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Additive preemptive possibilistic programming in assemble-to-order production planning: A case study

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Abstract. Additive Preemptive Possibilistic Linear Programming (APPLP) model for Assembly-to-Order (ATO) production planning is proposed in this research. Uncertainties in ATO environment make difficulties in creating a precise plan. So, most of the factories under these conditions always face problems of raw material shortage and unsatisfied demands, including the case study company. Imprecise operating costs and demands are considered in the proposed model. The model attempts to maximize profit by transforming the fuzzy objective function to three crisp objective functions, which are maximizing the opportunity of obtaining the higher profit, minimizing the risk of obtaining the lower profit and maximizing the most possible value of profit. Then, preemptive priority in the additive fuzzy programming is applied for achieving the satisfaction level of each priority that the decision maker satisfies. It makes easiness in finding a compromise solution and adjusting a satisfaction level. Alternative compromise solutions can be easily generated for the decision maker.

Keywords: master production schedule; assemble-to-order; rolling schedule; possibilistic linear programming; preemptive priority in the additive fuzzy programming

Introduction

Master Production Schedule (MPS) is a short-range planning. An effective MPS provides the basis for making good use of manufacturing resources, making customer delivery promises, resolving trade-offs between sales and manufacturing, and attaining the firm strategic objective (Jacobs *et al.*, 2011). In creating an MPS, the nature of the product and the market should be considered. Three basic production environments have been identified: Made-to-Stock (MTS), Made-to-Order (MTO) and Assemble-to-Order (ATO). Each of these environments affects the design of the MPS system. A manufacturing strategy where material and subassemblies are made or acquired according to forecasts, while the final assembly of products is delayed until customer orders have been received is generally referred to as ATO (Hsu and Wang, 2001). Precise forecasting of end-items is extremely difficult and stocking end-items is also high-risk. As a result, ATO firms try to maintain flexibility by starting to produce basic components and subassemblies into the production, but not starting final assembly until customer orders are received. In such environment, a rolling schedule method is commonly applied for supervising the newest market information, satisfying customer requirements and maintaining the lowest inventory (Hsu and Wang, 2001). In a rolling schedule process, at a period t, factory is requested to place their firm orders, FD_{tp0} and perform their demand forecasts for the next few periods. However, imprecise information of customer demands makes a problem in raw material purchasing.

The input data or parameters in real-world, such as demands, resources, costs and unit prices are often imprecise or fuzzy because some information is incomplete or unobtainable. Conventional mathematical programming cannot solve all problems those have imprecision. In dealing with imprecise data, some researchers may apply stochastic programming to solve. However, the main problem is the lack of computational efficiency and inflexible probabilistic doctrines in which the really imprecise meaning of the Decision Maker (DM) might be impossible to model (Wang and Liang, 2005). In 1976, Zimmermann, firstly, introduced fuzzy set theory into conventional Linear Programming problems (Zimmermann, 1991). Moreover, Zadeh (1978) presented the prominence of the theory of possibility, which is related to the theory of fuzzy sets by defining the concept of a possibility distribution as a fuzzy restriction, which acts as an elastic constraint on the value that can be assigned to a variable (Phruksaphanrat, 2011). He demonstrated the significance of the theory of possibility stems from the fact that much of the information on which human decision is based on is possibilistic rather than probabilistic in nature. In 1992, Lai and Hwang proposed a new approach to some of Possibilistic Linear Programming (PLP) problems, which coefficients of the objective are imprecise data. Fuzzy Linear Programming (FLP) is based on the subjective preferred concept for establishing membership functions with fuzzy data, while the PLP is based on the objective degree of event occurrence required to obtain possibilistic distributions with imprecise data. So, FLP techniques may not be applicable. PLP provides computational efficiency and flexibility (Wang and Lai, 2005). It also supports possibilistic decision-making in an uncertain environment. Wang and Lai (2005) proposed a PLP model for solving a single objective APP problem with imprecise demands, parameters and capacity. The fuzzy objective was converted to a Multiple Objective Linear Programming (MOLP) model using the method of Lai and Hwang (Lai and Wang, 1992). PLP approach simultaneously minimizes the most possible value of the imprecise total cost, maximizes the possibility of obtaining lower total cost, and minimizes the risk of obtaining higher total cost. Imprecise forecast demands were converted to crisp demands or constant demands by adopting weight average method. Other studies of PLP problems also use this strategy to solve their applications (Wang and Liang, 2005). However, simultaneous solving the MOLP problem by adjusting their membership functions is difficult and time-consuming because each objective has different range or scale to adjust. Moreover, one objective may be extremely important than the others. Especially, in a production planning problem, the total profit of the firm is extremely important. So, this objective should be set as the first priority and the remaining objective functions are solved accordingly. Weight additive model is useful when the DM wants to improve the solution of a multiple objective problem by applying weight to each objective function and solving them at the same time.

