Lecture Notes in Management Science (2014) Vol. **6**: 198–206
6th International Conference on Applied Operational Research, Proceedings

ISSN 2008-0050 (Print), ISSN 1927-0097 (Online)

Heuristic solution methods for the Fiber To The Home cabling problem

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Abstract. The deployment of Fiber To The Home (FTTH) technologies proves one of the most challenging issues for telecommunication operators. This paper focuses on the FTTH cabling problem which is modeled as a single-commodity flow problem with non-linear costs. Scalable heuristic approaches are presented and benchmarked on a real-life instances test set.

Keywords: fiber to the home; network design; heuristics

Introduction

Due to the development of bandwidth requiring services (HD television, video streaming), telecommunication companies are facing the necessity of network bandwidth upgrade. Most operators decided to replace, in the long run, their former copper wire line access network by a new optical one, engaging Fiber To The Home (FTTH) deployments in order to achieve this transformation in an efficient and sustainable way. The FTTH roll-out is an important challenge with huge economic stakes and human resources implications. From a decision-making perspective, this roll-out is a complex issue including several key steps. At a strategic level, one has to decide which type of architecture is to be adopted and its associated set of engineering or regulatory rules. At an operational level, the deployment scheme of minimum cost, compliant with the chosen architecture and rules, must be produced. France (network operators or the National Regulatory Authority) favored specific point to multi-point architectures named Passive Optical Network (PON). For the sake of clarity, we present here only a general two-level PON architecture, which corresponds to the one deployed in moderate density of population areas in France. This can be synthesized as follows (see. Figure 1): optical cables depart from the entry point of optical access network, called the Central Office (CO) and are routed within ducts of the civil engineering infrastructure. Along the way, these cables are potentially split into smaller size cables in concrete rooms, then designating Splicing Functional Point (SFP), before reaching the Distribution Points (DP) of the FTTH network.

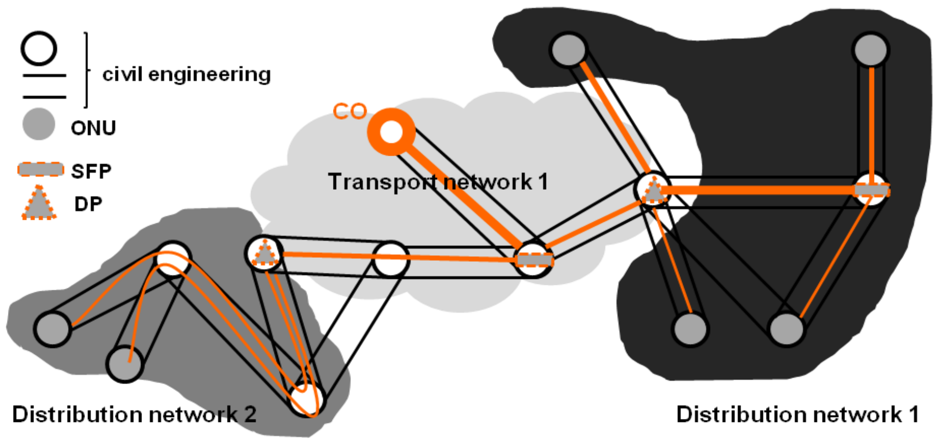

Fig. 1. A general two-level PON

DPs home optical equipments named splitters: these are passive equipments multiplying each incoming fiber into several ones, the maximum number of optical fibers potentially produced by a splitter being equal to its capacity. New cables departing from each DP are routed in the same way in order to serve Optical Network Units (ONUs), corresponding to the end-users groups. The final part between ONU and each end-user is achieved by routing individual mono-fiber cables. The part of the network from the CO to the DPs (resp. from a DP to its ONUs) is called the Transport network (resp. a Distribution network). With regards to engineering rules, the re-use of the existing civil engineering is a major requirement, meant to avoid trench digging costs: ducts are in practice large enough to deploy any set of cables while we assume that at most one cable can be split in a concrete room (i.e. one SFP per room). Finally both Transport and Distribution networks are most often required to be tree-like structured. Note that the two latest constraints are not motivated by technological reasons but meant to simplify the FTTH network future Operations, Administration and Maintenance.

From an operational point of view, the design of minimum cost PON deployment schemes remains a very complex task for at least two main reasons. First, it implies multiple interdependent decisions among which: (i) decide the grouping of end-users and location of ONUs, (ii) decide the location of DPs and their constitution (iii) decide the fiber paths and (iv) choose cables. Second, it involves multiple types of capital expenditures: equipments costs (for cables, optical splitters, splicing boxes, etc) as well as manpower costs (for ducts and concrete rooms status checking, cables laying, fiber splicing, etc).

