Lecture Notes in Management Science (2014) Vol. **6**: 228–238
6th International Conference on Applied Operational Research, Proceedings

ISSN 2008-0050 (Print), ISSN 1927-0097 (Online)

Multi-node offer stack optimization over electricity networks

Anthony Downward, Danny Tsai and Yibo Weng

University of Auckland, Auckland, New Zealand
a.downward@auckland.ac.nz

Abstract. In this work we examine the problem that electricity generators face when offering power at multiple locations into an electricity market. The amount of power offered at each node can affect the price at the other node, so it is important to optimize all offers simultaneously. Even with perfect information (i.e. known demand, and known offers from competitors) this is a non-convex bi-level optimization problem. We first show how this can be formulated as an integer program using special ordered sets of type 2 (SOS2) enabling this problem to be solved efficiently. We then extend this work to allow for uncertainty, and hence find the profit maximising offer stacks at each node (as opposed to a single quantity, as in the deterministic case). We demonstrate the intuition that we can gain from this model in a simple two-node example, and discuss extensions to this work such as the co-optimization of reserve and generation, as well as demand-side bidding.

Keywords: electricity markets; integer programming; bi-level optimization

Introduction

New Zealand electricity market

In the New Zealand electricity market, every half-hour generators submit offers, in the form of offer stacks (a collection of up to five tranches, each specifying a quantity of power and a corresponding price). These offers are used by the system operator (Transpower) to determine how much electricity each generator should produce (dispatch) as well as nodal power prices at every location in the country.

These dispatch quantities are determined through a large linear program, which has the objective of minimising the cost of generation, while meeting the demand over the country, while complying with network constraints. This model is known as SPD, which stands for scheduling, pricing and dispatch.

In this paper, we will present an optimisation model which strategic generators can use to determine what offer stacks to submit to the system operator in order to maximize their profits. Initially, we will present this model in the context of a single node, and show how an optimal offer stack can be constructed. We will then extend this model over networks, and present a two-node example to gain some intuition into the strategic incentives that networks provide. We conclude by discussing some extensions to this model.

We will initially introduce the simplest version of the SPD model: the single-node dispatch model. This will allow us to develop the method for constructing offers stacks clearly, before we extend the model to handle networks, where generators potentially offer simultaneously at multiple nodes, in section 5.

Single-node dispatch model

$$\begin{array}{lll} \min & \sum_{t \in T} p_t x_t & \\ \text{s.t.} & \sum_{t \in T} x_t = d & [\pi] \\ & 0 \le x_t \le q_t & \forall t \in T, \end{array}$$

where T is the set of tranches (quantities of electricity, q_t, offered in at a constant price, p_t) offered by generators, indexed by t; d is the demand at the node, and x_t is the quantity of tranche t dispatched by the system operator. Finally, π is the shadow price on the demand balance constraint and gives the market clearing price, for all power that is bought and sold.

This optimization problem seeks to minimize the cost of generation while ensuring enough generation is dispatched to meet the demand, while complying with the offered tranches. This is a uniform price auction, as discussed in Schweppe *et al*. (1988). In the next section we will use this single-node dispatch problem to try to understand the behaviour of a profit maximizing generation offering power into this market.

Single-node optimisation

A strategic generator, wishing to maximize its profits would consider how its actions (i.e. its choice of tranches) would affect the total quantity dispatched and also the market clearing price. This can be modelled as a bi-level optimisation problem, where the strategic generator chooses their offer stack, anticipating the optimal dispatch. In a deterministic setting, a generator with a single plant would essentially be choosing a quantity y to solve the following bi-level optimization model:

Single-node bi-level problem:

$$\begin{array}{lll} \max & y \times \pi - C(y) & \\ \text{s.t.} & 0 \leq y \leq \bar{y} & \\ & \min \quad \displaystyle\sum_{t \in T} p_t x_t & \\ & \text{s.t.} \quad \displaystyle\sum_{t \in T} x_t = d - y & [\pi] \\ & \qquad\quad 0 \leq x_t \leq q_t & \forall t \in T, \end{array}$$

where $\bar{y}$ is the capacity of the strategic generator's plant; and $C(y)$ is the cost of producing y units of power over the period. The embedded linear program gives the correct value of π, as a function of the generation y. This optimisation problem finds the offer quantity y for the strategic generator, which yields the maximum profit, given known offer stacks for each of the other generators, later we will incorporate uncertainty, but first we will show how to model the above as a MIP using special ordered sets of type 2 (SOS2).

