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Analyzing the benefits of manure separation using mathematical optimization

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Abstract. Optimization techniques are used extensively for strategic and operations planning in a large number of system engineering applications. We consider here the coupling of a crop planning, manure separation process and a nutrient management system for dairy farms. A nonlinear programming model is developed that determines optimal settings for each of these systems when coupled via a parametric herd size and farm layout. The model is at a full farm, or small farm system level. Numerical experiments are provided to illustrate the use of the model for exploring the interactions between environmental constraints and nutrient requirements and the logistic tradeoffs between manure processing and exogenous fertilization. Our results clearly show the benefits of manure separation in both of an environmental metric and an economic metric. Extensions that incorporate coupling of multiple optimization models are also discussed.

Keywords: animal manure separation; mathematical modelling; nonlinear optimization

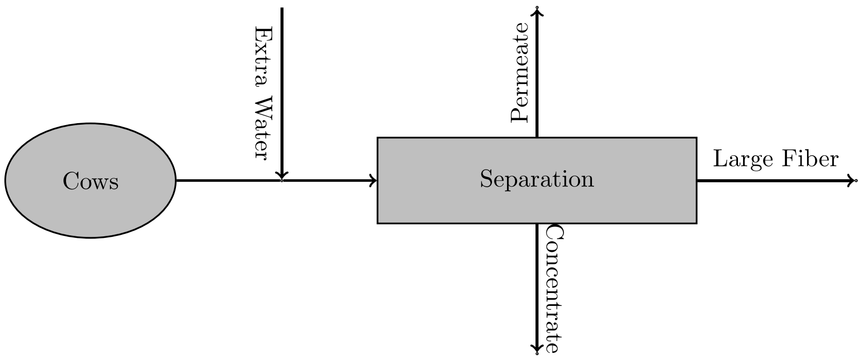
Introduction

In the dairy industry of Wisconsin, USA, the accumulation of manure from dairy cows is a major issue. The attraction of larger herd sizes and its accompanying economies of scale are offset against the need for land to produce feed, cost of nutrient requirements, the need to respect environmental effects such as leaching, chemical side effects and energy overuse, and the need to dispose of manure produced by the herd. All of these factors lead to a complex interacting system that involves a number of different components, each linked by a flow of physical materials and monetary concerns. We aim to build a strategic level model that addresses these interactions, and that will be used in connection with a number of other component system models. This necessitates a level of aggregation discussion, along with an understanding of how different variables in the strategic model feed more detailed models of separation systems, nutrient management models and planning systems for crop, storage and farming strategies. Such models will incorporate both stochastic effects and operational decisions and will be implemented as coupled models with the GAMS modeling system.

Our current model presented in this extended abstract is a nonlinear optimization problem that approximates and captures key features of farmers' behavior while trying to minimize the total operation cost. The nonlinearity arises mainly from simultaneously determining allocation of land pieces for different crops and making operational decisions. Our model exploits the main effects of manure separation, that different separation products have very different natures such as Nitrogen/ Phosphorus ratios and logistic efficiencies. We design two experiments, using our model, to illustrate the benefits of manure separation technology in an environmental metric and an economic metric. Relevant previous work includes Somda (2003) and Giasson etc. (2002). However, to the best of our knowledge, neither work involve a manure separation nor proposed an extensible framework for coupling several optimization models, where lies our main novel contributions.

Ultra filtration manure separation process

In this extended abstract we focus on manure separation by the ultra filtration technology. A very simplified flow chart that characterizes the main input/output is the following. It is worth noting that our model applies to any kind of separation technology. We chose ultra filtration simply because we have access to parameters corresponding to a real farm setting.



Raw manure flows out of cows/barn. With extra water added (cleaning separated bedding sand - a process we do not explicitly consider here), this augmented flow enters the ultra filtration manure separation process. The large fiber flow is a product that can be further processed as bio-feedstock, for example, in production of mulch and peat moss. Concentrate and permeate are liquid flows that need to be stored in pits, and can be later applied over farmland as a source of nutrients (we focus on Nitrogen (N) and Phosphorus (P) in this extended abstract). The volume ratio between concentrate and permeate is approximately 1:1, while the concentrate flow contains more solids and higher levels of N and P, those levels in permeate are much lower. Because of this difference, one can apply permeate on farmland using center pivot irrigation system, which is per volume significantly cheaper than applying concentrate or manure, where dragline or trucks are needed. Another benefit of applying permeate as a nutrient source on farmland is that it is considered to be more "Eco-friendly". The N:P ratio is approximately 5:1, instead of 2:1 in manure. As of the concentrate has more nutrients per unit volume, logistics is more efficient than un-separated manure. We are able to capture these insights in our optimization model.

