Integrated location distribution and investment model for new distribution centers using possibilistic programming

Chakkapat Chuanin and Busaba Phruksaphanrat

Abstract. Distribution Center (DC) plays a key role in reducing logistics cost for an organization. Moreover, it also can be used to increase competency in customer service. Normally, location, distribution and investment problems are considered separately. However, construction a new DC, not only investment cost but also location and distribution plans of DCs are necessary to be considered simultaneously in order to increase efficiency of a logistics system. So, in this research, an integrated location distribution and investment model for new DCs using multi-objective possibilistic mixed integer programming is proposed. Three objectives are considered: minimizing net present value of the total cost, maximizing customer service based on supplied quantities and maximizing customer service level based on ability to response customers. These are considered under uncertain cost coefficients. Possibilistic programming is used to deal with these uncertainties and ε-constraint method is applied to find a compromise solution. Decision maker can select the appropriate levels of service based on supplied quantities and the ability to response customers. Then, the suitable investment plan for each year, location decision and distribution plan can be generated concurrently. A case study is also illustrated to show the effectiveness on the proposed model.

Keywords: distribution centre; location; distribution; possibilistic programming

Introduction

Distribution center (DC) has played an important role in the supply chain in terms of reducing total logistics cost and increasing of customer service level (Nozick and Turnquist, 2001). In investment of a new DC, location decision is crucial because it affects the total cost of the supply chain system. This location problem involves
how to select locations of DCs from the potential set and how to transport products from the manufactory to customers via DCs. However, investment plans for each period are not included. Network location models and mixed integer programming models are normally used to solve these problems with total cost objective functions (Yang et al., 2007). Customer satisfaction should also be in concern for selecting the right place. This consideration is always a conflict with the cost objective function. Nozick and Turnquist, (2001) presented a modeling approach that provides an integrated view to study the effect of the number of DCs associated with service coverage by comparing the annual cost and number of DC for meeting a customer needs under precise information. However, most of the information is imprecise in nature. From literatures, there are three issues considered in location problems; to find the location that can minimize the total cost, to find the location that customer responsiveness can be improved and to find the location that has appropriate quality. Meanwhile, solution methods for investment decision problems always related to both qualitative and quantitative approaches. Most of them consider selection of the best location to invest, but they did not consider investment plans for each period (Kengpol, 2004; Wang, 2010; Awasthi, et al.2011). Most of them used multi-criteria decision making approaches to rank the proposed locations based on selected criteria. Investment plans were not determined in the proposed models.

Few researchers have focused on integrated location distribution and investment plans for new DCs. Moreover, uncertainties of this long term consideration should also be considered. In dealing with imprecise data, some researchers may apply stochastic programming to solve. However, the main problem is the lack of computational efficiency and inflexible probabilistic doctrines in which the really imprecise meaning of a Decision Maker (DM) might be impossible to model. In 1978, Zadeh presented the prominence of the theory of possibility, which is related to the theory of fuzzy sets by defining the concept of a possibility distribution as a fuzzy restriction, which acts as an elastic constraint on the values that can be assigned to a variable (Phruksaphanrat, 2011). He demonstrated the significance of the theory of possibility stems from the fact that much of the information on which human decision is based on is possibilistic rather than probabilistic in nature. In 1992, Lai and Hwang proposed a new approach to some of Possibilistic Linear Programming (PLP) problems. After that, much research related to PLP was presented (Pishvaee and Razmi, 2011).

In this research, an integrated location distribution and investment model for new DCs is proposed with 20 years planning horizon. Uncertainties of cost coefficients are also determined. Three objectives are considered; minimizing net present value of the total cost, maximizing customer service based on quantity supply and maximizing customer service based on the ability to response customers. So, multi-objective possibilistic mixed integer programming is constructed. Moreover, ε-constraint method which is the method that can provide an appropriate picture of the whole Pareto-optimal set for a DM, is then applied to find the compromise solution. Finally, a case study is used to evaluate the effectiveness of the proposed model.
Model formulation