In its current form, this problem would be extremely difficult to solve. Firstly, since the objective is bi-linear; and secondly we have an optimisation problem within the constraints of the bi-level program. One option for solving this could be to find the optimality (KKT) conditions of the dispatch problem, and embed those within the above optimization problem as either complementarity constraints, or using binary variables and big-M constraints. The latter method was implemented by Nates (2010), along with an approximation of the objective function and was found to be rather inefficient. Philpott *et al.* (2005) have also considered this problem using dynamic programming, albeit solely for a generator situated at a single node.

Piecewise-linear reformulation

In this paper, we will present an alternate approach which is both efficient and requires no approximation. Consider the dispatch problem presented earlier. From the optimality conditions of this problem, it can be seen that the solution can be found by simply ordering the tranches from the cheapest to the most expensive. Moreover, if there exists a partially dispatched tranche, then the market price must equal that tranche's offer price, otherwise the price must be greater than all fully-dispatched tranches and less than all undispatched tranches. These conditions on price can be shown to be equivalent to simply choosing a point on an offer stack sorted from cheapest to most expensive. Let us consider the simple offer stack shown in figure 1; this offer stack depicts 4 tranches.

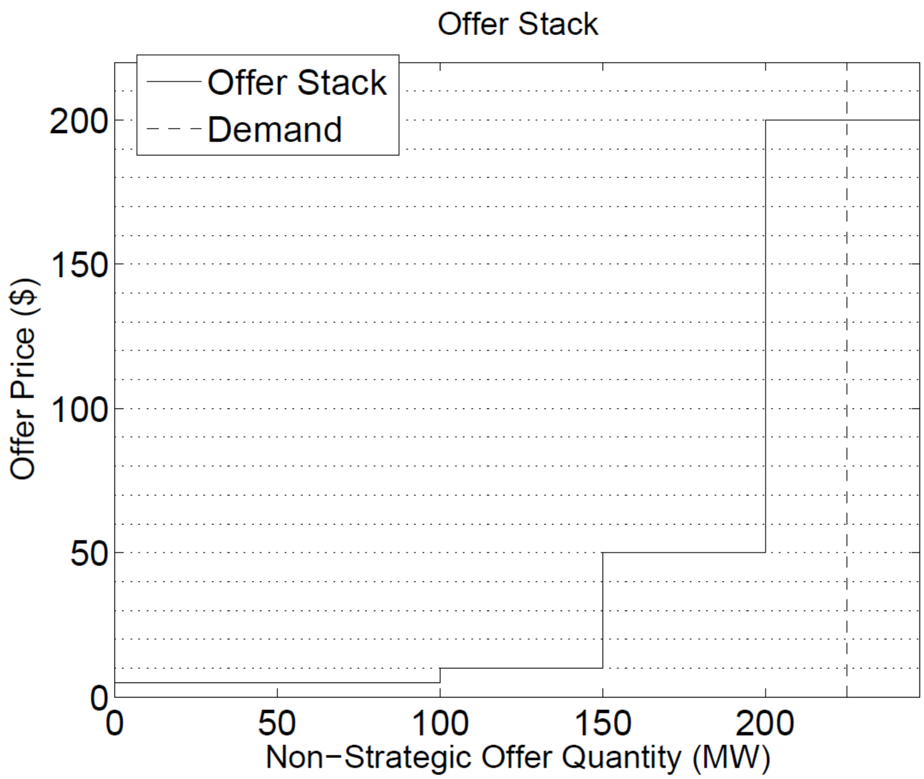

Fig. 1. Offer stack of non-strategic plant.