Modeling

We are interested in two scenarios: the base scenario, where farmers do not employ manure separation and simply spread manure (raw manure with certain amount of extra water added), and the separation scenario, where farmers spread concentrate and permeate. The model we present here implements these two scenarios by simple re-parameterization. To simplify our presentation and easily differentiate optimization variables and constant parameters, we use letters with a bar on top to denote constant parameters, and letters without a bar to denote optimization variables. Main parameters/variables and the full model are summarized below.

r	*Indices for manure products*
l	*Indices for land distance rings;*
k	*Indices for crops;*
$\bar{S}_l$	*Area of ring* l *;*
$\bar{V}_k$	*Nitrogen demand level;*
$\bar{F}_k$	*Phosphorus removal level;*
$\bar{p}_{r,i}$	*Size of portion i of source r;*
$S_{k,l}$	*Area for crop k in ring l;*
$T_{r,k,l}$	*Application level of source r.*
$\bar{N}_{r,i}$	*Nitrogen density of portion i or source r;*
$\bar{\mu}$	*Phosphorus rule-abiding level;*
$\bar{H}_r$	*Average Phosphorus density;*
$\bar{A}_{r,l}$	*Application cost of source r to ring* l *;*
$\bar{P}(N)$	*Market price of Nitrogen;*

$\bar{D}_k$	*Feed demand for crop k;*
$\bar{M}_k$	*Market price of crop k.*
ρ_k	*Splitting ratio for crop k;*
$\bar{R}_k$	*Production rate of crop k.*

$$\begin{aligned}
\mathbf{min}\quad & CostBackFill + CostFeed + CostApply + \lambda \cdot \bar{P}(N) \cdot TotLeach \\
s.t.\quad & S_{k,l} = \rho_k \bar{S}_l, \forall k, l, && (1) \\
& \sum_k \rho_k = 1, && (2) \\
& \bar{V}_r = \sum_k \rho_k \sum_l \bar{S}_l T_{r,k,l}, \forall r && (3) \\
& \bar{\mu}\bar{F}_k \geq \sum_r \bar{H}_r T_{r,k,l}, \forall k, l && (4) \\
& CostApply = \sum_r \sum_l \bar{A}_{r,l} \sum_k \rho_k \bar{S}_l T_{r,k,l} && (5) \\
& BackFill(l) = \sum_k \rho_k \bar{S}_l \sum_i \bar{p}_{1,i}[\bar{v}_k - T_{1,k,l}\bar{N}_{1,i}]_+, \quad (base\ scenario) && (6) \\
& BackFill(l) = \sum_{k,i,j} \rho_k \bar{S}_l \bar{p}_{1,i} \bar{p}_{2,j}[\bar{v}_k - T_{1,k,l}\bar{N}_{1,i} - T_{2,k,l}\bar{N}_{2,j}]_+, \quad (sep.\ scenario) && (7) \\
& CostBackFill = \sum_l [\bar{P}(N) + FertDistrCost(l)] \cdot BackFill(l) && (8) \\
& TotLeach = \sum_{l,k} \rho_k \bar{S}_l \sum_i \bar{p}_{1,i} g(\bar{v}_k + [\bar{v}_k - T_{1,k,l}\bar{N}_{1,i}]_+), \quad (base\ scenario) && (9) \\
& TotLeach = \sum_{l,k,i,j} \rho_k \bar{S}_l \bar{p}_{1,i} \bar{p}_{2,j} g([-\bar{v}_k + T_{1,k,l}\bar{N}_{1,i} + T_{2,k,l}\bar{N}_{2,j}]_+), \quad (sep.\ scenario) && (10) \\
& CostFeed = \sum_k \bar{M}_k[\bar{D}_k - \rho_k \bar{S}\bar{R}_k]_+ && (11) \\
& S_{k,l} \geq 0, \rho_k \geq 0, \ T_{r,k,l} \geq 0
\end{aligned}$$