So, in this research, weights are attached to differentiate the relative importance of objective functions in each priority. This method can reduce the problem of membership functions' adjustment. Then, the best compromise solution can be easily obtained.

Additive preemptive possibilistic programming

Indices:

t	planning horizon time period, $t = 1,2,\ldots,T$; p type of products, $p = 1,2,\ldots,P$
c	type of materials, $c=1,2,\ldots,C$; j type of production lines, $j=1,2,\ldots,J$

Parameters:

$\tilde{p}_p$	unit price of product p, (\$/unit)
$\widetilde{cs}_p$	stockout penalty cost per unit of unsatisfied demand for product p, (\$/unit)
$\widetilde{cc}_j$	idle capacity penalty cost per unit of production line j, (\$/unit)
$\widetilde{ci}_c, \widetilde{cm}_c$	inventory holding and purchasing cost per unit of material c, (\$/unit)
$\widetilde{cp}_p, \widetilde{co}_p$	regular and overtime production cost per unit of product p, (\$/unit)
l_c	acquisition lead time of material c, (period)
FD_{tpl_c}	forecast demand for product p at period t, performed by dealers at period t-l_c, (units)
FD_{tp0}	actual order quantities of product p at period t, (units)
CLX_j, CLO_j	capacity of regular and overtime productions per production line j, (units)
u_{pc}	units of material c required for one unit of product p, (units)
Q_c^{min}	the minimum order quantities of material c, (units)
I_c^{max}	the maximum inventory level of material c, (units)
NL_j^{max}	the maximum number of lines for production line type j,(lines)
k_{pj}	1, if product p is produced on production line j, otherwise, 0

Variables:

SS_c	safety stock level of material c, (units)
β_{tp}	service level of product p at period t, (% of order quantities)
X_{tp}, O_{tp}	regular and overtime productions of product p at period t, (units)
SO_{tp}	stockout quantities of product p at period t, (units)
AP_{tp}	actual production quantities of product p at period t, (units)
I_{tc}	inventory level of material c at the end of period t, (units)
$EI_{(t-1)c}$	estimated inventory level of material c at the end of period $t-1$, performed at period $t-l_c$, (units)
Q_{tcl_c}	order quantities of material c at period $t-l_c$, received at period t, (units)
TU_{tpc}	the total usage of material c for product p at period t, (units)
NL_j	the total number of production lines, (lines)