We can parameterize the offer stack, as a function of t, the distance travelled from the origin. These piecewise-linear functions will be called $p(t)$ and $q(t)$. Using these piecewise-linear functions, we can rewrite the optimisation problem for a generator as follows:

$$\begin{array}{ll} \max & y \times \pi - C(y) \\ \text{s.t.} & 0 \leq y \leq \bar{y} \\ & q(t) = d - y \\ & p(t) = \pi. \end{array}$$

Note that we have not shown the full formulation of the piecewise-linear functions; these can be modelled using special ordered sets of type 2 (Beale and Tomlin, 1970), or by utilizing the piecewise-linear functionality of a modelling language such as AMPL (with CPLEX). However, this model still has a bilinear term in the objective; this can be made linear, as follows:

$$\begin{aligned} y\pi &= \big(d - q(t)\big)p(t) \\ &= dp(t) - q(t)p(t), \end{aligned}$$

where $d - q(t)$ is the residual demand at price $p(t)$, or the demand remaining after all other tranches with prices less than $p(t)$ are dispatched. On the surface, this still appears nonlinear, since we have the term $q\ (t)\ p\ (t)$. However, recall that $\big(q\ (t);\ p\ (t)\big)$ is the parametric representation of a piecewise-constant function. This means that whenever $q\ (t)$ is increasing, $p\ (t)$ is constant, and vice-versa. Thus we can define: $pq\ (t) :=\ q\ (t) \times p\ (t)$, and $pq\ (t)$ will be a piecewise-linear function of t. This leaves us with the following formulation, which can be solved using a standard integer programming solver (so long as the cost function is also piecewise linear.

$$\begin{array}{ll} \max & dp(t) - pq(t) - C(y) \\ \text{s.t.} & 0 \le y \le \bar{y} \\ & q(t) = d - y. \end{array}$$

Constructing a profit-maximizing offer stack

So far, we have only considered maximizing profit for a single scenario; i.e., we knew what the demand would be, and what offers other generators would submit. This model results in a single optimal price and quantity which maximizes profit. In fact, when you submit your offer, you are not sure what offers have been submitted by other firms, and there will be uncertainty with respect to wind generation, as well as changes in demand. This means that instead of simply choosing a single quantity that maximizes profit, you want to submit an offer stack that maximizes expected profit. We will denote this uncertainty, for each scenario $\omega \in \Omega$ by a superscript, e.g. the demand in scenario ω would be d^{ω}.

In order for the offer stack that you submit to the market to be valid, it must be monotonically increasing. This requirement means that it is not possible to construct an offer that will be optimal for each realisation of the uncertainty. The lines in left graph in figure 2 are (inverse) residual demand curves for each scenario and the dots on the left plot show the optimal dispatch points for each scenario. It is clear that no monotonically increasing function passes through all these points, so instead we must seek to maximize the expected profit, given the distribution of possible scenarios; the offer stack which maximises this expected profit is shown on the right of figure 2. Note that in order to comply with the monotonicity requirement, lower profits are attained for certain scenarios.