Farm Land and Crops area. We model our farmland into five rings, with distances to barn of cows from 1 mile to 5 miles. Each ring comprises of land of certain size. We assume that the split ratios for different crops are the same in all rings, due to the necessity of crop rotation. Equation (1) and (2) formalize these assumptions. **Crops nutrient needs.** We assume that each crop *k* has a Nitrogen demand level denoted by $\bar{v}_k$ (lbs/acre) and Phosphorus removal level $\bar{F}_k$. After application of nutrient sources (manure or concentrate/permeate), we always backfill up to level $\bar{v}_k$ where the Nitrogen level received fall below $\bar{v}_k$, by using purchased commercial fertilizers. We do not backfill for Phosphorus as it is always assumed to be sufficient in soil. This assumption is actually valid in most dairy farms in Wisconsin, as existing fields have had too much phosphorus already due to the historical need to dispose of manure and apply for enough nitrogen. The crop removal level $\bar{F}_k$ is used as an upper bound of Phosphorus application, in an effort to reduce the Phosphorus level in long term and improve sustainability. In later analysis, we consider scenarios in which farmers strictly follow this restriction, in which farmers violate it to a certain degree and in which totally ignore this restriction (eco-unfriendly).

Density variation in nutrient sources. It was founded in the study of Cox (2012) that the manure applied over land is highly variable, i.e., the concentration of nitrogen applied varies from place to place, because of the storage in lagoons before storage, limited power of agitation. However, the separated manure products, i.e., concentrate and permeate considered here, have less variability in application. This fact could affect the optimal ways of spreading the raw manure or manure separated products. For example, on a specific piece of land, we tend not to spread highly variable products. To incorporate this factor into our model, we approximate the nitrogen concentration in the whole body of each nutrient source to spread with a finite distribution, i.e., for nutrient source *r*, $\bar{p}_{r,i}$ portion of it has nitrogen density $\bar{N}_{r,i}$ (in lbs per 1000 gallons), where

$$\sum_{i=1}^{n} \bar{p}_{r,i} = 1, \forall r .$$

The distribution $(\bar{p}_{r,i}, \bar{N}_{r,i}), i = 1,...,n$ for each *r* is constructed from data collected in Cox (2012).

Distribution of Nutrient sources. Equation (3) represents the total volume constraint of each nutrient source, i.e., we assume that the farmer needs to get rid of all raw manure/separated products, as otherwise the removal cost is extremely high. Note that this is a nonconvex bilinear constraint. Depending on whether farmers strictly follow the Phosphorus removal level guidance, which was parameterized by $\bar{\mu}$, we use equation (4) to represent the Phosphorus constraints for each crop k in each ring l.

Equations (5) – (11) calculate main cost associated in application of nutrient sources, back filling and environmental penalty/opportunity loss of leaching. The backfill amount (6) – (7) and the total leaching amount (9) – (10) are computed differently in the two scenarios of our concern. Note in these constraints,

$[x]_+ = \max(0, x)$ and $g(x) = 0.04x^2$ is the Nitrogen leaching curve fitted from field experiments. See Vanotti and Bundy (1994). In our GAMS implementation, we introduce variables for the expressions inside $[.]_+$, and replace the $[.]_+$ operation by two linear constraints without loss of generality. We skip the proof here in the interests of space and simpler notation. Finally the total cost is the summation of all monetary costs and optional penalty of leaching.

Parameterization and Implementation. Most of our parameters are chosen corresponding to the Larson Acres (http://www.larsonacres.com/, accessed on March 30, 2013) to our knowledge. We implement our model in the General Algebraic Modeling System (GAMS). This has the benefit that multiple different solvers can be utilized for solution, and while our results are computed using the CONOPT solver (Drud (1994)), we also ensured that the solutions found are globally optimal by checking the results using BARON (Tawarmalani and Sahinidis (2005)), a global optimization package. Both solvers determine the same optimal solutions in all our experiments, but CONOPT is typically an order of magnitude faster.

