_n \cdot y_{ijk} + s_{ijk} \quad \forall\, n \in N,\ k \in K_n,\ i \in I,\ j \in A_{in} \tag{12}$$

$$\sum_{\substack{n \in N, k \in K_n \\ i \in \mathfrak{F}_g, j \in A_{in}}} z_{ijklm} = \sum_{\substack{n \in N, k \in K_n \\ i \in \mathfrak{F}_g, j \in E_{in}}} z_{jiklm} \quad \forall\, l, m \in G,\ l \neq m,\ g \in G \setminus \{l, m\} \tag{13}$$

$$\sum_{n \in N, k \in K_n, i \in \mathfrak{F}_l, j \in E_{in}} z_{jiklm} = 0 \quad \forall\, l, m \in G,\ l \neq m \tag{14}$$

$$\sum_{n \in N, k \in K_n, i \in \mathfrak{F}_m, j \in A_{in}} z_{ijklm} = 0 \quad \forall\, l, m \in G,\ l \neq m \tag{15}$$

$$\sum_{n \in N, k \in K_n, i \in \mathfrak{F}_l, j \in A_{in}} z_{ijklm} = d_{lm} \quad \forall\, l, m \in G,\ l \neq m \tag{16}$$

$$v_{gklm} \geq \sum_{\substack{i \in \mathfrak{F}_g \\ j \in A_{in}}} z_{ijklm} - \sum_{\substack{i \in \mathfrak{F}_g \\ j \in E_{in}}} z_{jiklm} \quad \forall\, g, l, m \in G,\ l \neq m,\ g \neq m,\ n \in N,\ k \in K_n \tag{17}$$

$$x_{ijk}, a_{ijk} \in \mathbb{B} \quad y_{ijk}, r_{ik}, u_{ik}, z_{ijklm}, v_{gklm}, s_{ijk} \in \mathbb{R}_0^+$$

Constraints (1) and (2) guarantee that an arc is traversed by a line if and only if vehicles of the line serve that arc. Moreover, Constraints (2) ensure that a particular service level is achieved if an arc is served by a line. Constraints (3) make sure that a line leaves a station in at most two directions, typically the forward and the backward direction. Undesired splits of a line are prohibited in combination with Constraints (4) that enforce the number of outgoing connections to equal the number of incoming connections at a station. The vehicle flow is conserved by Constraints (5). This formulation generally allows a line to not necessarily follow the same path in both directions. Rather it is possible that some stations are visited only in one direction, which can also be seen in practice.

Constraints (6-8) ensure the connectivity of the lines and are inspired by the connectivity constraints by Miller et al. (1960) for the vehicle routing problem. Each station i along the line k has a label indicated by variable u_{ik}. The binary variable a_{ijk} indicates an increment of the label of station j w.r.t. the label of station i. Starting from station i, the label of any neighboring station along the same line has to be incremented w.r.t. i, i.e. $a_{ijk} = 1$ for at least one station j (Constraints 6). However, a_{ijk} may only take the value 1 if line k traverses arc (i, j) (Constraints 7). Finally, the actual increment of the label of station j w.r.t. station i is ensured by Constraints (8) in a way that u_{jk} is incremented w.r.t. u_{ik} if $a_{ijk} = 1$ and the reverse direction is also traversed by line k. If $a_{ijk} = 0$, u_{jk} may be decremented. The starting node σ_k is omitted in this constraint. Therefore, the model will try to increment the labels in the direction of the starting node, as otherwise this group of constraints yield a conflict, in particular $u_{ik} < u_{jk}$ and $u_{jk} < u_{ik}$. Hence, lines that are not connected with the starting node induce a conflict and are thereby prohibited. The last terms of the left-hand sides of Constraints (8) are needed to regard the case of lines incorporating segments that are traversed in only one direction.

Constraints (9) guarantee that the maximum fleet sizes are respected. Therefore, the number of vehicles serving an arc and turning at a station is weighted by the respective time requirement. Turns are identified by Constraints (10) if fewer vehicles leave a station in any other direction than the station where the vehicles originally came from. Constraints (11) ensure that the maximum utilization of an arc is not exceeded. Generally, the passenger flow should be less than the provided capacity for each arc and each line, as stated by Constraints (12). In order to guarantee the feasibility of the model a slack variable is added to the provided capacity, which is highly penalized in the objective function.

The passenger flow is modeled as a multi-commodity flow in a straight forward way: The passengers have to leave their origin in the number specified by the OD-matrix (Constraints 16). They must not return to their origin and must not leave the destination (Constraints 14 and 15). Constraints (13) ensure the flow conservation. Finally, the transfers are measured by Constraints (17).

The non-binary decision variables are allowed to take real values. This is reasonable for the practical application, as the OD-matrix is typically only an estimate of the real demand. Moreover, this model basically tries to support the decision of practitioners. With regard to the frequencies one has to note that they have to be rounded to convenient time units when it comes to the practical implementation. Moreover, adjustments might be needed to realize an integer fleet size per line.

Case study

The model is applied to the case of planning the replacement services for a blockade of the metro line U4 in Vienna due to modernization. Therefore, the network is reduced by excluding irrelevant nodes. The considered network incorporates 47 stations. There are four modes of transport, including metro, train, tramway and bus with 4, 3, 6 and 3 lines, respectively. The bus lines consist of one regular line and two extra lines. The available fleet size for metro, train, tramway and bus is 60, 20, 30 and 30, respectively. Walking passages can be easily incorporated by adding another mode of transport. While this extension may be beneficial in some cases, computational experiments showed that it could not improve the results of this particular scenario.