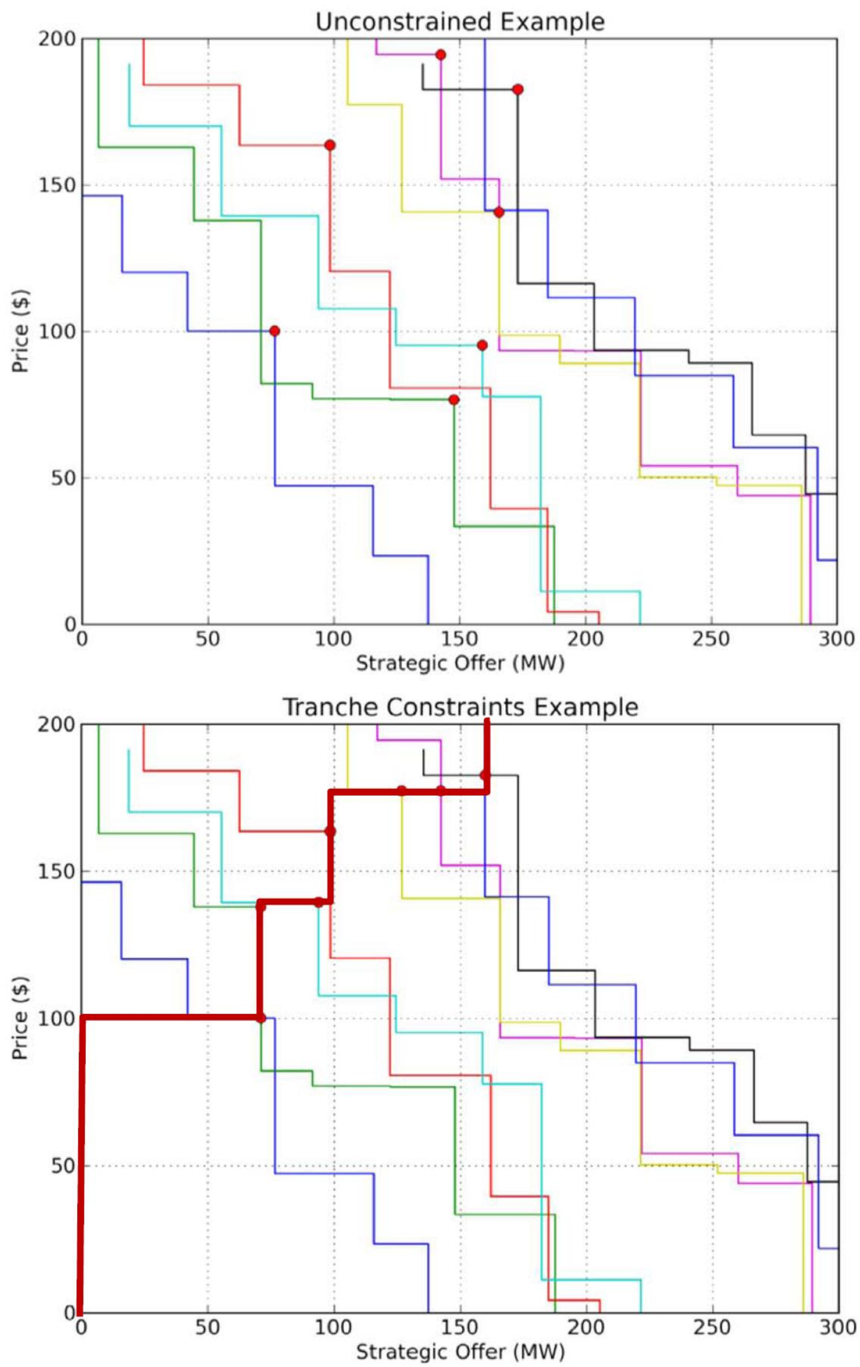

Fig. 2. Multiple scenarios, with and without monotonicity constraints.

We can construct this offer stack by adding additional constraints to the problem. If we know in advance that the offer stack will hit the various scenarios in the following order: $\{\omega_1, \omega_2, \omega_3 \ldots\}$, then ensuring monotonicity could be simply achieved by adding linear constraints, as follows:

$$y^{\omega_i} \le y^{\omega_{i+1}}, \quad i = 1 \ldots |\omega| - 1,$$
$$p^{\omega_i} \le p^{\omega_{i+1}}, \quad i = 1 \ldots |\omega| - 1.$$

A situation where you would be able to enforce the monotonicity in this manner is where the only uncertainty is coming from demand (in this case the offer stack will pass through the residual demand curves from the lowest demand to the highest. However, where it is not possible to predict the order of the scenarios, an integer programming approach is required. To do this we introduce a set of binary variables: z_{ij} which is set to 1 if scenario ω_j is passed through after scenario ω_i in the offer stack, and 0 otherwise. Finally, note that each scenario $\omega \in \Omega$ has a corresponding probability of ρ^{ω} . Thus we can create the following profit maximization problem.

