Pricing, product quality, and retail service in a three-echelon supply chain

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Abstract. This paper proposes a model of a three-echelon supply chain consisting of one supplier, one manufacturer, and one retailer, who serve customer demand that is sensitive to retail price, product quality, and retail service. By means of game theory, the firms’ optimal strategies regarding margins, quality level, and service level are determined analytically for three different scenarios: (a) a symmetrical distribution of power within the supply chain (Nash game); (b) a dominant manufacturer (Manufacturer Stackelberg game); (c) a joint profit maximization of the firms (Co-operation game). The solutions are analyzed on the base of a first numerical example, which proves that the presented model yields logically consistent results.

Keywords: three-echelon supply chain; game theory; pricing; quality; service

Introduction

The interaction between supply chain members constitutes not only an important issue of many managers in practice, but also an extensive field of academic research. Due to its complexity, the application of methods from operations research is very common—especially with regard to game theory, as it allows to study the various relationships between firms and the interdependencies between their decisions. A good overview of possible research directions and game-theoretical applications can be found in Leng and Parlar (2005). One direction deals with the correct setting of variables that influence consumer demand within a supply chain. Here, especially pricing is intensively studied (see, e.g., Choi, 1991), but also other demand drivers like advertising (see the review of Aust and Buscher, 2014), product quality (see, e.g., Hsieh and Liu, 2010; Xie et al., 2011; Zhu et al., 2007), and retail service (see, e.g., Lu and Liu, 2013; Tsay and Agrawal, 2000; Wu, 2010) are analyzed under different market settings and supply chain configurations. Thereby, one might recognize a certain trend of including more than one of these factors into consideration, as this allows more general insights into consumer behavior. However,
this is mostly limited to two factors like price and advertising or price and quality. The work of De Giovanni (2011) constitutes an exception, as it simultaneously considers pricing, advertising, and product quality in a dynamic model. Similarly, most works concentrate on competition between firms belonging to the same echelon of the supply chain or on the interaction between two echelons, while models dealing with more than two echelons are sparse (see Chung et al., 2011, for an analysis of pricing in a three-echelon supply chain). Hence, this paper shall contribute to fill this gap by proposing a model of a three-echelon supply chain, which is composed of one supplier, one manufacturer, and one retailer, who are able to influence consumer demand by the three factors price, quality, and service.

According to a market research by Ipsos in 2011, quality is the most important factor when buying a new car for about 45% of the interviewed Americans (http://www.ipsos-na.com/news-polls/pressrelease.aspx?id=5435, accessed 28 March 2014). However, the term quality covers various aspects of a product. For example, Garvin (1984) defines eight dimensions of quality, e.g., the performance of a product, its reliability and durability, or its conformance with given specifications. For the sake of simplicity, we follow Banker et al. (1998) and summarize these characteristics to a quality level that is appreciable to consumers and leads—ceteris paribus—to a higher demand.

Also customer service can be a decisive element of a buying decision. Fallah (2011) distinguishes three levels of service: first, pre-transaction elements provide an appropriate framework for the buying process, e.g., by well-founded technical information and consultation regarding the product; second, transaction elements are related to the actual buying process, like the convenience of the placing the order or detailed information on expected delivery time; lastly, post-transaction elements accompany the customer during his use of the product, like repair services or the dealing with complaints. For the sake of simplicity, this is also summarized to a service level during the further study, which positively influences consumer demand.

The remainder of this paper is organized as follows: First, we introduce a three-echelon supply chain model and define profit and demand functions, together with an explanation of the relevant decision variables and market parameters. On that base, three game scenarios with different distribution of power and relationship between supplier, manufacturer, and retailer are analyzed and closed-form expression are derived for firms’ margins, quality levels, and service level. Then, a numerical example is used to prove if the results of our model are logically consistent. In the end, some concluding remarks are given, together with an outlook on the intended next steps in that research.
Model formulation

As illustrated in Figure 1, we consider a three-echelon supply chain consisting of one supplier, one manufacturer, and one retailer selling a single product to end consumers. The manufacturer purchases a raw or primary material from the supplier, which is then further processed within his own production of the final product. Without loss of generality, we assume that one unit of raw material is needed to manufacture one unit of the final product. Furthermore, both production processes of supplier and manufacturer influence the quality of the final product according to the quality level chosen, which are denoted by \( q \) and \( Q \), respectively (please see Table 1 for a complete listing of mathematical variables and parameters that are used throughout the paper). Similarly to Aust (2012), we assume that the total quality of the final product is characterized by \((q + Q)\).