Indices:
- \( d \) index of investment location of a distribution center, \( d = 1, 2, 3, \ldots, D \)
- \( f \) index of factories, \( f = 1, 2, 3, \ldots, F \)
- \( n \) index of investment plans, \( n = 1, 2, 3, \ldots, N \)
- \( m \) index of customer groups, \( m = 1, 2, 3, \ldots, M \)
- \( t \) index of years, \( t = 1, 2, 3, \ldots, T \)

Parameters:
- \( \bar{I}_{dn} \) initial investment cost of investment plan \( n \) at location \( d \) in the year that the plan is set
- \( \bar{O}_{dn} \) annual operating cost of investment plan \( n \) at location \( d \)
- \( \bar{D}_{dn} \) salvage value of investment plan \( n \) at location \( d \) in the year that the plan is set
- \( \bar{O}_f \) annual operating cost of factory \( f \)
- \( \bar{C}_h \) holding cost per unit
- \( \bar{C}_p \) capacity of factory \( f \) and capacity of DC \( d \) in the year \( t \) of plan \( n \)
- \( \bar{C}_{ds}, \bar{C}_{df} \) logistics cost per unit from factory \( f \) to DC \( d \), from DC \( d \) to customer group \( m \) and from factory \( f \) to customer group \( m \)
- \( \bar{C}_s,m, \bar{C}_f,m \) service cost of DC \( d \) and factory \( f \) for serving customer group \( m \)
- \( \bar{D}_{min}, \bar{D}_{max} \) the minimum and the maximum demand of customer group \( m \) in year \( t \)
- \( SV^1, SV^2, SV^3 \) service levels based on the degree of customer order’s response
- \( \sim \) represents imprecise information

Variables:
- \( S_{fmt} \) amount of products that serve from factory \( f \) to customer group \( m \) in year \( t \)
- \( S_{dmn} \) amount of products that serve from DC \( d \) to customer group \( m \) in year \( t \)
- \( S_{fm} \) amount of products that serve from factory \( f \) to DC \( d \) in year \( t \)
- \( X_{dn} \) is 1 if investment plan \( n \) is selected to locate at location \( d \), 0 otherwise
- \( LV^1_{dtn}, LV^2_{dtn}, LV^3_{dtn} \) is 1 if DC \( d \) can serve customer group \( m \) with service level 1, 2 or 3 in year \( t \), respectively, 0 otherwise
- \( LV^1_{fmt}, LV^2_{fmt}, LV^3_{fmt} \) is 1 if factory \( f \) can serve customer group \( m \) with service level 1, 2 or 3 in year \( t \) respectively, 0 otherwise.

In this research, three objective functions are considered: minimizing net present value of the total cost, maximizing customer service level based on supplied quantities and maximizing the ability to response customers, which are represented by (1) (2) and (3), respectively. Fixed cost, variable cost logistic cost, holding cost and service cost are described in (4)-(8). Equations (9) and (10) represent distribution
constraints. Capacity constraints are shown in (11)-(13). Limits of customers’ needs those should be served, represented by (14)-(15). Equations (16)-(19) present service level constraints.

\[
\begin{align*}
\text{Minimize } Z_1 &= \sum_{d,n} \left( \sum_{t} (F_{d,n}(t=0) + \frac{F_{d,n}(t=1)}{(1+\text{rate})^t}) \right) \\
\text{Maximize } Z_2 &= \left[ \sum_{f} \sum_{m} \left( \sum_{t} S_{dmt} + \sum_{t} S_{fmt} \right) \right] / (M \times T) \\
\text{Maximize } Z_3 &= \sum_{f} \sum_{m} \left( \sum_{d} \sum_{t} S_{dmt} + \sum_{d} \sum_{t} S_{fmt} \right) \left( \sum_{f} \sum_{t} \frac{S_{dmt}}{S_{fmt}} \right) / (M \times T) \quad (3)
\end{align*}
\]

