

# Heuristic schemes for the efficient utilization of 3D printing stereolithography apparatus

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**Abstract.** 3D Printing (3DP) technologies are increasingly being employed for the production of consumer products and mechanical components in the manufacturing sector, because of the advantages they exhibit as far as fabrication speed and flexibility are considered. This shift of focus in the application of 3DP technologies puts a new emphasis on the study of some of the process planning problems and issues that are related with the cost efficient use of 3DP systems and the quality of their products. As a result, the packing or platform layout optimization problem for the simultaneous fabrication of different parts has been identified as one of the most crucial tasks encountered in the process planning phase of 3DP. In the present paper a study of this problem that focuses on 3DP technologies that due to technical or quality reasons exclude the fabrication of a part on top of another, e.g. Stereolithography (SL) is presented. The methodologies discussed in the paper, employ a heuristic optimization technique (Simulated Annealing) in conjunction with two placement schemes, appropriately adapted to the problem. The reliability of the methodologies under discussion is evaluated via a case study concerning representative “real-world” parts/objects with quite general free form geometry.

**Keywords:** stereolithography; nesting problem; 2D packing; heuristic optimization; 3D printing

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## Introduction

During recent years the capabilities of 3DP technologies with respect to reliability and materials have significantly improved, to the point that they are increasingly employed for the production of actual end-use products of highly complex geometry or highly customized products in small numbers in relatively short times. On the other hand the cost per part associated with 3DP methods is relatively high. Therefore, a series of process

planning problems have to be examined in order to achieve a cost effective utilization of the 3DP technologies in the industrial manufacturing setting. The selection of build parameters, such as build orientation and layer thickness/pattern, and the efficient utilization of the machine time and workspace have been recognized as the most important among the process planning problems (Munguia, 2008).

In the present paper the efficient utilization of an 3DP machine is thoroughly discussed through the case of determining the optimum placement layout of different parts in an SL machine workspace. The problem is tackled employing a heuristic approach (Simulated Annealing) in conjunction with two different placement schemes, which are based on established nesting and 2D packing methodologies appropriately adapted to 3DP technologies that due to technical or quality reasons exclude the fabrication of a part on top of another. The reliability of the methodologies under discussion is evaluated via a case study concerning representative “real-world” parts/objects with quite general free form geometry.

## **Problem Description**

In order to design an effective optimization approach, the specific characteristics and constraints of the 3DP technology under investigation should be first considered. The most basic relevant constraint in the context of a SL technology seems to be the requirement for support structures which are used to prevent part/layer drifting and deformations. Due to this, parts in SL technologies are usually placed aside each other, after of course a proper build orientation for each one is chosen as proposed by the many authors (Canellidis *et al*, 2009; Pandey *et al*, 2007). The geometrical interaction between the parts being packed is, therefore, limited to the x-y plane only, and the resulting packing problem essentially becomes two-dimensional (2D packing).

Another methodological concern is the apparent trade-off between the quality of the optimization solution and the required computational time. For the problem under investigation significant parameters that affect the quality/time trade-off, besides the choice of a specific optimization technique, are the number of orientations allowed per part, the complexity of the information needed concerning the geometry of a part being nested, as well as the selection/construction of a specific placement and interference checking strategy.

The complexity of the nesting task increases considering the infinite feasible placement positions for each part if a free orientation scheme is selected, that considers rotations of the parts about all three axes. That is why the examination of a limited set of acceptable orientations, is proposed in several studies (Gogate & Pande, 2008; Canellidis *et al*, 2006)

All things considered, there are two important methodological decisions that have to be made in order to address any specific instance of the 2D nesting problem, namely the selection of an optimization scheme, i.e. how to guide effectively the search in the solution space of all possible layouts/solutions, and that of a placement scheme, i.e. how a specific layout is constructed through the placement of the various parts. The characteristics and details regarding the placement and optimization schemes will be discussed in the following sections.