According to Banker et al. (1998), De Giovanni (2011), and Gurnani et al. (2007), these quality levels cause both variable and fixed quality costs, whereby variable costs depend linearly on quality and quantity with cost coefficients \( \lambda_s \) and \( \lambda_m \) (with \( \lambda_s, \lambda_m \in \mathbb{R} \)), while fixed costs incur with a quadratic relation to the quality level with cost parameters \( \mu_s \) and \( \mu_m \) (with \( \mu_s, \mu_m \geq 0 \)). The latter assumption is motivated by the observation in practice that investments in quality often increase progressively for further improvements, like in the case of new machines with lower manufacturing tolerances. As apparent, we allow that quality level may have either increasing or decreasing influence on variable production costs: One could imagine the first case when the quality investment refers to a more acute monitoring, while the second case may occur after an investment in new machines.

The retailer carries out the sales of the product, which also includes additional customer services like pre-sales advisory services, after-sales technical support, etc., which are collectively represented by a service level \( s \). As these services are often part of the retailer’s business philosophy and are offered for all products, we only assume quantity-independent service costs \( \sigma s^2 \), which depend quadratically on the service level \( s \) with a service cost parameter \( \sigma \) with \( \sigma \geq 0 \) (cf. Lu and Liu, 2013; Tsay and Agrawal, 2000; Wu, 2012). This disproportionate increase of service costs again refers to the commonly diminishing returns on such activities.

As visible from Fig. 1, each echelon individually charges a price for its product, i.e., the supplier’s price \( r \) for one unit of raw material, the manufacturer’s wholesale price \( w \) as well as the retailer’s retail price \( p \) for one unit of the final product. Letting \( D \) denote the quantity demanded by the consumers and assuming that the whole demand can be
fulfilled by each echelon, we are able to formulate the profit functions of supplier ($\Pi_s$), manufacturer ($\Pi_m$), and retailer ($\Pi_r$):

\begin{align}
\Pi_s &= (r - \lambda_s q)D - \mu_s q^2 \\
\Pi_m &= (w - r - \lambda_m Q)D - \mu_m Q^2 \\
\Pi_r &= (p - w)D - \sigma s^2.
\end{align}

For example, the supplier realizes a sales volume of $rD$, out of which he has to bear quality-related variable production costs $\lambda_s qD$ as well as fixed costs $\mu_s q^2$. As previous analyses of supply chain interaction revealed, it is favorable from modelling perspective to directly consider the players' margins instead of the charged prices (cf. Aust and Buscher, 2012). Hence, we introduce manufacturer's margin $M = w - r$ and retailer's margin $m = p - w$, which leads to the following equation for the retail price:

$$p = r + M + m. \quad (4)$$

The demand quantity $D$ of the end consumers depends on three factors: retail price $p$, product quality $q + Q$, and retailer's service level $s$. We assume the following linear demand function, which is widely used in literature and represents a good compromise between reality and mathematical tractability (see Huang et al., 2013, for a general overview of demand functions in decision modeling; Banker et al., 1998, and Xie et al., 2011, for quality demand; and Lu and Liu, 2013, and Wu, 2010, for service demand):