Subject to:

\[
\begin{align*}
\text{Fix}_d &= X_{d,n} \{ I_{d,n} + \bar{O}_{d,n} - \bar{D}_{d,n} \} \quad (4) \\
\text{Var}_d &= L_{d,n} + H_{d,n} + S_{d,n} + \bar{O}_{d,n} \quad (5) \\
L_{f} &= \sum_{f} \sum_{m} C_{dmt} S_{f} + \sum_{d} \sum_{m} C_{d} S_{dmt} + \sum_{d} \sum_{f} C_{d} S_{fmt} \quad (6) \\
H_{f} &= \{ \bar{C}_{d} \sum_{m=1}^{M} \sum_{t=1}^{T} S_{dmt} \} \quad (7) \\
S_{d} &= \sum_{f} \sum_{m} \sum_{t} S_{dmt} \quad (8) \\
\sum_{m} X_{d,n} \leq 1, \text{ for all } d \\
\sum_{m} S_{dmt} \leq \text{Gap}_{d} \text{, for all } d, t \\
\sum_{m} S_{fmt} \leq \text{Gap}_{f} \text{, for all } f, t \\
\text{Gap}_{d} &= X_{d,n} \{ \text{ Gap}_{d,n} \}, \text{ for all } d, t \\
D_{dmt} \cdot \text{Dmax} \geq \sum_{t} S_{dmt} \geq D_{dmt} \cdot \text{Dmin}, \text{ for all } f, t \\
D_{fmt} \cdot \text{Dmax} \geq \sum_{t} S_{fmt} \geq D_{fmt} \cdot \text{Dmin}, \text{ for all } f, t \\
\sum_{f} D_{dmt} + \sum_{f} D_{fmt} = 1, \text{ for all } t \\
L_{v1,dmt} + L_{v2,dmt} + L_{v3,dmt} \leq 1, \text{ for all } m, d, t \\
L_{v1,dmt} + L_{v2,dmt} + L_{v3,dmt} \leq S_{dmt}, \text{ for all } m, d, t \\
L_{v1,fmt} + L_{v2,fmt} + L_{v3,fmt} \leq S_{fmt}, \text{ for all } f, d, t \\
X_{d,n} \cdot D_{dmt} + D_{fmt} \cdot L_{v1,dmt} + L_{v2,dmt} + L_{v3,dmt} \leq S_{dmt}, \text{ for all } f, d, t \\
L_{v1,fmt} + L_{v2,fmt} + L_{v3,fmt} \leq S_{fmt}, \text{ for all } f, d, t \\
X_{d,n} \cdot D_{dmt} \cdot D_{fmt} \cdot L_{v1,dmt} \cdot L_{v2,dmt} \cdot L_{v3,dmt} \cdot L_{v1,fmt} \cdot L_{v2,fmt} \cdot L_{v3,fmt} \text{ is binary} \\
\text{Others variables } \geq 0
\end{align*}
\]

Integrated location distribution and investment model for new distribution centers

In the proposed multi-objective mixed integer programming model, uncertainties exist in many places. Flexible mathematical programming model should be used. In this research, possibilistic programming approach is applied. There are two phases to solve this multi-objective possibilistic mixed integer programming (MOPMIP). Firstly, the MOPMIP is converted to an equivalent auxiliary crisp model. Then, ε-constraint method is applied to find the final compromise solution.
The equivalent auxiliary crisp model

Imprecise information is represented by triangular membership functions, which exist in (1), (4)-(8) and (11)-(13). According to Pishvaee and Razmi, (2001) the equivalent auxiliary crisp model can be formulated as follows.

\[
\begin{align*}
\text{Min} & \quad W_1 = \sum_{t} \left( \sum_{f} (F_t x_{mf}(z=0) + \epsilon (x_{mf(z)+} + x_{mf(z)-})) \right) \\
\text{Max} & \quad W_2 = \left[ \sum_{d} \sum_{m} \left( x_{dmt} \right) \right] \left( M \times T \right) \\
\text{Max} & \quad W_3 = \left[ \sum_{d} \sum_{m} \left( x_{dmt} \right) \right] \left( M \times T \right)
\end{align*}
\]

