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A mathematical programming approach for solving the problem of fixing prices and augmenting capacity for two competing suppliers

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Abstract. The decision to produce internally or outsource the manufacturing of certain goods is critical because of its effects on the utilized capacity, the production costs, the quality of products and the customer service. A bad performance of external suppliers represents a potential risk for an organization; however the internal production of low-value products involves the use of resources (physical and human) that may be used to manufacture products of higher value. Original Equipment Manufacturers (OEM) in the automotive sector need to identify reliable suppliers to who outsource the manufacturing of parts and components. In the case these suppliers do not have enough capacity to produce all the goods demanded, the OEMs may incentive them to invest in additional capacity by offering to increase the volume of production subcontracted.

Keywords: outsourcing; manufacturing; bi-level mathematical programming; simulated annealing

Introduction

The problem attended by this work is the following: there is a unique buyer who wants to outsource the production of basic components (collision automotive parts) to competing suppliers. At the beginning of the planning period these suppliers do not have enough capacity to satisfy the demand of auto-parts, but they can gradually increase their capacity if the production volume subcontracted by the buyer is large enough to justify the investment. At the end of the production horizon, this additional capacity may be used to manufacture similar auto-parts for other automobile models then there is no loss of the investment. The suppliers must decide the selling price of the auto-parts, which depends on internal production costs. The supplier with the lowest price offer will get an increment in the production volume from the buyer, but to fabricate this extra amount of goods, a possible augment in capacity should be made. At each production period, the additional capacity may result in economies of scale and the possibility to decrease further the price to the OEM. In the case none of the suppliers be able to offer competitive prices (below the cost of producing internally) or wish to increase capacity, the buyer will need to manufacture all the auto-parts but with high opportunity costs. The buyer also may decide to keep an inventory of auto-parts to take advantage of the extra capacity of the suppliers during some period and then avoid the internal manufacturing of auto-parts in subsequent periods in the case suppliers decide not to increase capacity furthermore. Inventory costs are applied under this situation.

The previous problem has been solved by using economic models describing the decisions of the suppliers. This work takes a different perspective by proposing a two-level mathematical programming problem that optimizes the decisions of buyer and suppliers. The objective of the buyer is to minimize the total cost of purchase of the auto-parts along the planning horizon while assuring the demand at each period will be satisfied. This problem is similar as the stated by Basnet and Leung (2005) in the context of lot sizing problem with supplier selection. The objective of the suppliers is to maximize their utilities by offering low prices and gradually increasing their capacity in order to gain additional production volumes. Because the suppliers compete for more production, the price-capacity strategy of a supplier should be the best response to the strategy chosen by the other supplier; otherwise there is no equilibrium and the price war will result in a utility loss for both suppliers. The problem of best response assumes the supplier *i* tries to maximize its utility after taking into consideration the strategy of supplier j because a unilateral strategy always represents a utility reduction. The best-response model may be formulated as a bi-level mathematical programming model (Perakis and Sood 2004) because at the higher level both suppliers elect their best price-capacity strategy, while at the lower level the buyer evaluates these strategies to decide how much production will be assigned to each supplier.

The Karush-Kuhn-Tucker (KKT) conditions were applied to reduce the original bi-level mathematical programming problem to a one level mathematical programming problem (as described by Anandalingam 1988; Sinha and Sinha 2002). However, the resulting problem becomes more difficult to solve because it is a non-linear mathematical programming problem with a series of equilibrium equations. We used a meta-heuristic algorithm based on a moving neighborhood to explore the set of possible strategies of a supplier and select the one that represents the best response and leads to an economical equilibrium. A population based simulated annealing algorithm developed by Van Hentenryck and Vergados (2007) was used to explore the space of strategies available to each supplier.

This algorithm defines a neighborhood within the space of valid strategies for the supplier *i* and then under well-defined rules it changes the location of the neighborhood to regions where the strategies provide better gains for the supplier. The initial centroid of the neighborhood was the middle point of the interval of valid prices and capacities at each period and its radius is a percentage of the range of values for each decision variable.

Once an initial population of strategies was generated, the problem of best response is solved for each one of given the strategies. The supplier's utility is the corresponding performance measure, and a new centroid is defined by computing a weighted average of the individual strategies where the weights are the associated performances or utilities. The algorithm was implemented by using an integration of software: a) GAMS to solve the best response problem and 2) MATLAB to execute the population based simulated annealing and explore a region in the space of strategies where the performance is good enough in terms of the price setting and the capacity planning competition.

Five test problems were generated to test the effectiveness of the proposed algorithm. Each test problem resulted from changing the demand; the production costs of the auto-parts for each supplier; the initial capacity of the suppliers, the price of increasing this capacity at each period and the maximum increment in capacity allowable at each period. For all the test problems the initial solutions have a poor performance and result in economic losses for supplier i. This happens because the increments in capacity are not compatible with the price strategy; initially supplier i makes higher investments in capacity to gain additional volumes of production subcontracted but due to the need to offer competitive prices, the monetary sales do not compensate its capacity's increments. As more promising regions are explored, the utilities of supplier i improve and economic gains are achieved as number of iterations of the proposed algorithm goes on. The difference between the utilities of both suppliers also reduces as the number of search iterations increases and best strategies are found. The winning supplier is defined in terms of its initial advantages: large initial capacity and lower production costs. However, the best price-capacity expansion strategy always results in a utility for both suppliers while the buyer gets consecutive price reductions and eventually subcontracts all the auto-parts production to the suppliers. This small set of test problems show the advantages of formulating the make-or-buy decision as a bi-level mathematical programming problem and how a buyer may improve the performance of suppliers (price reductions and increased capacity) by using competitive mechanisms.

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