The proposed ATO production planning model can be represented as follows:

$$Max = \sum_{t=1}^{T}\sum_{p=1}^{P}\widetilde{p_p} \times AP_{tp} - \sum_{t=1}^{T}\sum_{p=1}^{P}\widetilde{cs_p} \times SO_{tp} - \sum_{t=1}^{T}\sum_{p=1}^{P}(\widetilde{cp_{pj}} \times X_{tp} + \widetilde{co}_p \times O_{tp}) - \sum_{t=1}^{T}\sum_{c=1}^{C}\widetilde{ci_c} \times I_{tc} - \sum_{t=1}^{T}\sum_{c=1}^{C}\widetilde{cm}_c \times Q_{tcl_c} - \sum_{t=1}^{T}\sum_{j=1}^{J}\widetilde{cc_j}\left[NL_j(CLX_j + CLO_j) - \sum_{p=1}^{P} AP_{tp} \times k_{pj}\right] \tag{1}$$

$$AP_{tp} = \beta_{tp} \times FD_{tp0} \tag{2}$$

$$0 \leq \beta_{tp} \leq 1 \tag{3}$$

$$AP_{tp} = X_{tp} + O_{tp} \tag{4}$$

$$SO_{tp} = FD_{tp0} - AP_{tp} \tag{5}$$

$$TU_{tpc} = AP_{tp} * u_{pc} \tag{6}$$

$$\sum_{p=1}^{P} X_{tp} \times k_{pj} \leq CLX_j \times NL_j \tag{7}$$

$$\sum_{p=1}^{P} O_{tp} \times k_{pj} \leq CLO_j \times NL_j \tag{8}$$

$$NL_j \leq NL_j^{max} \tag{9}$$

$$I_{tc} = I_{(t-1)c} + Q_{tcl_c} - \sum_{p=1}^{P} TU_{tpc} \tag{10}$$

$$I_{tc} \leq I_c^{max} \tag{11}$$

$$EI_{(t-1)c} = I_{(t-lc)c} + \sum_{i=1}^{l_c-1} Q_{t-(l_c-i)c} - (\sum_{i=1}^{l_c-1}\sum_{p=1}^{P} FD_{t-(l_c-i)p} \times u_{pc}) \tag{12}$$

$$Q_{tcl_c} = SS_c + (\sum_{p=1}^{P} FD_{tpl_c} \times u_{pc}) - EI_{(t-1)c} \tag{13}$$

$$Q_{tcl_c} \leq Q_c^{min} \tag{14}$$

$$AP_{tp}, SO_{tp}, X_{tp}, O_{tp}, I_{tc}, EI_{(t-1)c}, TU_{tpc}, Q_{tcl_c}, SS_c, NL_j \geq 0 \; and \; integer \tag{15}$$

The main objective of the proposed ATO production planning is the profit, which composes of income minus costs as shown in (1). The actual production quantity of product p at a period t is shown in (2). The service level of each product is between 0 and 1 as in (3). Equation (4) represents that the actual production quantities of product p at a period t equal to regular and overtime production of product p at period t. Equation (5) expresses stock out derived strictly from the different between actual production quantities and firm order quantities of product p at a period t. The total material usage for each product is represented by (6). The capacity limits of regular and overtime productions express in (7) and (8). Limitation of the number of production lines is shown in (9). Equation (10) shows inventory balance equation. It is calculated from the inventory levels of material c at a period t-1 and quantities of received material c, deducted by actual usage of material c at period t. The number of inventory level should not be greater than the maximum available inventory capacity as shown in (11). The estimated inventory level of order quantities for material c is calculated from inventories of material c at the end of period $t-l_c$ and the remaining inventories of material c, which are used during purchasing lead time is shown in (12). Equation (13) is used to compute order quantities of material c, Q_{tcl_c}, which is purchased at period $t-l_c$. It is derived from safety stocks, gross demands in one period advanced of ending estimated inventory level. The order quantities should be more than the minimum order quantities as in (14).