Analysis

We illustrate, in the following two experiments, that our optimization model can be used to explore the complex interaction among profit-oriented farmers, economical factors and environmental constraints.

Positive Effect of Separation in Reducing Nitrogen Leaching. We run the model under the base scenario (no separation) and the separation scenario with different Phosphorus violation rules (by choosing different $\bar{\mu}$ in the previous model. Our results clearly show the farmers' motivation of violating the Phosphorus regardless of the increasing Nitrogen leaching amount, and the positive effect of separation in reducing Nitrogen leaching.

Figure 1 shows how the total cost and total Nitrogen leaching amount change when farmers allow themselves to violate the Phosphorus constraints to a certain degree, while not using manure separation. The horizontal axis represents farmers' Phosphorus violation level, i.e., paramete$\bar{\mu}$r in our model. $\bar{\mu} = 1$ corresponds to 0% Phosphorus violation, and $\bar{\mu} = (1 + 200\%) = 3$ correspond to 200% Phosphorus violation. Clearly farmers have the incentive to violate the Phosphorus bound because of the reducing cost. In practice, this corresponds to dumping relatively large amount of manure over a relatively small area (closer to the barn of cows) because transportation cost to distribute manure to far-away land is high, and this is a driving force in our model. Accordingly, violating the Phosphorus constraints increases the amount of total Nitrogen leaching. When a farmer is allowed to violate the Phosphorus bound for 200%, the total Nitrogen leaching amount is about 70% more than that of when the farmer obey the Phosphorus bound.

When employing ultra filtration to separate manure, though the trends of curves are similar, the environmental risk of extra Nitrogen leaching is significantly reduced. Figure 2 shows that farmers still have the incentive to violate the Phosphorus bound because of cost, but when a farmer violates the Phosphorus bound for 200%, the total Nitrogen Leaching only goes up about 13%, versus 70% in the non-separation scenario. This effect is due to the fact that the permeate flow is a more eco-friendly nutrient source, the concentrate flow is logistically more efficient (per amount of nutrient), and a portion of Nitrogen (about 10%) goes into the dry part (Large Fiber flow).

Cost Saving by varying herd size and Phosphorus Violation. We now concentrate on farmers' cost saving due to the introduction of ultra filtration separation. The cost incurred by installing and running the separation system is amortized yearly (Cox (2012)). Table 1 summarizes our results. In both scenarios, we report the total cost for different parameterization of herd size and Phosphorus violation. We find that the percentage of cost saving is in general larger for farmers with more cows. This is expected because with larger herd size (more manure), the manure-related operation is more important, which can be improved by introducing a manure separation system. Another effect is the saving percentage is higher for farmers who obey the Phosphorus bound. This shows separation monetarily "compensates" this eco-friendly behavior.