Single-node offer stack construction:

$$\begin{array}{lll}
\max & \displaystyle\sum_{\omega \in \Omega} d^{\omega} p^{\omega}(t^{\omega}) - p q^{\omega}(t^{\omega}) - C(y^{\omega}) & \\
\text{s.t.} & q^{\omega}(t^{\omega}) = d^{\omega} - y^{\omega} & \forall \omega \in \Omega \\
 & 0 \le y^{\omega} \le \bar{y} & \forall \omega \in \Omega \\
 & y^{\omega_j} \le y^{\omega_i} + M z_{ij} & \forall \omega_i, \omega_j \in \Omega, i \ne j \\
 & p^{\omega_j}(t^{\omega_j}) \le p^{\omega_i}(t^{\omega_i}) + M z_{ij} & \forall \omega_i, \omega_j \in \Omega, i \ne j \\
 & z_{ij} + z_{ji} = 1 & \forall \omega_i, \omega_j \in \Omega, i \ne j \\
 & z_{ij} \in \{0,1\} & \forall \omega_i, \omega_j \in \Omega, i \ne j.
\end{array}$$

In this model, we now seek to maximize expected profit (as opposed to the profit for a particular scenario). The last four constraints enforce that the sequence of dispatch points on the offer curve are increasing monotonically. This is done using a big-M type constraint to enforce and relax constraints depending on the order in which the offer stack passes through each of the scenarios. Finally, we ensure that if $z_{ij} = 1$ then $z_{ji} = 0$, and the z is binary. This model can be used to determine the offer stack that a single generator can submit at a single node in order to maximize its profit.

Bidding at multiple locations over a transmission network

In the previous section, we have assumed that all offers and demand occur at a single node. In this section, we will detail how the generator offer model can be extended to the case where there is an underlying network. The dispatch problem in this case must be extended, to include transmissions lines and capacities; specifically, we now have a set of nodes N and a set of arcs A, as shown below. Following this we will show how the offer stack construction formulation extends to allow us to optimise the generation of multiple plants located over a network. The model that we have developed is called the *Offer Model over Electricity Networks* (OMEN).

Network dispatch model

$$
\begin{array}{llll}
\min & \sum_{t\in T} p_t x_t & & \\
\text{s.t.} & \sum_{t\in T_n} x_t - \sum_{i|ni\in A} f_{ni} + \sum_{i|in\in A} f_{in} = d_n & [\pi_n] & \forall n \in N \\
& \sum_{ij\in A} l_{k,ij} f_{ij} = 0 & [\lambda_k] & \forall k \in L \\
& -K_{ij} \le f_{ij} \le K_{ij} & [\eta_{ij}^-, \eta_{ij}^+] & \forall ij \in A \\
& 0 \le x_t \le q_t & & \forall t \in T,
\end{array}
$$

where $T_n \subseteq T$ is the subset of tranches that are offered at node n, and L is the set of loops in the network. The first constraint simply ensures that each node in the network satisfies a power balance requirement (i.e., the generation at a node plus the net flow into the node equals the demand. The second constraint ensures that the physical loop-flow laws are respected. Finally, the third constraint ensures that the power flow along the line ij is less than the capacity of the line in both directions.

OMEN: Offer Model over an Electricity Network

$$
\begin{array}{lll}
\max & \sum_{n\in N} [d_n p_n(t_n) - p q_n(t_n) - C_n(y_n)] - \sum_{ij\in A} [\eta_{ij}^+ + \eta_{ij}^-] K_{ij} & \\
\text{s.t.} & 0 \le y_n \le \overline{y_n} & \forall n \in N \\
& q_n(t_n) = d_n - y_n & \forall n \in N \\
& -p_i(t_i) + p_j(t_j) - \eta_{ij}^+ + \eta_{ij}^- + \sum_{k\in L} \lambda_k l_{k,ij} = 0 & \forall ij \in A \\
& \sum_{ij\in A} l_{k,ij} f_{ij} = 0 & \forall k \in L \\
& 0 \le K_{ij} - f_{ij} < 2K_{ij}(1 - u_{ij}^+) & \forall ij \in A \\
& 0 \le K_{ij} + f_{ij} < 2K_{ij}(1 - u_{ij}^-) & \forall ij \in A \\
& 0 \le \eta_{ij}^+ \le M u_{ij}^+ & \forall ij \in A \\
& 0 \le \eta_{ij}^- \le M u_{ij}^- & \forall ij \in A \\
& u_{ij}^+, u_{ij}^- \in \{0,1\} & \forall ij \in A.
\end{array}
$$

From the optimality conditions of the problem and using the piecewise-linearization, we construct the following mixed-integer program to maximize the profit of a firm who owns multiple generation plants for a given scenario. (It is possible to extend this formulation to allow for multiple scenarios, but for ease of understanding we will present this model for only a single scenario; see Weng (2013) for the full formulation.)