$$D = \alpha - \beta p + \gamma (q + Q) + \delta s. \quad (5)$$

<table>
<thead>
<tr>
<th>Variables</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$ Price of raw material</td>
<td>$\alpha$ Initial base demand</td>
</tr>
<tr>
<td>$w$ Wholesale price</td>
<td>$\beta$ Price sensitivity</td>
</tr>
<tr>
<td>$p$ Retail price</td>
<td>$\gamma$ Quality sensitivity</td>
</tr>
<tr>
<td>$M$ Manufacturer’s margin</td>
<td>$\delta$ Service sensitivity</td>
</tr>
<tr>
<td>$m$ Retailer’s margin</td>
<td>$\lambda_s$ Variable quality cost parameter</td>
</tr>
<tr>
<td>$q$ Supplier’s quality level</td>
<td>$\mu_s$ Fixed quality cost parameter</td>
</tr>
<tr>
<td>$Q$ Manufacturer’s quality level</td>
<td>$\sigma$ Service cost parameter</td>
</tr>
<tr>
<td>$s$ Retailer’s service level</td>
<td></td>
</tr>
<tr>
<td>$D$ Demand</td>
<td></td>
</tr>
<tr>
<td>$\Pi_i$ Profits of echelon $i$</td>
<td></td>
</tr>
</tbody>
</table>
Here, parameter $\alpha$ denotes the market’s initial base demand, parameter $\beta$ characterizes
the price sensitivity of the consumers, and parameter $\gamma$ (\( \delta \)) can be interpreted as the
consumers sensitivity to product quality (service). With Eqs. (4) and (5), we can reformulate
the profit functions given in Eqs. (1) – (3) as follows:

\[
\begin{align*}
\Pi_s(r, q) &= (r - \lambda_s q)[\alpha - \beta(r + M + m) + \gamma(q + Q) + \delta s] - \mu_s q^2 \\
\Pi_m(M, Q) &= (M - \lambda_m Q)[\alpha - \beta(r + M + m) + \gamma(q + Q) + \delta s] - \mu_m Q^2 \\
\Pi_r(m, s) &= m[\alpha - \beta(r + M + m) + \gamma(q + Q) + \delta s] - \sigma s^2.
\end{align*}
\]

**Nash game**

At first, we consider an equal distribution of power within the supply chain where no
firm is able to dominate its partners and every decision is made without cooperation.
Hence, every player tends to maximize his own profit independently of the others’ actions.
This situation corresponds to a Nash game, which leads to an equilibrium where no
player can obtain higher profits without the other players’ outcomes being reduced. In
order to find this equilibrium, one has to determine each player’s best-response functions,
which solely depend on his counterparts’ decision variables. Therefore, we have three
individual optimization problems:

\[
\max_{r, q \geq 0} \Pi_s(r, q) \quad \text{and} \quad \max_{M, Q \geq 0} \Pi_m(M, Q) \quad \text{and} \quad \max_{m, s \geq 0} \Pi_r(m, s).
\]

After setting each first order partial derivative of the profits functions stated in Eqs.
(6) – (8) to zero, we can solve the resulting system of equations. This procedure leads to
the Nash equilibrium, which is given in the following proposition:

**Proposition 1:** An equal distribution of power within the supply chain can be characterized
by a Nash equilibrium with the following solutions:

\[
\begin{align*}
\sigma &= \frac{2\alpha \mu_s \mu_m \sigma + 2\mu_s - \beta \lambda_s^2}{\Psi_1} \quad \text{for the supplier}; \\
\gamma &= \frac{2\alpha \mu_s \mu_m \sigma + 2\mu_m - \beta \lambda_m^2}{\Psi_1} \quad \text{for the manufacturer}; \\
m &= 2\alpha \mu_m \mu_s \sigma / \Psi_1 \quad \text{for the retailer}; \quad \Psi_1 = -\gamma^2 \sigma (\mu_s + \mu_m) + 2\beta \gamma \sigma (\lambda_m \mu_s + \lambda_s \mu_m) - (\delta^2 - 8\beta \sigma) \mu_m \mu_m - \beta^2 \sigma (\lambda_m^2 \mu_s + \lambda_s^2 \mu_m).
\end{align*}
\]

**Manufacturer Stackelberg game**

In this section, we analyze a situation where the manufacturer obtains the channel power
and dominates his partners, which is a common scenario in many industries. For example, in
the automotive industry, we find few large Original Equipment Manufacturers (OEM)
like Toyota or Volkswagen sourcing parts from a multitude of different and often sub-
stitutable suppliers on the one hand; on the other hand, retailers mostly act on the base of dealership contracts with OEM, which can contain an extensive set of regulations concerning the product mix offered by the retailer, the realization of advertising campaigns, or the standards of after-sales service, etc.