This paper focuses on the Fiber To The Home cabling problem, which consists in finding an optimal set of cables in each duct assuming that decisions related to ONUs, DPs and fiber paths have already been taken and cannot be revised. We point out here that splitting a cable into several ones includes two major field operations.

First, a splicing box must be installed at the original cable extremity in order to "plug" new cables, inducing equipments costs; then light fibers (e.g. fibers that are actually used) must be spliced properly, inducing manpower costs. Figure 2 illustrates some cabling options and associated field operations/sources of cost.

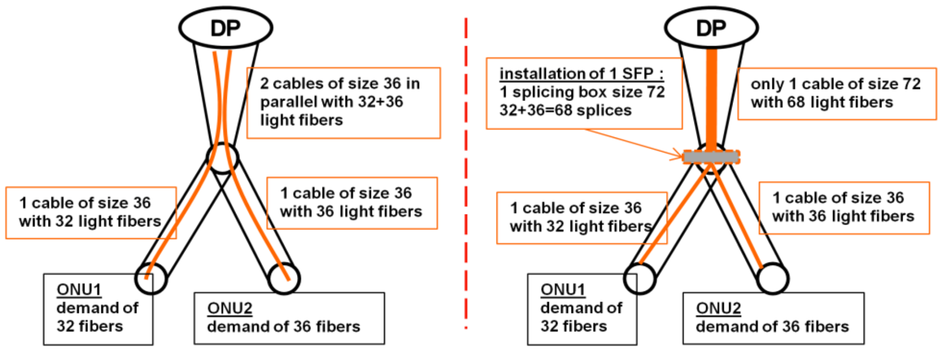

Fig. 2. Two cabling options (prolonging cables in parallel versus grouping cables)

The aim of this paper is to design and compare scalable heuristics for the FTTH cabling problem with non-linear cable splitting costs. The remainder of this paper is organized as follows. Next section presents a synthetic view of the related literature. The following Section is dedicated to the modeling and solving of the problem: beside a modeling framework, several heuristics are proposed and analyzed. Numerical results are reported in next Section to assess the relative efficiency of the approaches, before concluding.

Literature review and position of the paper

Fixed access network design is being widely studied for decades, see Gollowitzer and Ljubic (2011) and Gollowitzer *et al*. (2013) for recent works.

In recent years, PON design has gained interest and multiple variants have been investigated. In Kim *et al*. (2011), the authors introduce a 2 and 3-level PON design problem within a tree-like civil engineering infrastructure while considering non-linear fiber cost hypotheses. They propose an arc-path mixed integer formulation whose exact solving by branch-and-bound proves limited to instance sizes of a couple of nodes, and derived several relaxation-based heuristics. This problem has been further investigated in Hervet and Chardy (2013), where authors propose two exact solution methods: first a branch-and-bound approach based on an arc-node reformulation of the problem, and second, a dynamic programming approach which proves scalable for real-size instances. Chardy *et al*. (2012) focus on a 3-level PON design problem with linear fiber costs, no assumption being made on the civil engineering infrastructure. They propose an arc-node mixed integer model and integrate several graph reduction properties as well as valid inequalities to enhance their branch-and-bound based solution method. Finally, in Bley *et al*. (2013), the authors propose a heuristic approach for the 2-level PON problem with linear fiber costs based on Lagrangian relaxation and the introduction of specific cuts. As it can be noted, FTTH cabling issues have been little investigated though an abundant literature related to PON optimization. To our knowledge, only two works consider cables in PON design (see. Kim *et al*. (2011) and Hervet and Chardy (2013) cited above), but considering a restrictive framework, notably imposing a single cable per duct and null splitting costs.

The main contribution of this paper is to introduce a general PON cabling problem, embedding the complexity of real cabling cost structures and the variety of cabling options. Second, dedicated scalable heuristics are designed and assessed on real instances.

Mathematical framework

Problem modeling

Let $G = (V, A)$ be an arborescence representing a civil engineering infrastructure, with a leave-to-root (denoted by *root*) orientation. Then, for any node $i \in V$, we respectively note $\Gamma^+(i)$, $\Gamma^-(i)$ and $G(i)$ the father node of i (null for node *root*), its set of son nodes and the sub-graph rooted in i. Then the distance between any couple of nodes $i, j \in V^2$ is denoted by d_{ij} and $V^{SFP} \subset V$ denotes the subset of nodes compliant with the installation of a Splitting Function.