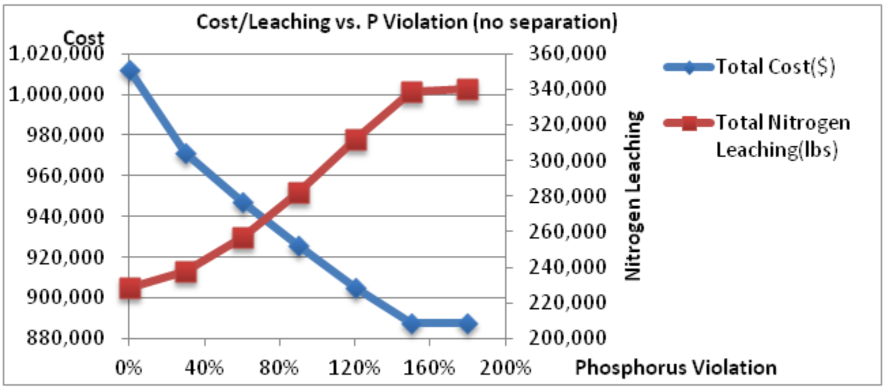


Fig. 1. Farmer's Cost and Nitrogen Leaching while not using separation

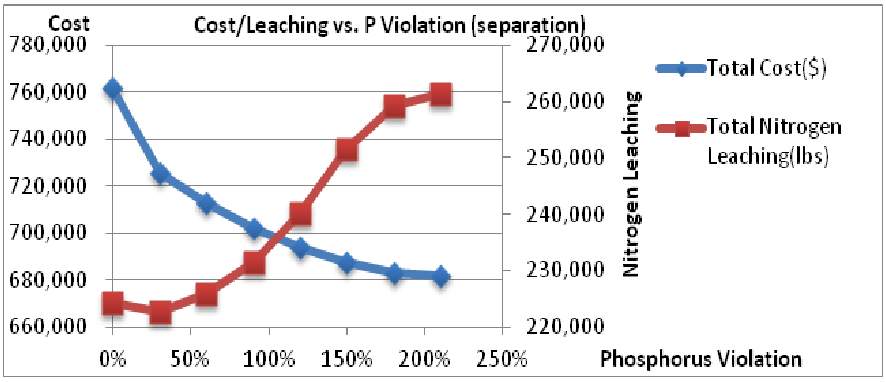


Fig. 2. Farmer's Cost and Nitrogen Leaching while using separation

Further Extension. We remark that our model has evolved significantly after the initial submission of this extended abstract. One large improvement is that we incorporated the time axis into our decision space. We extended this model with a novel Markov chain approach that captures the practical consideration of crop rotations, and turn this model into a long-term decision-aid tool over the (theoretically) infinite time horizon. The Markov chain approach can be seen as a concrete mathematical realization of the vague term "sustainability". Another benefit of this Markov Chain approach is that the complexity of the optimization problem remains tractable for modern nonlinear optimization solvers.

Table 1. Cost savings due to Manure Separation

	Without Separation			With Separation			Total Saving			Percentage of saving		
Herd size	0% P Viol	100% P Viol	Ignore P	0% P Viol	100% P Viol	Ignore P	0% P Viol	100% P Viol	Ignore P	0% P Viol	100% P Viol	Ignore P
2500	822,668	742,469	739,480	781,849	733,888	723,173	40,819	8,581	16,307	4.96%	1.16%	2.21%
2750	915,334	830,619	813,452	850,510	794,927	779,783	64,824	35,692	33,669	7.08%	4.30%	4.14%
3000	1,011,861	918,769	887,431	919,176	855,967	836,394	92,685	62,802	51,037	9.16%	6.84%	5.75%
3250	1,269,853	1,168,378	1,122,883	1,150,943	1,078,465	1,054,463	118,910	89,913	68,420	9.36%	7.70%	6.09%
3500	1,669,587	1,559,717	1,500,094	1,526,686	1,442,694	1,414,264	142,901	117,023	85,830	8.56%	7.50%	5.72%
3750	2,073,278	1,951,056	1,877,342	1,902,429	1,806,922	1,774,063	170,849	144,134	103,279	8.24%	7.39%	5.50%
4000	2,487,029	2,342,395	2,254,603	2,278,172	2,171,151	2,133,863	208,857	171,244	120,740	8.40%	7.31%	5.36%

We have built another short-term decision model, which is a mixed-integer linear optimization problem. The rationale behind is the long-term model provides an optimal practice in the Markovian equilibrium, however planning the “route” to move from current farm status to achieve that equilibrium is itself an optimization problem, which is solved in this short-term model.

The coupling/interaction of these two models form a basis for a decision-aid system for dairy farms, which can be used in strategic designing and evaluating new practice in dairy industry. For example, the flow of large fiber (not valued in current model) can be used to as bio-feedstock in producing mulch or peat moss. The flow of concentrate can be used to produce protein products. Also, both larger fiber and concentrate flows can be pelletized as logistically efficient fertilizers. The Accelerated Renewable Energy research group in University of Wisconsin-Madison is actively exploring/analyzing these possibilities. These alternatives, if proved to be practical, would bring in extra revenue sources and can be very easily incorporated into our model.

Conclusion

Our current model is built to uncover main effects in the operation of the dairy farm system and will be used to explore the economic viability of newly developed separation technologies. While we have demonstrated the economic and environmental benefits of manure separation above using our model, there are huge opportunities to incorporate a much richer variety of products, and to optimize how these products would be strategically utilized in a real farm setting. The amount of uncertainty in the data of these models necessitates a course scale analysis of the form we have carried out above, but the interactions between different components of the system must be considered and the work outlined here has demonstrated that in a simple, stylized but realistic framework.

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