A Stackelberg game is used to model this asymmetric distribution of power. It is characterized by a sequential process of decision making, where one player (the Stackelberg leader) acts first, while the other player(s) are only able to react to this action in the second step. Of particular importance is the underlying assumption of complete information, which enables the Stackelberg leader to anticipate the actions of his follower(s) and to include this knowledge into his decision. As there is no relationship between supplier and retailer, the Nash game between them remains unchanged.

Mathematically, we apply backward induction and start with the Nash game between supplier and retailer, which corresponds to the following two optimization problems:

\[
\begin{align*}
\max \Pi_s(r, q) \quad & \text{s.t. } r, q \geq 0 \quad \text{and} \quad \\
\max \Pi_r(m, s) \quad & \text{s.t. } m, s \geq 0.
\end{align*}
\]

Solving these problems by setting the first order partial derivatives \(\partial \Pi_s / \partial r, \partial \Pi_s / \partial q, \partial \Pi_r / \partial m, \text{ and } \partial \Pi_r / \partial s\) to zero leads to the best-response functions of supplier and retailer, which constitute the constraints of the manufacturer’s decision problem:

\[
\begin{align*}
\max \Pi_m(M, Q) \quad & \text{s.t. } r = (\alpha - \beta M + \gamma Q)(2\mu_s + \gamma \lambda_s - \beta \lambda_s^2)\sigma / \Psi_2 \\
q &= (\alpha - \beta M + \gamma Q)(\gamma - \lambda_s^2)\sigma / \Psi_2 \\
m &= (\alpha - \beta M + \gamma Q)2\mu_s\sigma / \Psi_2 \\
s &= (\alpha - \beta M + \gamma Q)\delta \mu_s / \Psi_2 \\
M, Q &\geq 0 \\
\text{with } \Psi_2 &= -\gamma^2 \sigma + (2\beta \gamma \sigma - \beta^2 \sigma)\lambda_s + (6\beta \sigma - \delta^2)\mu_s.
\end{align*}
\]

By means of the constraints, the decision variables of supplier and retailer can be eliminated from the manufacturer’s profit function. From \(\partial \Pi_m / \partial M = 0\) and \(\partial \Pi_m / \partial Q = 0\), we can determine the manufacturer’s profit-maximizing strategy \((M, Q)\) as well as the complete Manufacturer Stackelberg equilibrium, which is given in the following proposition:

**Proposition 2:** A dominant manufacturer within the supply chain can be characterized by a Manufacturer Stackelberg equilibrium with the following solutions:

\[
\begin{align*}
r &= \alpha \mu_m \sigma(\gamma \lambda_s + 2\mu_s - \beta \lambda_s^2) / \Psi_3 \quad \text{and} \quad q = \alpha \mu_m \sigma(\gamma - \beta \lambda_s) / \Psi_3 \quad \text{for the supplier;} \\
M &= \alpha(-\gamma^2 \sigma \mu_m + \beta \gamma \sigma (\lambda_m \mu_s + 2\lambda_s \mu_m) - (\delta^2 - 6\beta \sigma)\mu_m \mu_s - \beta^2 \sigma (\lambda_m^2 \mu_m + \lambda_s^2 \mu_m)) / \beta \Psi_3 \quad \text{and} \quad Q = \alpha \mu_m \sigma (\gamma - \beta \lambda_m) / \Psi_3 \quad \text{for the manufacturer;} \quad \text{and} \\
m &= 2\alpha \mu_m \mu_s \sigma / \Psi_3 \quad \text{and} \quad s = \alpha \mu_m \mu_s \sigma / \Psi_3 \quad \text{for the retailer;} \quad \text{with} \\
\Psi_3 &= -\gamma^2 \sigma (\mu_m + 2\mu_m) + 2\beta \gamma \sigma (\lambda_m \mu_s + 2\lambda_s \mu_m) - (2\delta^2 - 12\beta \sigma)\mu_m \mu_s - \beta^2 \sigma (\lambda_m^2 \mu_m + 2\lambda_s^2 \mu_m).
\end{align*}
\]
Cooperation