Considering the initial fiber setting, we denote by $d_l \in \mathbb{N}^{+*}$, the demand in fibers at any leaf $l \in V^{leaf}$. Let $F = \{F^1, \dots, F^p\}$ the set of available cable sizes with (w.l.o.g) $d_l \leq F^p, \forall\, l \in V^{leaf}$. With regards to the cabling cost structure, we denote by $C^F = \{\mathrm{C}_1^F, \dots, \mathrm{C}_p^F\}$, assumed positive concave increasing w.r.t F, their associated deployment costs (note that $\mathrm{C}_i^F\ i = 1, .., p$ are per unit length costs which represent equipment and/or manpower laying costs). We then denote by $B = \{B^1, \dots, B^h\}$ the set of available splicing boxes of respective sizes $n_b \in \mathbb{N}$, $\forall b = 1, .., h$: we assume $n_h \geq F^p$ as cables are necessarily split in a splicing box of greater size. A positive concave increasing unitary cost structure, denoted by $C^B = \{\mathrm{C}_{n_1}^B, \dots, \mathrm{C}_{n_h}^B\}$ is associated to B. Moreover, we note $C^S = \{\mathrm{C}_{n_1}^S, \dots, \mathrm{C}_{n_l}^S\}$ the positive convex decreasing unitary splicing costs structure (assumption $n_1 = 1$) such that the unitary splicing cost when performing n splices at a Splicing Functional Point is given by $\mathrm{C}_{n'}^S$ with $n' = argmax_{n_i\,|i=1,..,l}\ (n_i \leq n)$.

Finally, with regards to decision variables, let the set $Cables$ represent a feasible cabling scheme. We take for notative convention to index any cable by an identifier k so that $Cables_{ij} = \{c^{k_1}, \dots, c^{k_{n_{ij}}}\}$ denotes the set of portions/chunks of cables going through arc $(i, j) \in A$.

Heuristics solution methods

Two heuristic approaches are presented in this Section. For notative convenience, we first introduce the following functions/operators:

$\boldsymbol{capa}(c)$ which denotes the size of the cable c

$\boldsymbol{f_u}(c)$ which denotes the number of light fibers conveyed by cable c: $\boldsymbol{f_u}(c) \in \{1, \dots, \boldsymbol{capa}(c)\}$

$\boldsymbol{Create}(n, k)$ which creates a cable c^k of size $argmin_{F^k \in F}\ (n \le F^k)$ containing n light fibers

First we want to design a heuristic emulating the cabling policy imposed in Kim *et al*. (2011) and Hervet et Chardy (2013), which will be referred as the **max&asap grouping heuristic**. This cabling scheme consists in systematically choosing, in each duct, the cable of minimum size superior to the number of light fibers to be deployed in the duct, without questioning the relevance of installing Splicing Functions. Considering the framework described in the previous Section, this scheme can be straight-forward adapted as follows:

<u>initialization (leaf nodes):</u> all direct arcs from a leaf to its father node are supplied with a chunk of cable of immediate greater size than the demand at the leaf.

<u>recursion (intermediate nodes):</u> 2 cases must be considered. If the node is not authorized to home a SFP, the chunks of cables that are deployed between the node and its son nodes are prolonged up to the father node. Otherwise, we install a SFP grouping the maximum number of cables coming from the son nodes (independently of the son node); then we first deploy a chunk of this new cable between the node and its father node, and second we prolong the chunks of the cables that have not been grouped up to the father node.

<u>termination (root node):</u> no treatment is required at the root node.

The algorithm is detailed in Algorithm 1.

Second we propose a heuristic method based on a local cost optimization in order to decide at each node of the "best" strategy, considering a compromise between the cost of the Splicing Functional Point (if one is installed) and the cost prolonging un-grouped cables to an upper point of the node-to-root path.

<u>initialization (leaf nodes):</u> all direct arcs from a leaf to its father node are supplied with a chunk of cable of immediate greater size than the demand at the leaf.

<u>recursion (intermediate nodes):</u> 2 cases must be considered. If the node is not authorized to home a SFP, the chunks of cables that are deployed between the node and its sons are prolonged up to the father node. Otherwise, we iteratively test the opportunity of installing a Splicing Functional Point grouping the k chunks of cable of smaller capacity coming from the son nodes, as well as the "no-SFP" option. The cost evaluation is done for each tested option by summing the SFP cost (if any) and the cost of deploying a new cable (if any) plus the one of deploying the prolonged chunks of cables to a point of the path to the root. Note that this cost computation remains an estimation of the future cost of the "upper part of" cables as we cannot know where these cables will be actually grouped (and even if they will). The cheaper option is retained; let us say installing a Splicing Functional Point grouping the n cables of smaller size amongst the N cables coming from the son nodes. Then we first deploy a chunk of the new

cable between the node and its father node, and second we prolong the chunks of the $N-n$ cables of greater capacity up to the father node.
termination (root node): no treatment is required at the root node.