This work assumes that a triangular possibility distribution can be stated as the degree of occurrence of an event. Imprecise unit prices, $\tilde{p}_i = (p_i^p, p_i^m, p_i^o)$ and cost coefficients, $\tilde{A}_i = (A_i^p, A_i^m, A_i^o)$ are presented by triangular possibility distributions. These kinds of imprecise information exist in the objective function due to price fluctuations, material obsolescence and the ambiguity of costs. Lai and Hwang (1992) referred to portfolio theory and converted the fuzzy objective with a triangular possibility distribution into three crisp objectives. According to their method, the objective function can be fully defined by three prominent points (Z^p, 0), (Z^m, 1) and (Z^o, 0) as shown in Fig. 1. The imprecise objective function can be maximized by pushing the three prominent points towards the right. Because of the vertical coordinates of the prominent points being fixed at either 1 or 0, the three horizontal coordinates are the only considerations (Wang and Liang, 2005). Consequently, solving the imprecise objective requires maximizing Z^m, maximizing Z^o-Z^m and minimizing Z^o-Z^m, simultaneously. These problems involve maximizing the most possible value of the imprecise profit, Z^m, minimizing the risk of obtaining the lower profit, (Z^m-Z^p), and maximizing the possible of obtaining the higher profit, (Z^o-Z^m). Three new crisp objective functions, (Z_1, Z_2, Z_3) can be constructed as follows.

$$
\begin{aligned}
Max\ Z_1 = Z^m & \\
= & \sum_{t=1}^{T}\sum_{p=1}^{P} p_p^m \times AP_{tp} - \sum_{t=1}^{T}\sum_{p=1}^{P} cs_p^m \times SO_{tp} - \sum_{t=1}^{T}\sum_{p=1}^{P} (cp_p^m \\
& \times X_{tp} + co_p^m \times O_{tp}) - \sum_{t=1}^{T}\sum_{c=1}^{C} cm_c^m \times Q_{tcl_c} - \sum_{t=1}^{T}\sum_{c=1}^{C} ci_c^m \times I_{tc} \\
& - \sum_{t=1}^{T}\sum_{j=1}^{J} cc_j^m \left[NL_j\left(CLX_j + CLO_j\right) - \sum_{p=1}^{P} AP_{tp} \times k_{pj} \right]
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
Min\ Z_2 = Z^m - Z^p \\
&= \sum_{t=1}^{T}\sum_{p=1}^{P}(p_p^m - p_p^p) \times AP_{tp} - \sum_{t=1}^{T}\sum_{p=1}^{P}(cs_p^m - cs_p^p) \times SO_{tp} \\
&- \sum_{t=1}^{T}\sum_{p=1}^{P}(cp_p^m - cp_p^p) \times X_{tp} + (co_p^m - co_p^p) \times O_{tp}) \\
&- \sum_{t=1}^{T}\sum_{c=1}^{C}(cm_c^m - cm_c^p) \times Q_{tcl_c} - \sum_{t=1}^{T}\sum_{c=1}^{C}(ci_c^m - ci_c^p) \times I_{tc} \\
&- \sum_{t=1}^{T}\sum_{j=1}^{J}(cc_j^m - cc_j^p) \times \Bigg[NL_j\big(CLX_j + CLO_j\big) \\
&- \sum_{p=1}^{P} AP_{tp} \times k_{pj} \Bigg]
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
Max\ Z_3 = Z^o - Z^m \\
&= \sum_{t=1}^{T}\sum_{p=1}^{P}(p_p^o - p_p^m) \times AP_{tp} - \sum_{t=1}^{T}\sum_{p=1}^{P}(cs_p^o - cs_p^m) \times SO_{tp} \\
&- \sum_{t=1}^{T}\sum_{p=1}^{P}(cp_p^o - cp_p^m) \times X_{tp} + (co_p^o - co_p^m) \times O_{tp} \\
&- \sum_{t=1}^{T}\sum_{c=1}^{C}(cm_c^o - cm_c^m) \times Q_{tcl_c} - \sum_{t=1}^{T}\sum_{c=1}^{C}(ci_c^o - ci_c^m) \times I_{tc} \\
&- \sum_{t=1}^{T}\sum_{j=1}^{J}(cc_j^o - cc_j^m) \times \Bigg[NL_j\big(CLX_j + CLO_j\big) \\
&- \sum_{p=1}^{P} AP_{tp} \times k_{pj} \Bigg]
\end{aligned}
\tag{18}
$$