**Optimization Scheme.** The nesting problem is NP complete problem. The classical (exact) optimization algorithms are, therefore, inefficient for such problems and one has to resort to heuristic approaches. Among the most popular heuristic approaches is Simulated Annealing (SA) (Dreo, 2006). SA is a generalization of the Monte Carlo method that was motivated by an analogy to the thermodynamics of annealing in solids. A SA optimization starts with a Metropolis Monte Carlo simulation at an initial high temperature  $T$ . The control parameter of temperature  $T$  is an artificial parameter, acting as a source of stochasticity. In order for the method to be able to escape local optima SA takes not only downhill moves, but also permits uphill moves with an assigned probability:

$$P(\Delta C) = \exp\left(-\frac{\Delta C}{T}\right) \quad (1)$$

where,  $\Delta C$  refers to the change of the cost of the objection function. Temperature  $T$  represents the willingness of a system to accept a state that is worse than the current in order to escape possible local optima. After a sufficient number of Monte Carlo steps, the temperature is decreased based on the Cooling schedule which defines the cooling speed to anneal the problem from a random solution to a good, frozen one. The Metropolis Monte Carlo simulation is then continued. This process is repeated until the final temperature is reached.

As it was pointed earlier, the optimization of the build volume is achieved via the dense nesting of parts, to be fabricated, on the 3DP machine platform, in order to minimize the unoccupied areas (trim loss). Thus, we are dealing with a minimization problem and the cost function may be defined as the percentage of the area of the platform that is unused by the, say  $n$ , parts

$$f = \frac{\sum_{i=1}^n \text{projection\_Area\_of\_the\_}i\text{th\_part}}{\text{Fabrication\_Platform\_Area}} \quad (2)$$

The cooling schedule is being considered to be the most important factor in a SA algorithm. It is composed by the starting temperature and the rules to determine when and how much the temperature should be reduced and when annealing should be terminated. In the present paper a dynamic polynomial-time cooling schedule, proposed by Aarts & Van Laarhoven (1985), has been adopted as the most promising solution. A stable value for the initial temperature can be obtained by generating a fixed number of transitions and accepting all the increases:

$$T = \frac{\overline{\Delta C^+}}{\ln(m2/(m2*x - m1*(1-x)))} \quad (3)$$

where  $C$  is defined as the cost of the objection function,  $m1$  as the number of the transitions occurred resulting in decrease of the objection function ( $\Delta c \leq 0$ ),  $m2$  as the number of the transitions occurred resulting in increase of the objection function ( $\Delta c > 0$ ),  $x$  the initial acceptance rate and finally  $\Delta C^+$  is defined as the average increase in cost over  $m2$  transitions.

After the estimation of the initial temperature a decrement rule must be established. Keeping a record of the cost values of the configurations  $\pi_1, \dots, \pi_j$  that occur during the generation of the  $k_{th}$  Markov Chain (the inner loop), where  $j$  is the length of the  $k_{th}$  Markov Chain, we are able to approximate the probability distribution of the cost values of the  $k_{th}$  Markov Chain by a normal distribution with mean  $\mu_k$  and variance  $\sigma_k^2$  given by:

$$\mu_k = \frac{1}{j} \sum_{i=1}^j C(\pi_i) \quad (4)$$

$$\sigma_k^2 = \frac{1}{j} \sum_{i=1}^j C^2(\pi_i) - \mu_k^2 \quad (5)$$

Therefore, the decrement rule for the temperature may be expressed as:

$$T_{K+1} = T_k / (1 + T_k * \ln(1 + \delta) / 3 * \sigma_k) \quad (6)$$

where  $\delta$  is called the distance parameter. The choice of  $\delta$  determines how closely the algorithm will approximate a globally minimum state. Moreover,  $\delta$  controls the computational effort that is going to be needed to reach an approximate globally minimum configuration. Finally the SA algorithm ends when temperature reaches zero.

**Placement Strategy.** The issue of placement strategies/rules and interference checks in nesting is quite important, because it influences not only for the quality of the solution but also the required computational effort. The geometrical representation needed for performing the actual nesting, is the projection of each one of the 3D parts considered to be fabricated on the platform. The solution space, even in the case of 2D nesting, is large. Moreover, the projections of the parts may be freely rotated along the building direction (i.e. the z-axis) under any angle, as this kind of rotation does not affect the original fabrication orientation of the 3D models, which has been already adopted. Thus, appropriate care must be taken in every step of the packing procedures to alleviate the computational effort needed. The first action to be taken is to reduce the number of points needed to describe the geometry of a part projection. Thus, each projection is being offset by a suitable threshold and then the number of points of the projection curve is reduced utilizing the Douglas–Peucker algorithm (Douglas and Peucker, 1973). In the context of the present paper, two different placement schemes are discussed, the first employing direct trigonometry techniques, while the second is based at the notion of the No-Fit Polygon (NFP).

