Using multi-start biased randomization of heuristics to solve the vehicle routing problem with clustered backhauls

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Abstract

We consider the Vehicle Routing Problem with Backhauls (VRPB), where delivery and pick-up customers are to be served from a central depot. In particular, the group or cluster of delivery customers has to be served before the first pickup customer can be visited. Thus, the problem belongs to a sub-class called VRP with Clustered Backhauls (VRPCB). Our resolution procedure uses a multi-start approach designed to avoid the local minima and to be easily parallelizable. The algorithm employs a biased-randomized version of the classical savings heuristic, together with some local search processes. During the solution-construction process, the edges that connect one delivery customer with a pick-up customer are penalized to be chosen at a later stage. The savings list of edges is randomized using a skewed probability distribution. Some classical benchmark instances for the VRPB were selected in order to compare the efficiency of our approach.

Introduction

The Capacitated Vehicle Routing Problem (CVRP) is a very well-known model to help making decisions in logistics and transportation issues (Toth and Vigo, 2014). The classical version of the CVRP aims at generating the minimum cost set of routes for a fleet of vehicles that have to satisfy the customers’ requests. In this context, allowing not only deliveries but also pick-ups, then we are considering the General Pick-up and Delivery Problem (Savelsbergh and Sol, 1995). Thus, all goods delivered have to be loaded at the depot and all goods picked up have to be shipped to it. This class of problems is called Vehicle Routing Problem with Backhauls (VRPB). This paper focus on one of the VRPB variants: the VRP with Clustered Backhauls (VRPCB). Customers are either delivery or pick-up customers but cannot be both. Additionally, it is imposed that the cluster of delivery customers has to be served before the first pick-up customer can be visited. This cluster constraint is practically motivated by the fact that vehicles are often rear loaded. Hence, the on-board load rearrangement required by a mixed service is difficult—or even impossible—to carry out at customer locations. VRPB takes advantage of the unused capacity of a vehicle on the trip back to the depot and so, sender customers are considered as well. Our interest in the VRPB is motivated both by its great practical and theoretical importance. From the practical perspective, VRPB is frequently encountered by large companies who must transport goods from their production sites to other inter-company sites (linehauls). At the same time the production site must be supplied from vendors (backhauls) and/or other inter-company production sites located within the same geographic region (Goetschalckx and Jacobs-Blecha, 1989; Goetschalckx, 2011). A common example of backhaul loop can be found on linehaul customers that must return empty containers to their respective senders. Another example can be found in the grocery industry, where supermarkets and shops are the linehaul customers and grocery suppliers are the backhaul customers (Ubeda et al., 2011). In the retail industry, large companies own many outlets to be supplied from the depot, and at the same time, the depots must be resupplied by the vendors located in the same region by visiting backhaul customers in distribution routes (Golden et al., 1985). It has been widely recognized that in this mixed distribution-collection context, a significant saving in transportation costs can be achieved. Other applications can be considered: return of empty bottles, pallets, used batteries, etc.; delivery of new appliances accompanied by the pick-up of the old ones; recovery of defective or obsolete products, etc. The VRPB is also considered into the field of reverse logistics and Green Logistics (Demir et al., 2014) that raise the necessity of bi-directional product flows. Significant cost reduction can be achieved by combining linehaul with backhaul customers as this results in less empty routes back to the depot and therefore in the reduction of gas emissions, as shown in Ubeda et al. (2011), and Parragh et al. (2008).
The traditional approximation to solve this problem has been: firstly, solve the linehaul problem; secondly, solve the backhaul problem and thirdly, mix somehow both. Thus the first and the following one are the same and have extensively been documented in the literature, whereas the third one is specific for the backhauling problem. Deif and Bodin (1984) proposed the first heuristic algorithm in this area. These authors adapted the Clarke and Wright (1964) algorithm (CWS) to the VRPB case, where the cluster and precedence constraints (backhauls customers are only served when all deliveries to linehaul customers have been completed), reduce the number of feasible solutions to be considered. In other words, the final solution can only contain one combined edge that shifts the route from delivery to pickup mode. This edge is called interface. The saving of incorporating edge \((i, j)\) to the route is calculated as a piecewise function, where \(pS\) represents a penalty cost for being an interface and \(s_{ij}\) represents the traditional saving value as defined in Clarke and Wright (1964):

\[
S'_{ij} = \begin{cases} 
   s_{ij} - p \cdot S & \text{if edge } (i, j) \text{ is an interface} \\
   s_{ij} & \text{otherwise} 
\end{cases}
\]