Subject to:

\[
\begin{align*}
\text{Fix}_n & = X_{dn} \left( \frac{c^{\text{pes}}_{dm} + 2 c^{\text{mos}}_{dm} + c^{\text{opt}}_{dm}}{4} \right) + \left( \frac{c^{\text{pes}}_{dm} + 2 c^{\text{mos}}_{dm} + c^{\text{opt}}_{dm}}{4} \right) - \left( \frac{c^{\text{pes}}_{dm} + 2 c^{\text{mos}}_{dm} + c^{\text{opt}}_{dm}}{4} \right) \\
\text{Var}_r & = L_r + H_r + S_r + \left( \frac{c^{\text{pes}}_{fmt} + 2 c^{\text{mos}}_{fmt} + c^{\text{opt}}_{fmt}}{4} \right) \\
L_r & = \sum_{f} \left( \frac{c^{\text{pes}}_{f} + 2 c^{\text{mos}}_{f} + c^{\text{opt}}_{f}}{4} \right) S_{fmt} + \sum_{d} \sum_{m} \left( \frac{c^{\text{pes}}_{dmt} + 2 c^{\text{mos}}_{dmt} + c^{\text{opt}}_{dmt}}{4} \right) S_{dmt} + \sum_{d} \sum_{m} \left( \frac{c^{\text{pes}}_{dmt} + 2 c^{\text{mos}}_{dmt} + c^{\text{opt}}_{dmt}}{4} \right) \\
H_r & = \left( \frac{c^{\text{pes}}_{f} + 2 c^{\text{mos}}_{f} + c^{\text{opt}}_{f}}{4} \right) \sum_{d=1}^{D} \sum_{m=1}^{M} S_{dmt} \\
S_r & = \frac{L_{dmt}}{\text{CS}_{dmt}} - \frac{L_{dm}}{\text{CS}_{dm}} + \left( \frac{L_{dm}}{\text{CS}_{dm}} \right) \\
\text{CS}_{fmt} & = \frac{c^{\text{pes}}_{fmt} + 2 c^{\text{mos}}_{fmt} + c^{\text{opt}}_{fmt}}{4} + \left( \frac{c^{\text{pes}}_{fmt} + 2 c^{\text{mos}}_{fmt} + c^{\text{opt}}_{fmt}}{4} \right) + \left( \frac{c^{\text{pes}}_{fmt} + 2 c^{\text{mos}}_{fmt} + c^{\text{opt}}_{fmt}}{4} \right) + \left( \frac{c^{\text{pes}}_{fmt} + 2 c^{\text{mos}}_{fmt} + c^{\text{opt}}_{fmt}}{4} \right) \\
\sum_{m} \left( c^{\text{pes}}_{fmt} \right) & = \alpha \left( \frac{c^{\text{pes}}_{fmt} + c^{\text{opt}}_{fmt}}{2} \right) + (1-\alpha) \left( \frac{c^{\text{pes}}_{fmt} + c^{\text{opt}}_{fmt}}{2} \right), \text{ for all } f, t \\
\text{Cap}_{dn} & = X_{dn} \left( \frac{c^{\text{pes}}_{dmt} + c^{\text{opt}}_{dmt}}{2} \right) \left( \frac{c^{\text{pes}}_{dmt} + c^{\text{opt}}_{dmt}}{2} \right), \text{ for all } d, t
\end{align*}
\]

\(\epsilon\)-constraint method for finding compromise solution

The \(\epsilon\)-constraint is known as a posteriori method that is able to provide an appropriate picture of the whole Pareto-optimal set for a DM. So, this method is applied to find the compromise solution of the MOPMIP problem. Positive ideal solution (PIS) and negative ideal solution (NIS) of objectives are used to define a membership function of each objective as follows (Phruksaphanrat, 2011).

\[1, \text{ if } W_1 < W_{1,\text{PIS}}\]
\[0, \text{ if } W_{2,3} < W_{2,3,\text{NIS}}\]
\[ \mu_1(x) = \frac{W_1^{PIS} - W_1^{NIS}}{W_1^{PIS} - W_1^{NIS}}, \text{ if } W_1^{PIS} \leq W_1 \leq W_1^{NIS}, \]
\[ \mu_{2,3}(x) = \frac{W_{2,3}^{PIS} - W_{2,3}^{NIS}}{W_{2,3}^{PIS} - W_{2,3}^{NIS}}, \text{ if } W_{2,3}^{NIS} \leq W_{2,3} \leq W_{2,3}^{PIS} \]
\[ 0, \text{ if } W_1 > W_1^{NIS} \]
\[ 1, \text{ if } W_{2,3} > W_{2,3}^{PIS} \]

where \( \mu_i(x) \) represents the satisfaction degree of \( i \)th objective function.