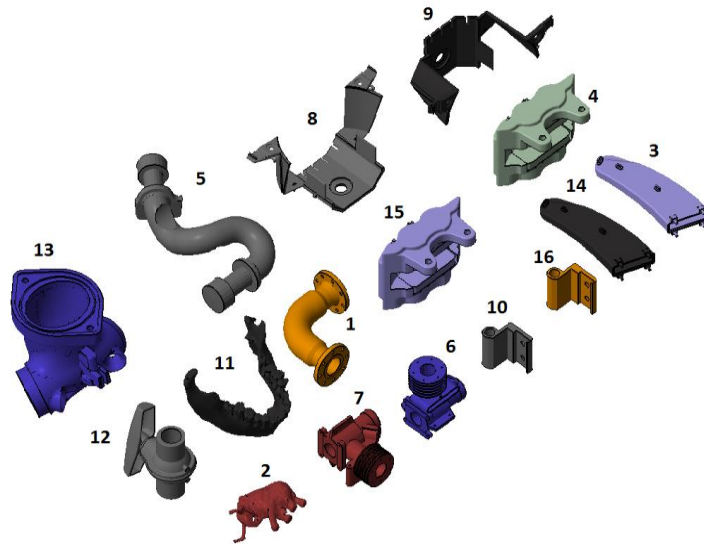
**Direct Trigonometry Method.** In order to deal with the complexity of the problem, the first placement strategy dynamically chooses a limited set of orientation instances for each projection. That is, for each projection the Minimum Bounding Rectangle (MBR) is being calculated and it is oriented such as each edge to be parallel or vertical to the fabrication platform edges, minimizing its orthogonal footprint to the fabrication platform. Then each corresponding projection can be nested rotated by 0, 90, 180 or 270 degrees regarding the above initial orientation (around the Z-axis) without increasing the orthogonal footprint. Having created the possible orientations instances of each projection the

placement policy attempts to nest as many as possible of them in the fabrication platform by favoring positions near the bottom-lower and side of the platform and as far to the left as possible. This is implemented as a two step procedure. The first step aims at placing relatively quick a part near to the platform origin such that no collision happens utilizing only the MBR of each part projection. The second step aims at moving the already placed part/projection as close as possibly to the rest of the nested parts and the origin of the platform, utilizing a ray casting approach. Specifically the extent of new possible movements is determined by employing casting of rays from the vertices of the incoming polygon and the already nested polygons and evaluating the minimum distance that the part can travel nearer to the origin of the platform (Canellidis *et al.*, 2013).

**No Fit Polygon Method.** In order to diminish the frequent use of the expensive ray-casting technique in the nesting procedure the second placement method is based on the concept of the No-Fit Polygon (NFP). The NFP can be used to determine all arrangements that two arbitrary polygons of standard orientation may assume so that the two shapes touch. The development of a procedure for deriving an analytical representation of the NFP is trivial when it concerns convex shapes but can be quite challenging and computational expensive for concave shapes. It entails the confrontation of numerous degenerated cases deriving from the generality of the shapes considered. On the other hand, it is adequate to obtain a rough representation of the NFP boundaries by point sampling in order to find the optimum geometric interrelation between two highly irregular polygons. Thus, in the present packing placement strategy, a method proposed by the authors in (Canellidis *et al.*, 2013) for obtaining an estimation (as a point cloud) of the actual NFP boundary of two highly irregular shapes, is utilized. Then, in order to compute the distance between two polygons (i.e. A and B) only one ray is needed to be casted from the reference point of polygon B to the approximate NFP of the two polygons ( $NFP_{AB}$ ). Moreover, to accelerate the packing process the NFPs of all pair combinations of the available objects are computed prior to the initiation of the SA heuristic process.