The additional MOLP problem can be changed into an equivalent single objective LP problem using the fuzzy decision–making of Bellman and Zadeh and Zimmermann's fuzzy programming method (Zimmermann, 1991). The positive ideal solution (PIS) and negative ideal solution (NIS) of these objectives are used to define membership functions as follows (Phruksaphanrat, 2011).

$$\mu(Z_1) = \begin{cases} 1 & \text{if } Z^m > Z_1^{PIS} \\ \dfrac{Z^m - Z_1^{NIS}}{Z_1^{PIS} - Z_1^{NIS}} & \text{if } Z_1^{NIS} \le Z^m \le Z_1^{PIS} \\ 0 & \text{if } Z^m < Z_1^{NIS} \end{cases} \tag{19}$$

$$\mu(Z_2) = \begin{cases} 1 & \text{if } (Z^m - Z^p) < Z_2^{PIS} \\ \dfrac{Z_2^{NIS} - (Z^m - Z^p)}{Z_2^{NIS} - Z_2^{PIS}} & \text{if } Z_2^{PIS} \le (Z^m - Z^p) \le Z_2^{NIS}, \\ 0 & \text{if } (Z^m - Z^p) > Z_2^{NIS} \end{cases} \tag{20}$$

$$\mu(Z_3) = \begin{cases} 1 & \text{if } (Z^o - Z^m) > Z_3^{PIS} \\ \dfrac{(Z^o - Z^m) - Z_3^{NIS}}{Z_3^{PIS} - Z_3^{NIS}} & \text{if } Z_3^{NIS} \le (Z^o - Z^m) \le Z_3^{PIS}, \\ 0 & \text{if } (Z^o - Z^m) < Z_3^{NIS} \end{cases} \tag{21}$$

Then, the complete equivalent multi-objective model for solving the ATO production planning problem can be formulated. The better way to adjust the level of satisfaction for different priorities of objectives can be done by preemptive priority (Phruksaphanrat, 2011) and weight additive model is also useful for reducing time of computation. Then, the complete additive preemptive possibilistic programming in ATO production planning model can be shown as follows:

$$Lex\ max\ =\ \{\mu_1(Z_1), \frac{\sum_{k=1}^{2}\Delta Z_k \mu_k}{\sum_{k=1}^{2}\Delta Z_k}, \frac{\sum_{k=1}^{3}\Delta Z_k \mu_k}{\sum_{k=1}^{3}\Delta Z_k}\} \tag{22}$$

Subject to:

$\mu_k \ge \mu_k^* \quad , k = 1,2$

(2)-(15),(19)-(21),

where, $\Delta Z_k = Z_k^{PIS} - Z_k^{NIS}\ \ \forall_k , k = 1,2, \dots, K$, if the objective function is maximize objective. $\Delta Z_k = Z_k^{NIS} - Z_k^{PIS}\ \ \forall_k , k = 1,2, \dots, K$, if the objective function is minimize objective. μ_k are membership functions of the objectives that are rank to be the kth priority. μ_k^* is the desirable achievement degrees for the *kth* priority membership function.

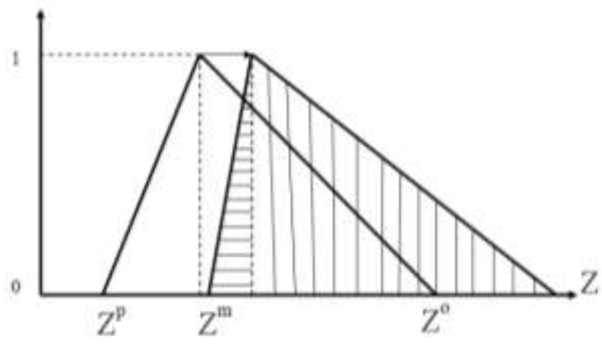

Fig.1. Triangular possibility distribution of $\tilde{Z}$ (the profit)

Numerical example

A real case study of the company which produces the electronic components is illustrated. The planning horizon is six months. There are 16 models of products. Unit prices, regular production costs, overtime production costs, stock out penalty costs, purchasing material costs, inventory material costs and idle capacity penalty costs are imprecise, represented by triangular possibility distributions.