This heuristic will be as referred as local-op heuristic in the rest of the article and is detailed in Algorithm 2.

The $\boldsymbol{computeCost}$ function is detailed in Algorithm 3.

Algorithm 1 max&asap grouping heuristic

Inputs
$G=(V,A)$ with V assumed sorted in decreasing order w.r.t depth
$d_l \in \mathbb{N}, \forall\, l \in V^{leaf}$
$index = 0$ (incremental index meant to manage new cables identifiers)
Step1: Initialization
for $(l \in V^{leaf})$ **do**
  $Cables_{l\Gamma^+(l)} = \{\boldsymbol{create}(d_l\,, index)\}$
  $index \leftarrow index + 1$
endfor
Step2: Recursion
for $(i \in V \backslash V^{leaf} \backslash root)$ **do**
  $\{c^{k_1}, \dots, c^{k_{n_i}}\} = \cup_{i' \in \Gamma^-(i)}\ Cables_{i'i}$ (assumed sorted in increasing order w.r.t size)
  if $i \notin V^{SFP}$ **then**
    $Cables_{i\Gamma^+(i)} = \{c^{k_1}, \dots, c^{k_{n_i}}\}$
  else
    $epa = max\{s = 1, .., n_i |\ \sum_{h=1}^{s} \boldsymbol{f_u}(c^{k_h}) \le F^p\}$
    $Cables_{i\Gamma^+(i)} = \{\boldsymbol{create}(\sum_{h=1}^{epa} \boldsymbol{f_u}(c^{k_h})\ , index)\} \cup_{h=epa+1,..,n_i} c^{k_h}$
    $index \leftarrow index + 1$
  endif
endfor

Algorithm 2 local-op grouping heuristic

Inputs
$G=(V,A)$ with V assumed sorted in decreasing order with respect to depth
$d_l \in \mathbb{N}, \forall\, l \in V^{leaf}$
$index = 0$ (incremental index meant to manage new cables identifiers)
$node^{up}(i)$, generic function returning an intermediate node between i and the root
Step1: Initialization
for $(l \in V^{leaf})$ **do**
  $Cables_{l\Gamma^+(l)} = \{\boldsymbol{create}(d_l\,, index)\}$
  $index \leftarrow index + 1$

endfor
Step2: Recursion
for $(i \in V \backslash V^{leaf} \backslash root)$ **do**
$\{c^{k_1}, \ldots, c^{k_{n_i}}\} = \cup_{i' \in \Gamma^-(i)}\ Cables_{i'i}$ (assumed sorted in increasing order w.r.t size)
$costNoSFP = \sum_{h=1}^{n_i} \left(\mathrm{C}^F_{\alpha_h} \,\middle|\, \boldsymbol{capa}(c^{k_h}) = F^{\alpha_h} \right). d_{i,node^{up}(i)}$
if ($i \notin V^{SFP}$ **or** $\left|\{c^{k_1}, \ldots, c^{k_{n_i}}\}\right| = 1$ **) then**
$Cables_{i\Gamma^+(i)} = \{c^{k_1}, \ldots, c^{k_{n_i}}\}$
else
$epa = argmin_{s=2,..,n_i} \boldsymbol{computeCost}(i, node^{up}(i), \{c^{k_1}, \ldots, c^{k_{n_i}}\}, \mathrm{s})$
if $\boldsymbol{computeCost}(i, node^{up}(i), \{c^{k_1}, \ldots, c^{k_{n_i}}\}, \mathrm{epa}) \leq costNoSFP$ **then**
$Cables_{i\Gamma^+(i)} = \{\boldsymbol{create}(\sum_{h=1}^{epa} \boldsymbol{f_u}(c^{k_h}),\ index)\} \cup_{h=epa+1,..,n_i} c^{k_h}$
$index \leftarrow index + 1$
else
$Cables_{i\Gamma^+(i)} = \{c^{k_1}, \ldots, c^{k_{n_i}}\}$
endif
endif
endfor