Then, the equivalent multi-objective crisp model can be converted to a single objective based on \( \varepsilon \)-constraint method as follows. In the model, the levels of \( \varepsilon_2 \) and \( \varepsilon_3 \) are varied until the DM is satisfied.

\[
\text{max } \mu_1(x) \\
\text{Subject to:} \\
\mu_2(x) \geq \varepsilon_2, \\
\mu_3(x) \geq \varepsilon_3, \\
x \in F(x) \text{ and } \varepsilon_1, \varepsilon_2 \in [0,1]
\]

**Implementation and evaluation**

In this section, the integrated model as well as the usefulness of the proposed solution method is investigated via the data of a case study. The manufacturer firm has 11 customer zones. It has to deliver approximately 7,000 boxes of products per week from the factory to all of customer groups. Demands of customers increase dramatically nearly 20% per year. Currently, customer satisfaction is low due to low frequency of response. So, construction of new DCs is considered for the firm within 20 years planning horizon at 10% interest rate. The firm’s manager considers seven investment plans for each location. There are 3 potential locations to open new DCs in order to serve customers with at least 0.7 or 70% service level.

To estimate possibility distributions of imprecise parameters, firstly the data is gathered and then at the consensus session, the experts and the firm’s manager determined three prominent values (i.e., the most likely, the most pessimistic and the most optimistic values) of the triangular fuzzy parameters according to available data and their knowledge. The fuzzy data of ability to distribute products of each investment plan is represented in Table 1. Fuzzy investment cost for plan 3 and 4 are shown in Table 2. Logistics cost, holding cost, annual operating cost and capacity of the factory are also represented by possibilistic distributions.

Service level in the third objective function, \( W_3 \) is evaluated by frequency of delivery per week to groups of customer. Current delivery frequency for each group of customer is once a week. So, in the proposed model service level is set as follows.

Delivery once a week, service level is set to 50%, \((SV^1 = 50\%)\).
Delivery two times per week, service level is set to 75%, \((SV^2 = 75\%)\).
Delivery three times per week, service level is set to 100%, \((SV^3 = 100\%)\).
Table 1. Possibilistic Distributions of distribution ability of each investment plan.

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-5</td>
<td>(0.8, 0.9, 1.5)</td>
<td>(0.6, 0.67, 0.8)</td>
<td>(0.41, 0.45, 0.6)</td>
<td>(0.18, 0.22, 0.3)</td>
<td>(1.6, 1.8, 2.1)</td>
<td>(3.2, 3.6, 4.2)</td>
<td>(4.5, 5.6, 8)</td>
</tr>
<tr>
<td>6-10</td>
<td>(1.5, 1.8, 2.1)</td>
<td>(1.2, 1.35, 1.5)</td>
<td>(0.8, 0.9, 1.1)</td>
<td>(0.4, 0.45, 0.6)</td>
<td>(1.6, 1.8, 2.1)</td>
<td>(3.2, 3.6, 4.2)</td>
<td>(4.5, 5.6, 8)</td>
</tr>
<tr>
<td>11-15</td>
<td>(4.4, 4.8, 6)</td>
<td>(2.7, 3.3, 5)</td>
<td>(2.2, 2.4, 2.7)</td>
<td>(1.1, 1.2, 1.8)</td>
<td>(3.2, 3.6, 4.2)</td>
<td>(6.8, 7.2, 7.6)</td>
<td>(10, 12, 14)</td>
</tr>
<tr>
<td>15-20</td>
<td>(10, 12, 15)</td>
<td>(8.9, 11)</td>
<td>(5.6, 8)</td>
<td>(2.5, 3.4, 5)</td>
<td>(3.2, 3.6, 4.2)</td>
<td>(6.8, 7.2, 7.6)</td>
<td>(10, 12, 14)</td>
</tr>
</tbody>
</table>