Goetschalckx and Jacobs-Blecha (1989) started with the composite heuristics proposing an algorithm that uses the known procedure of space-filling curve transformation. Anily (1996) introduced a cluster-first route-second algorithm. Toth and Vigo (1999) introduced the idea of symmetric and asymmetric costs between two customers and proposed another cluster-first route-second algorithm for solving both the symmetric and asymmetric VRPB. Wade and Salhi (2002) introduced a new problem where according to the users’ experience, the vehicle capacity, the type of products and the type of vehicle used, it is possible to determine the position along a route from which the first backhaul customer may be visited.

Literature since 2000 is mainly devoted to metaheuristic methods. This is the case of Osman and Wassan (2002), who were the first ones in developing a Tabu Search (TS) algorithm for the VRPB. They proposed two route construction methods which are based on savings, insertion and assignment approaches. The results are then used with their Reactive Tabu Search (RTS) algorithm. Similar approaches were performed by Brandao (2006) with his TS algorithm, and Wassan (2007), who developed a TS enhanced by adaptive memory programming. Ropke and Pisinger (2006) unified VRPB problems as extension of what they called Rich Pick and Delivery Problem with Time Windows (PDPTW). Wassan et al. (2009) and Gajpal and Abad (2009) use an ant colony system to solve the problem. Zachariadis and Kiranoudis (2012) solve the problem by a metaheuristic local search approach. These authors explore the neighborhoods by exchanging variable-length customer sequences. Vidal et al. (2014) introduce a solution for multi-attribute vehicle routine problems. Attributes are characteristics or constraints that describe the specifications of real vehicle routing problem applications. For the tested cases, the algorithm matches or outperforms the current best known solutions. Finally, Cuervo et al. (2014) propose an oscillating local search heuristic with the main contribution of allowing transitions between feasible and infeasible portions of the solution space. Our literature review can be complemented with the thorough revision done by Toth and Vigo (2014).

**The proposed algorithm**

The high-level flowchart of our algorithm is shown in Figure 1. For each customer, the initial solution assigns one round-trip from the depot to that specific customer. Once this dummy solution is in place, an initial base solution is obtained through the savings algorithm by Clarke and Wright (1964). This initial calculation is adapted to backhauls using Deif and Bodin’s (1984) approach. This adaptation has been detailed in the Literature Review section. The idea behind this adaptation is to delay the eligibility of the interface links so that both formed linehaul and backhaul routes are “mature” enough to be combined. By penalizing the interface links savings value, the order of the savings list is altered. Deif and Bodin (1984) concluded that the best results in the adaptation of the algorithm were obtained by linearly decreasing the original saving of the interface edges with a quantity obtained by multiplying the maximum saving value by a numerical factor belonging to \([0.05, 0.20]\). The value we have used in our approach is 0.085 as this has obtained the best results among other values. Once the base solution is in place, a multi-start process is initiated. As discussed in Juan et al (2010), by performing a biased randomization of the savings list, edges are selected in a different order at each iteration of the multi-start process. At this point, a new feasible solution is ready. The obtained route can be improved by using the cache and splitting techniques described in Juan et al (2011). The process starts again with a new Randomized CWS solution until the multi-start finishing criteria is met. Notice that the use of this approach with randomized savings list contributes to escape from local minima.
To the best of our knowledge, the approach we have designed in this paper using multi-start biased randomization has never been used to solve VRPB. The algorithm we propose is based on an adaptation of the algorithm introduced by Juan et. al (2011). As the main contribution of this paper, we present a hybrid approach that has proved to be an efficient procedure for obtaining quasi-optimal solutions in small- and medium-size VRPB, namely: (a) it is relatively simple to implement, which facilitates its use in practical applications; (b) it is a robust and flexible methodology that can be easily adapted to consider additional constraints and costs; (c) it is able to generate a set of alternative good solutions in a reasonable time period; and (d) it does not require any complex fine-tuning process.