Lastly, we assume that the supply chain members cooperate and make their decision together with the objective of a maximization of the total supply chain profit. This joint profit maximization also corresponds to a vertical integrated firm, which fulfills all considered tasks on its own, and is often used as a benchmark solution for other game scenarios.

We start with the supply chain’s total profit function $\Pi_{SC}$, which can be calculated via the sum of the firms’ individual profit functions given in Eqs. (6) – (8):

$$\Pi_{SC} = (p - \lambda_s q - \lambda_m Q)[\alpha - \beta p + \gamma (q + Q) + \delta s] - \mu_s q^2 - \mu_m^2 Q^2 - \sigma s^2. \tag{12}$$

Please note that the individual margins are replaced by the retail price $p$ (see Eq. (4)), as we concentrate on the total profit instead of the actual profit split between supplier, manufacturer, and retailer. Hence, the optimization problem depends on the four decision variables retail price $p$, quality level $q$ and $Q$ of supplier and manufacturer, respectively, and retailer’s service level $s$:

$$\begin{align*}
\max \Pi_{SC}(p, q, Q, s) \\
s.t. \quad p, q, Q, s \geq 0.
\end{align*} \tag{13}$$

Setting the first order partial derivatives $\partial \Pi_{SC}/\partial p$, $\partial \Pi_{SC}/\partial q$, $\partial \Pi_{SC}/\partial Q$, and $\partial \Pi_{SC}/\partial s$ to zero and solving the resulting system of equations results in the solution given in the following proposition:

**Proposition 3:** A joint profit maximization of the entire supply chain can be characterized by a Cooperation with the following solutions:

$$p = \alpha \sigma [2 \mu_m \mu_s + \gamma (\lambda_m \mu_m + \lambda_s \mu_m) - \beta (\lambda_m^2 \mu_s + \lambda_s^2 \mu_m)]/\Psi_s; q = \alpha \mu_m \sigma (\gamma - \beta \lambda_s)/\Psi_s; Q = \alpha \mu_s \sigma (\gamma - \beta \lambda_m)/\Psi_s; \quad \text{and} \quad s = \alpha \mu_m \mu_s \sigma \Psi_s; \quad \text{with} \quad \Psi_s = -\gamma^2 \sigma (\mu_s + \mu_m) + 2 \beta \gamma \sigma (\lambda_m \mu_s + \lambda_s \mu_m) - (\delta^2 - 4 \beta \sigma) \mu_s \mu_m - \beta^2 \sigma (\lambda_m^2 \mu_s + \lambda_s^2 \mu_m).$$

A first numerical example

Due to the complexity of the derived closed-form expressions, we base our analysis on a numerical example. To get a first impression if the model yields logically consistent results, we start with the following (arbitrary) set of parameters: $\alpha = 100$, $\beta = 2$, $\gamma = 1$, $\sigma = 0.75$, $\lambda_s = 0.1$, $\lambda_m = 0.2$, $\mu_s = 0.5$, $\mu_m = 1$ and $\sigma = 1$. Here, consumers’ sensitivity decreases from retail price over product quality to service ($\beta > \gamma > \delta$). Regarding quality costs, we can see that the supplier has lower cost coefficients (both variable and fixed costs) compared to the manufacturer, and that variable costs parameters $\lambda_s$ and $\lambda_m$ are positive, i.e., a higher quality level also results in higher production costs. This framework yields the solutions given in Table 2.
Table 2. Numerical example.