The above formulation builds on the earlier model, with the third constraint giving the relationship between prices at either end of a transmission line, which come about from the dual constraint corresponding to the primal flow variables. The final 5 constraints model the complementary slackness conditions associated with line capacity constraints in the network dispatch problem.

Two-node network example

To understand some of the incentives that firms have when offering at multiple nodes, let us first consider a simple two-node network (A--B), joined by a single line. The strategic firm has a plant at each node, with the plant at node A having slightly higher marginal costs. At each node there are a number of other non-strategic generators submitting offers, and what we wish to do is find a pair of offer stacks (one at each node) for the strategic firm that maximises its profit over both nodes. To understand the effect that a line can have, we will examine the optimal offer strategy under three line capacities: a capacity of 0MW, which is equivalent to two separate markets; an infinite capacity, which is equivalent to a single unified market; and a 50MW line, which may or may not become congested, depending on how the strategic firm offers. The final of these cases is the most interesting as the other two reduce to single-node problems.

Figure 3 shows the optimal offer stacks of the firm at node A (left) and node B (right). The line marked with a square is the optimal offer when the line is 0MW. We can see that the prices offered at the nodes are different, and power is withheld at both nodes; this is because the two markets are independent. At the other extreme, when the line is infinitely large (the offer stack marked with a triangle), we see that the optimal strategy is to fully utilise the cheaper generator (at node B), and withhold at node A in order to maintain a higher price (which is the same at both nodes). Interestingly, the optimal strategy for when the line is 50MW (marked with a circle), does not lie between these extremes; instead, the firm withholds at node B and attempts to be fully dispatched at node A – this causes the transmission line between nodes A and B to become congested, with the price at node B higher than that of node A. This occurs because the residual demand elasticity is lower at node B meaning that prices increase more rapidly when withholding.