Experimental results

The algorithm described has been implemented as a Java application. As preliminary results, we have solved 23 instances of the set generated and introduced by Goetschalckx and Jacobs-Blecha (1989) in order to test the effectiveness of the proposed method. The set of benchmark is available in the webpage, http://www2.isye.gatech.edu/~mgoetsch/lineback.html.

Table 1 shows the result corresponding to the number of vehicles chosen by our algorithm for the benchmarks containing more than 65 nodes. A standard personal computer, Intel R Core TM i7 CPU M 640 @ 2.80 GHz, 2.79 GHz, 3.42 GB RAM.

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**Fig. 1.** Flowchart for the proposed methodology
Operating system Microsoft Windows XP Professional Version 2002 Service Pack 3, was used to perform these tests. Results of these tests are summarized in Table 1, which contains the following information for each instance: name of the instance, vehicle capacity, number of linehaul customers, number of backhaul customers, cost given by the CWS solution, best known solution according to Zachariadis and Kiranoudis (2012), our Best solution, number of routes in the BKS, number of routes in our solution and computation time in seconds of best solution obtained. Each instance was run 25 times and only best solutions are reported. First of all, notice that negative gaps in column ‘Gap’ of Table 1 correspond to improvements with respect to the best-known solution so far. Moreover, this has been obtained in just a few seconds. Notice that in these two instances (I1 and L1) the best solution provided by our algorithm uses one route less than the best-known solution so far. The mean gap considering all the instances is 0.01% and in our opinion this can be consider an interesting result. Another important point to consider here is that the presented algorithm is relatively simple and easy to adapt to real-life problems and restrictions. It has been applied successfully to all tested instances without requiring any special fine-tuning or set-up process.