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>r</th>
<th>M</th>
<th>m</th>
<th>p</th>
<th>q</th>
<th>Q</th>
<th>s</th>
<th>D</th>
<th>(\Pi_s)</th>
<th>(\Pi_m)</th>
<th>(\Pi_r)</th>
<th>(\Pi_{SC})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nash</td>
<td>15.66</td>
<td>15.37</td>
<td>14.50</td>
<td>45.52</td>
<td>11.60</td>
<td>8.35</td>
<td>5.44</td>
<td>28.99</td>
<td>352.99</td>
<td>401.32</td>
<td>390.68</td>
<td>1,145.00</td>
</tr>
<tr>
<td>Stackelberg</td>
<td>10.82</td>
<td>26.05</td>
<td>10.02</td>
<td>46.90</td>
<td>8.02</td>
<td>3.01</td>
<td>3.76</td>
<td>20.05</td>
<td>168.76</td>
<td>501.13</td>
<td>186.78</td>
<td>856.66</td>
</tr>
<tr>
<td>Cooperation</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>39.33</td>
<td>27.60</td>
<td>10.35</td>
<td>12.94</td>
<td>69.00</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1,724.88</td>
</tr>
</tbody>
</table>

According to this example, firms’ margins are very similar in the Nash equilibrium, while the manufacturer can enforce a considerably higher margin when obtaining Stackelberg leadership. Interestingly, the retail price is only marginally affected by this shift of margins, as the followers’ margins are reduced to nearly the same degree. Obviously, Cooperation is the most favourable scenario for consumers, as it leads to the lowest retail price. Furthermore, the supplier makes use of his lower quality-related cost parameters and sets higher quality levels than the manufacturer in each considered equilibrium. From a consumer’s point of view, the Cooperation is again the most desirable setting, because quality and service levels are higher than in any other game, while the lowest quality and service levels result in the Stackelberg game.

Obviously, the resulting profits are of particular importance to the supply chains members. Here, we can state that—similarly to the analysis of margins—a Nash equilibrium yields a balanced division of profits within the supply chain. In contrast, the dominance of the manufacturer increases his own profit, though, but drastically diminishes the profits of supplier and retailer as well as the entire supply chain’s profit. Despite of the lowest price and by far the highest expenditures for quality and service (for example, the supplier has quality costs—which can be calculated via \(\lambda_qD + \mu_qq^2\)—of 571.24 in Cooperation game compared to 48.22 in Manufacturer Stackelberg game), Cooperation leads to the highest joint profit for the supply chain members. The reason for this can be seen in demand quantity, which is more than twice as high as in the Nash equilibrium. Thereby, the loss of a lower retail price and higher expenditures related to quality and service are clearly outweighed.

Conclusions

In this paper, we analyzed pricing, quality and service decisions of a three-echelon supply chain consisting of supplier, manufacturer, and retailer. Game theoretic solution concepts allowed us to compare different forms of interaction and distribution of power within the supply chain: a Nash game, where firms make their decisions on margins, quality levels, and service level non-cooperatively and under an equal distribution of power; a Manufacturer Stackelberg game, where the manufacturer obtains the channel leadership and dominates supplier and retailer; and lastly, a Cooperation between the three players with the objective of a joint profit maximization, which could also be seen as a vertical integration. We derived closed-form solutions of each considered equilibrium and analyzed a first numerical example, which showed that our model yields logically consistent results.
Obviously, further numerical tests and sensitivity analyses are necessary to prove the correctness of the proposed model also for wider ranges of parameter values and to provide further managerial insights into the interrelations of price-, quality-, and service-decisions of supplier, manufacturer, and retailer. Furthermore, the inclusion of additional game settings like a supplier- or retailer-leadership could be of interest, as this would increase the significance of our findings also for other industries where, e.g., retailers obtain the dominant position like in the food retailing sector.

References