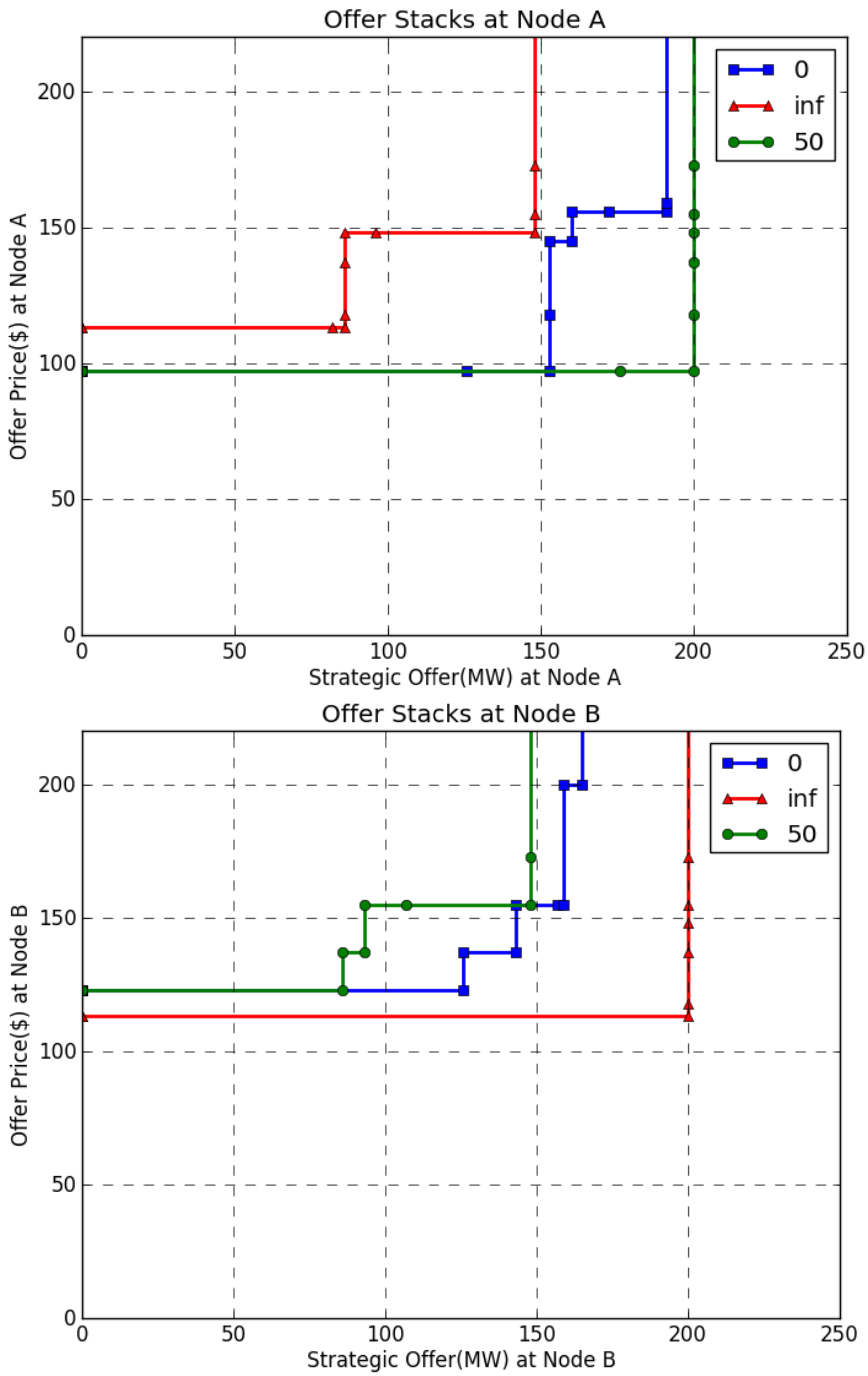

Fig. 3. Profit maximizing offer stacks at node A (left) and node B (right).

This behaviour could not have been captured using traditional single-node models, since the line is not becoming congested in all scenarios, and thus the curves must be constructed simultaneously. Furthermore, understanding these strategic incentives that exist over networks is important not only for generators, by also for the regulator (in New Zealand, the Electricity Authority); this is because in order to have a well-functioning and efficient market, the correct incentives should be in place. Particularly, this may be useful when considering line upgrade proposals, since the change in incentives can be identified. For examples, the grid investment test, overseen by the Electricity Authority (2010) states that competition benefits should be taken into account when considering grid upgrade proposals.

Extensions

We have implemented this model using real New Zealand generation and transmission data (for the full New Zealand network). However, this is not presented here due to space constraints. Non-convex cost functions and contract positions for gentailers (generators/ retailers) have also be incorporated. Transmission losses, mathematically can be included in the model, however, this significantly increases the size of the model needing to be solved. We are currently investigating methods to quickly find good incumbents to speed up the branch and bound process. Finally, we are intending to add the reserve market to this model, and consider how one might offer into the energy and reserve market simultaneously, either as a large consumer or a generator.

References

Beale EML and Tomlin JA (1970) Special facilities in a general mathematical programming system for non-convex problems using ordered sets of variables. Proceedings of the Fifth International Conference on Operational Research: 447-454. Tavistock Publications, London.

Electricity Authority (2010). Grid investment test. https://www.ea.govt.nz/dmsdocument/623.

Nates G (2010). Generator energy nodal offering model. Master's Thesis, University of Auckland. http://hdl.handle.net/2292/5971.

Philpott A, Zakeri G, and Pritchard G (2005). Non-parametric estimation of market distribution functions in electricity pool markets", Mathematics of Operations Research 34.

Schweppe FC, Caramanis MC, Tabors RD, and Bohn RE (1988). Spot pricing of electricity. Kluwer Academic Publishers.

Weng Y (2013). Generator offer optimisation in the New Zealand electricity market. Part 4 Project, Department of Engineering Science, University of Auckland.