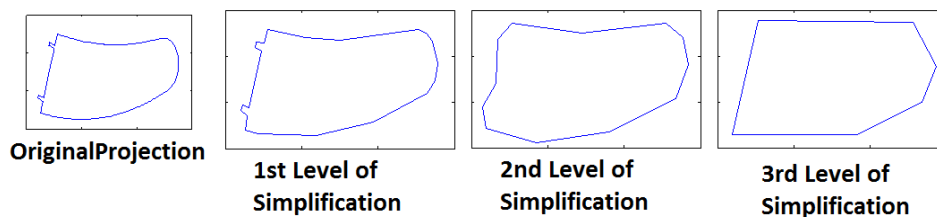
## Computational Results and Discussion

The nesting methodologies described has been implemented using Matlab R2008a. In order to investigate the performance as well as to check how the nesting methodologies are adapted to the operational characteristics of the SL technology, a representative test case was attempted. The 16 ‘real-world’ objects/parts considered in the test case are presented in Fig. 1. The test case simulates the nesting procedure in a fabrication platform of 250 mm x 250 mm wide.

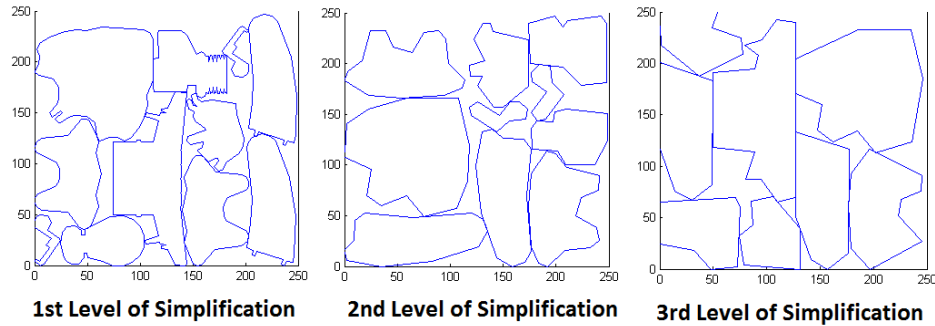


**Fig. 1.** Parts examined in the SL test case.

The projections of the parts on the platform were computed after slicing the STL files and Boolean union of the corresponding slices. Due to the relatively high level of detail and complexity of the STL files the obtained 2D projections consists of a large number of points per polygon. Utilizing the 2D projections with the afore mention nesting methodologies, without any further processing, has been proved (Canellidis *et al.*, 2013) to be computational expensive. Thus, a reduction of the level of detail of the projected polygons is required before the application of the nesting procedures. Geometry simplification is performed through the reduction of the number of polygon points employing the Douglas–Peucker algorithm, following an initial offset of the original geometry by a certain threshold, to avoid possible intersections between parts due to the simplification procedure. Three different abstract representations of the initial 2D projections were constructed in order to investigate the effect of the geometry used to the performance of the nesting methodologies. The first abstract representation level follows more precisely the geometry of the original projections while the other two are more abstract utilizing even less points to describe the projections to be packed. In Fig.2 the original and the three simplified projection for the part 3 are presented.



**Fig. 2.** Schematic representation of the 3rd part projections



**Fig. 3.** "Optimum" layout arrangements for the SL test case using different geometry detail.

From the Table 1 it can be observed that utilizing more abstract representations (up to the second level) for the method of the Direct Trigonometry seems to be beneficiary, as the computational time is significantly reduce without compromising the quality of the results. This is probably due to the reduction in the use of the expensive ray-casting technique that is repeatedly utilized in the nesting procedure. Moreover, the highly irregular geometry of the parts doesn't allow the facile creation of layouts that present extensive exact matches between the features of the projections (e.g. exact match between a concave and convex areas of 2 polygons), thus the nesting procedure cannot be benefit from the excess geometrical information that an more accurate representation possess.

**Table 1.** Results of the SL test case.

Packing Method	1st Abstraction Level		2nd Abstraction Level		3rd Abstraction Level	
	Total computational time (s)	Platform area coverage (%)	Total computational time (s)	Platform area coverage (%)	Total computational time (s)	Platform area coverage (%)
Direct Trigonometry	14.850	60,7	6.825	57,2	6.085	52,8
NFP	2.750	63,9	2.854	53,2	2.281	55,8

The second packing methodology (see Fig. 3.) doesn't seems to benefit from the use of more abstract representations as the use of the expensive direct trigonometry routines are already set to the minimum. Thus, it can be observed that we obtain worse results as the abstraction level increases while the computational time is approximately the same as it depends solely on the number of the projection to be packed rather that their geometry. On the other hand, it must be pointed out that the process were the NFPs of all pair combinations of the available objects are computed prior to the initiation of the heuristic process is significantly accelerated due to the reduced geometrical complexity.

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