Algorithm 3 $computeCost$ function

Inputs
i : potential SFP node and source of chunk of cables to be evaluated
j : extremity node to be taken into account as the extremity of chunk of cables to be evaluated $\{c^{k_1}, \ldots, c^{k_{epa}}, \ldots, c^{k_{n_i}}\}$: list of cables (assumed sorted in ascending order w.r.t size)
epa : index (in the list) of the last cable to be regrouped
Computation
$cost = 0$
no_splice $= \sum_{h=1}^{epa} \boldsymbol{f_u}(c^{k_h})$ *% computation of the number of needed splices*
$cost += no\_splice\,.\,\mathrm{C}^S_{n'}$ with $n' = argmax_{n_i \mid i=1,..,l}\,(n_i \leq no\_splice)$ *% adding splicing cost*
$\alpha = \min\{k = 1, .., p \mid no\_splice \leq F^k\}$ *% computation of the size of the cable to be split*
$cost += \mathrm{C}^F_{\alpha}\,.\,d_{ij}$ *% adding estimation of the cost of the "upper part" of the new cable*
$n_\alpha = \min\{k = 1, .., h \mid F^\alpha \leq n_k\}$ *% computation of the size of the cheapest compliant splicing box*
$cost += \mathrm{C}^B_{n_\alpha}$ *% adding the splicing box cost*
$cost += \sum_{h=epa+1}^{n_i} \left(\mathrm{C}^F_{\alpha_h} \,\middle|\, \boldsymbol{capa}(c^{k_h}) = F^{\alpha_h} \right). d_{ij}$ *% cost of prolonged cable chunks*
return $cost$

Numerical tests

The objective of this Section is to compare the heuristic solution methods described in the previous Section and to assess their scalability and efficiency. Preliminary results are given on the Transport network of 11 French moderate density of population areas. For each instance, we store the number of concrete rooms and Distribution Points (i.e. demand nodes) in respective columns "$|V^T|$" and "no DPs", see. Table1. Three heuristics are analyzed:

the **max&asap grouping heuristic**.

the **local-op heuristic** with $node^{up}$ instantiated with the constant function returning the root node.

the **local-op heuristic** with $node^{up}$ instantiated with the function returning, for each node i the first node of V^{SFP} along the path from i to $root$.

Results for these heuristics are respectively stored in columns "$H_{max\&asap}$", "$H_{loc-opt-root}$" and "$H_{loc-opt-next}$". We report for two indicators: the solution cost ("cost" column, in €) and the number of Splicing Functional Points that have been installed ("no SFP" column).

Table 1. Test instances description and numerical results

Instance	*features*		$H_{max\&asap}$		$H_{loc-opt-root}$		$H_{loc-opt-next}$	
	$\|V^T\|$	*no DPs*	*cost*	*no SFP*	*cost*	*no SFP*	*cost*	*no SFP*
Net_T_1	27	5	**60268**	1	**60268**	1	**60628**	1
Net_T_2	74	9	146217	5	**140033**	3	144212	3
Net_T_3	51	10	154877	3	**150703**	2	**150703**	2
Net_T_4	46	17	225889	13	**207555**	11	216987	5
Net_T_5	76	20	266416	23	**245208**	16	255260	8
Net_T_6	139	46	569510	20	**552926**	18	557406	12
Net_T_7	152	32	424147	18	**398726**	16	412815	12
Net_T_8	274	73	580738	22	**561062**	18	562463	12
Net_T_9	274	73	478371	17	**442975**	16	461755	9
Net_T_10	346	69	1011085	37	**927496**	29	1000808	22
Net_T_11	274	73	901076	26	882216	21	**879010**	15

The main observation is that both instantiations of the **local-op heuristic** outperforms (in cost) the **max&asap grouping heuristic** which can be seen as a reference "naive" heuristic on all instances. More precisely, $H_{loc-opt-root}$ performs best on 10 out of 11 instances, with an average gain of 5.2% and a maximum of 9.0% obtained on Net_T_10. Moreover let us note that all instances have been solved within a few seconds, which confirms the scalability of the methods and their potential use in an operational context. Moreover, in such context, running the 2 local-op heuristics would prove a reliable strategy.

Conclusions

This paper deals with a Passive Optical Network cabling problem which is a key sub problem of the whole Fiber To The Home deployment optimization problem. A mathematical framework has been proposed for the modeling of this problem embedding realistic cost structure and cabling policies, as well as two kinds of heuristic solution methods. Tests results from experimentation conducted on real instances clearly show the economical benefit that operators could expect by using less constrained cabling policies and the design of decision-aid tools integrating dedicated cabling algorithms: nevertheless these are preliminary results that need to be enriched. The main research avenue for the future should be the design of exact solution methods of this kind of problems.

Acknowledgments— This work has been entirely supported by Orange Labs.

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