Table 1. Comparative results for the VRPB

<table>
<thead>
<tr>
<th>Instance</th>
<th>Q</th>
<th>NLH</th>
<th>NBH</th>
<th>CW</th>
<th>BKS</th>
<th>OurBest</th>
<th>Gap</th>
<th>K</th>
<th>Our K</th>
<th>Time</th>
</tr>
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<tr>
<td>H1</td>
<td>400</td>
<td>45</td>
<td>23</td>
<td>285.618</td>
<td>268.933</td>
<td>268.933</td>
<td>0.00%</td>
<td>6</td>
<td>6</td>
<td>49</td>
</tr>
<tr>
<td>H2</td>
<td>5100</td>
<td>45</td>
<td>23</td>
<td>271.493</td>
<td>253.365</td>
<td>253.365</td>
<td>0.00%</td>
<td>5</td>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>H3</td>
<td>6100</td>
<td>45</td>
<td>23</td>
<td>269.395</td>
<td>247.448</td>
<td>247.448</td>
<td>0.00%</td>
<td>4</td>
<td>4</td>
<td>51</td>
</tr>
<tr>
<td>H5</td>
<td>7100</td>
<td>45</td>
<td>23</td>
<td>263.617</td>
<td>246.121</td>
<td>246.121</td>
<td>0.00%</td>
<td>4</td>
<td>4</td>
<td>53</td>
</tr>
<tr>
<td>I1</td>
<td>3000</td>
<td>45</td>
<td>45</td>
<td>364.968</td>
<td>350.246</td>
<td>349.156</td>
<td>-0.31%</td>
<td>10</td>
<td>9</td>
<td>80</td>
</tr>
<tr>
<td>I2</td>
<td>4000</td>
<td>45</td>
<td>45</td>
<td>320.635</td>
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<td>7</td>
<td>87</td>
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<tr>
<td>I4</td>
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<td>45</td>
<td>45</td>
<td>309.547</td>
<td>295.988</td>
<td>295.988</td>
<td>0.00%</td>
<td>6</td>
<td>6</td>
<td>94</td>
</tr>
<tr>
<td>J1</td>
<td>4400</td>
<td>75</td>
<td>19</td>
<td>345.697</td>
<td>335.007</td>
<td>335.006</td>
<td>0.00%</td>
<td>10</td>
<td>10</td>
<td>88</td>
</tr>
<tr>
<td>J2</td>
<td>5600</td>
<td>75</td>
<td>19</td>
<td>326.193</td>
<td>310.417</td>
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<td>8</td>
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<tr>
<td>J3</td>
<td>8200</td>
<td>75</td>
<td>19</td>
<td>294.206</td>
<td>279.219</td>
<td>279.219</td>
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<td>6</td>
<td>6</td>
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<tr>
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<td>6600</td>
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<td>19</td>
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<tr>
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<td>394.071</td>
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<td>10</td>
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<tr>
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<td>362.130</td>
<td>362.129</td>
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<td>8</td>
<td>8</td>
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</tr>
<tr>
<td>K4</td>
<td>6200</td>
<td>75</td>
<td>38</td>
<td>378.999</td>
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<td>0.00%</td>
<td>7</td>
<td>7</td>
<td>171</td>
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<tr>
<td>L1</td>
<td>4400</td>
<td>75</td>
<td>75</td>
<td>448.498</td>
<td>417.896</td>
<td>417.304</td>
<td>-0.14%</td>
<td>10</td>
<td>9</td>
<td>189</td>
</tr>
<tr>
<td>L2</td>
<td>5000</td>
<td>75</td>
<td>75</td>
<td>443.219</td>
<td>401.228</td>
<td>402.153</td>
<td>0.23%</td>
<td>8</td>
<td>8</td>
<td>155</td>
</tr>
<tr>
<td>L4</td>
<td>6000</td>
<td>75</td>
<td>75</td>
<td>412.740</td>
<td>384.636</td>
<td>385.923</td>
<td>0.33%</td>
<td>7</td>
<td>7</td>
<td>148</td>
</tr>
<tr>
<td>M2</td>
<td>5200</td>
<td>100</td>
<td>25</td>
<td>436.978</td>
<td>396.917</td>
<td>397.407</td>
<td>0.12%</td>
<td>10</td>
<td>10</td>
<td>111</td>
</tr>
<tr>
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<td>6200</td>
<td>100</td>
<td>25</td>
<td>409.948</td>
<td>375.696</td>
<td>376.159</td>
<td>0.12%</td>
<td>9</td>
<td>9</td>
<td>110</td>
</tr>
<tr>
<td>M4</td>
<td>8000</td>
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<td>25</td>
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<td>348.140</td>
<td>0.00%</td>
<td>7</td>
<td>7</td>
<td>147</td>
</tr>
<tr>
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<td>5700</td>
<td>100</td>
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<td>465.112</td>
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<tr>
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<td>100</td>
<td>50</td>
<td>437.896</td>
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<td>394.337</td>
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<td>9</td>
<td>9</td>
<td>152</td>
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<tr>
<td>N5</td>
<td>8500</td>
<td>100</td>
<td>50</td>
<td>411.866</td>
<td>373.476</td>
<td>375.718</td>
<td>0.60%</td>
<td>7</td>
<td>7</td>
<td>189</td>
</tr>
</tbody>
</table>

Gap Avg: 0.01%

Table: Instance: name of the instance; Q: vehicle capacity; NLH: number of linehaul customers
NBH: number of backhaul customers; CW: Clarke&Wright, OurBest: Our Best solution K; number of routes in the BKS solution
Our K; number of routes in our solution, Time in seconds of OurBest

Conclusion

This paper focuses on the Vehicle Routing Problem with Backhauls which is found in practical applications of some real-world transportation activities. The methodology here presented develops a multi-start biased randomization combined with a memory learning mechanism and splitting strategies. It would probably be the first randomized-oriented algorithm that is able to compete with other methodheuristics in solving the VRPB. Moreover, thanks to the multi-start nature of the constructive approach, many alternate solutions can be generated according to the experiment needs. Thus, the decision
 maker can evaluate different solutions by considering not only some aprioristic criteria but also other ones known in a later stage. The algorithm presented here is easily parallelizable and does not require any particular fine-tuning or configuration process. These properties make this procedure an interesting alternative to other more complex approaches. A set of standard benchmarks has been used to test our algorithm, and the computational results obtained show that our approach is able to provide high quality solutions in real time to most of the tested benchmarks.

Acknowledgments—This work has been supported by the Spanish Ministry of Economy and Competitiveness (grant TRA2013-48180-C3-P), and by the Ibero-American Programme for Science and Technology for Development (CYTED2014-515RT0489). Likewise we want to acknowledge the support received by the CAN Foundation (Navarre, Spain) (Grant CAN2014-3758).

References