For this study, only the OD-matrix for the metro stations was provided. However, the demand for the metro lines refers to the largest share of the total demand for public transport in Vienna. Still, when looking at the results one has to be aware that extra vehicles are needed on the non-metro lines for their regular demand. As the network is reduced, the OD-matrix has to be adjusted. Therefore it is assumed that the passengers are able to move in the undisturbed/unconsidered part of the system without limitations. Additionally, for simplicity it is assumed that the passengers enter or exit the considered part of the system where the respective shortest path given a generally undisturbed system would suggest. The OD-demand of the respective pair of stations in the considered part is increased accordingly. For example, assume that the shortest path of the passengers traveling from station *A* to station *C* according to the OD-matrix is along the stations *A-B-C* in an undisturbed system. If station *A* is outside but *B* and *C* are inside the considered system, the OD-matrix is adjusted in a way that the passengers traveling from *A* to *C* enter the considered system at *B*. The entry in the OD-matrix corresponding to *B-C* is therefore increased by *A-C*. The model is solved via CPLEX within a few hours. The solution is shown in the Figures 1 and 2. The blockade occurs along the section *Huetteldorf - Schoenbrunn*. The black arcs refer to the newly established bus lines, while the gray arcs refer to all other lines. Figure 1 illustrates the routes of the new bus lines, where high frequencies are indicated by thick arcs. The two new bus lines replace the blocked metro section. One of them should even go as far as *Westbahnhof*, a major hub.

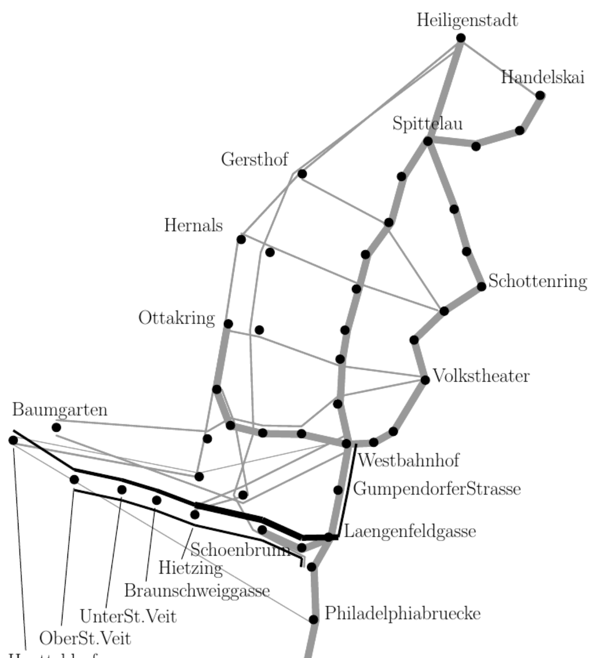

Figure 1. Frequencies

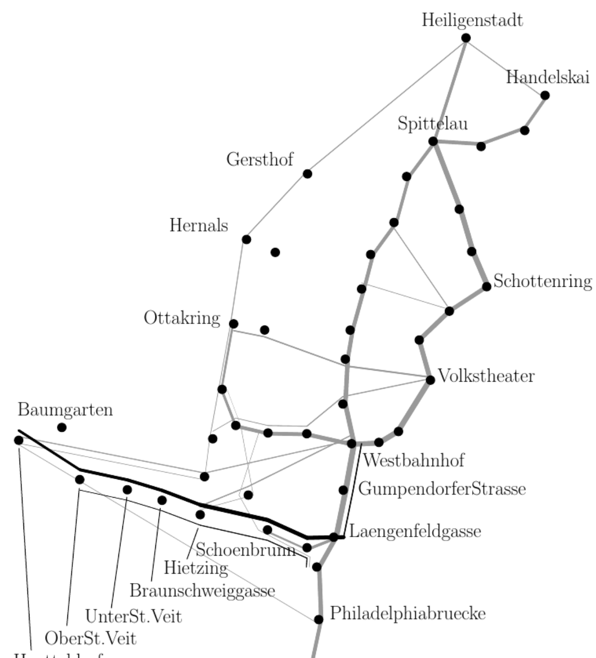

Figure 2. Passenger flow

Table 1 shows the headways of the new bus line *new1*. It can be seen that some buses have to turn short in order to establish the suggested headways. The headways of the other new bus line are 10 min on every arc. In Figure 2 the passenger flow is shown. A high demand along the arcs is represented by thick arcs. Clearly, the working metro lines show a high passenger flow. From the figure it can also be seen that some passengers that are affected by the metro blockade resort to other modes of transport than the new bus lines. In particular, some passengers are supposed to travel along a train line from *Huetteldorf* to *Westbahnhof*.

Table 1. Headways of the new bus lines (*new1*)

Huetteldorf – Ober St.Veit	10 min
Ober St.Veit – Unter St.Veit	6 min
Unter St.Veit – Braunschweigg.	6 min
Braunschweigg. – Hietzing	6 min
Hietzing – Schoenbrunn	4 min
Schoenbrunn – Meidling Hauptstr.	4 min
Meidling Hauptstr. – Laengenfeldg.	4 min
Laengenfeldg. – Gumpendorfer Str.	10 min
Gumpendorfer Str. – Westbahnhof	10 min

The results so far referred to the daily demand of passengers in the system. Hence, they give a good recommendation about the line plan for the average case. However, the passenger flow varies significantly over the day. Therefore, when it comes to the headway planning of the actual operation, one has to take the hourly OD-matrices into account. For that purpose, we fixed the routes of the lines suggested for the average case and computed the headways on the basis of the hourly OD-matrices for four different times of the day, including morning peak, midday off-peak, evening peak and late evening off-peak. The results confirm the intuition that the headways have to be reduced considerably during the rush hours compared to the off-peak situation. Due to the extent of the results, they cannot be presented in detail.

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