The PIS of each objective is \$2,979,461.6, \$333,578.2 and \$552,887.3, respectively and the NIS of each objective is \$1,764,338.8, \$449,055.3 and \$471,361.8, respectively. In this paper, two cases are selected to illustrate the results. In case 1, the profit is allowed to be reduced 1% of the PIS. Then, the compromise solution is obtained. The satisfaction levels of each objective are 0.990, 0.117 and 0.807, respectively. In case 2, profit is allowed to be reduced 5% of the PIS. The satisfaction levels of each objective are changed to 0.989, 0.126 and 0.821, respectively. Relaxing of the first objective function causes improvement in the remaining objective functions. So, the risk of obtaining the lower total profit and opportunity to obtain the higher total profit are improved.

Profit, income and costs of each case are compared with the current production plan as shown in Table 1. APPLP model can generate the better production plans than the present plan, which has higher profit for both cases (most-likely results). The proposed plans by APPLP model have more income and less cost than the current plan. All types of costs of the proposed plans are less than the existing plan. Three solutions of each case are shown; pessimistic, most-likely and optimistic values. This information is useful for DM. The solutions can be easily generated and selected based on the preference of the DM, which is convenient for the DM.

Table 1. Profit, income and cost of each production plan.

List	*Production plan*		
	Current plan	*APPLP model*	
		(case 1)	*(case 2)*
Profit ($)		3,504,093.30	3,504,477.98
	2,633,074.17	2,965,792.62	2,967,310.34
		2,531,339.04	2,531,723.73
Income ($)		15,036,226.74	15,036,226.74
	14,522,850.73	14,800,437.39	14,800,437.39
		14,528,550.80	14,528,550.80
Total cost ($)		11,532,133.44	11,531,748.75
	11,889,776.55	11,834,644.77	11,833,127.05
		11,997,211.76	11,996,827.07
- Production cost ($)		4,295,372.48	4,294,987.79
	4,447,661.04	4,418,382.08	4,416,864.37
		4,499,923.61	4,499,538.92
- Stockout penalty cost ($)		3,164.75	3,164.75
	28,004.75	3,282.53	3,282.53
		3,327.65	3,327.65
- Purchasing material cost ($)		7,109,850.50	7,109,850.50
	7,255,154.55	7,285,731.58	7,285,731.58
		7,366,688.58	7,366,688.58
- Inventory cost ($)		506.74	506.74
	544.40	551.47	551.47
		574.81	574.81
- Idle capacity penalty cost		123,238.98	123,238.98
	158,411.82	126,697.11	126,697.11
		126,697.11	126,697.11
Safety stock level of material A (unit)	-	197,568	197,568
B	-	160,396	160,396
C	-	55,497	55,497
D	-	18,579	18,579

Concluding remarks

This research presents an Additive Preemptive Possibilistic Linear Programming (APPLP) model for MPS problem under uncertain environments. The proposed APPLP approach attempts to maximize the most possible total profit, minimize the risk of obtaining the lower total profit and maximize the opportunity to obtain the higher profit by setting the satisfaction level of each objective additively and orderly. This method can reduce the problem of adjusting membership functions of existing PLP approach and reduce time of computation. DM can easily manipulate to find the best compromise solutions based on his or her own preferences. Relaxing the satisfaction level of the first objective function is done to find the better solution for the DM because the lower risk and the higher opportunity plans can be generated. More information about optimistic, most-likely and pessimistic values of each plan are also known.

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