To implement the modified ε-constraint method to the MOPMIP, the satisfaction degree of maximizing customer service based on supplied quantities, $\mu_2(x)$ and maximizing customer service based on the ability to response customers, $\mu_3(x)$ are used as side constraints, which are set to 0.7. The satisfaction degree of minimizing net present value of the total cost, $\mu_1(x)$ is proposed to be the objective function of the model. The results are reported in Table 3 with the different level of $\alpha$, i.e., 0, 0.5 and 1. These different levels mean pessimistic, most-likely and optimistic views, respectively. $\alpha$–level is related to the capability to distribute products to customers. If $\alpha$–level is set to the high level, it means that the firm has high ability to distribute their products to customers. So, if the satisfaction levels of service level based on supplied quantities and ability to response customers can be reduced, then the net present value of the total cost will be decreased. This result confirms that minimizing net present value of the total cost is conflict with the other two objective functions. Selected investment plan is changed according to the $\alpha$–level and the satisfaction levels. For example, at $\alpha$–level equals to 0.5 and satisfaction levels of service level based on supplied quantities and ability to the response equal to zero only plan 3 is selected to invest at the location DC1. But if the satisfaction levels are increased to 0.33, then plan 4 is selected to invest at locations DC1 and DC3. Next, if the satisfaction levels are increased to 1, afterward plan 4 is selected to invest at locations DC1 and DC3 and plan 3 are selected to invest at location DC2.
Table 3. The summary results at different level of \( \alpha \).

<table>
<thead>
<tr>
<th>( \alpha )-level</th>
<th>Satisfaction level</th>
<th>Objective function value</th>
<th>selected plan</th>
<th>Distribution plan</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \mu(x) )</td>
<td>( \mu(x) )</td>
<td>( W_1 )</td>
<td>( W_2 )</td>
</tr>
<tr>
<td>1.0</td>
<td>0.67</td>
<td>0.67</td>
<td>30,093</td>
<td>90</td>
</tr>
<tr>
<td>0.87</td>
<td>0.33</td>
<td>0.33</td>
<td>26,528</td>
<td>80</td>
</tr>
<tr>
<td>0.5</td>
<td>0.67</td>
<td>0.67</td>
<td>40,133</td>
<td>100</td>
</tr>
<tr>
<td>0.86</td>
<td>0.33</td>
<td>0.33</td>
<td>27,234</td>
<td>80</td>
</tr>
<tr>
<td>0.61</td>
<td>0.33</td>
<td>0.33</td>
<td>30,936</td>
<td>90</td>
</tr>
<tr>
<td>0.88</td>
<td>0.33</td>
<td>0.33</td>
<td>27,398</td>
<td>80</td>
</tr>
</tbody>
</table>

(* unit \( \times 1,000 \) Baht, ** %)

At \( \alpha \)-level = 0.5, PIS and NIS of \( W_1 = (25,569, 40,133) \), \( W_2 = (100, 70) \), \( W_3 = (100, 70) \).

At \( \alpha \)-level = 1, PIS and NIS of \( W_1 = (24,469, 39,910) \), \( W_2 = (100, 70) \), \( W_3 = (100, 70) \).

At \( \alpha \)-level = 0, PIS and NIS of \( W_1 = (25,570, 40,843) \), \( W_2 = (100, 70) \), \( W_3 = (100, 70) \).

\( \alpha \)-level at 0.5 is selected for the case study firm. So, the firm’s manager can select the suitable plan by selecting plan 4 for locations DC1 and DC3 with the net present value approximately 30 million Baht and service level based on supply’s quantities and ability to response customer equal to 90%. This is the most appropriate plan for the DM because it is worth for investment. Location DC1 will be served for customer groups 4, 7, 8, 9. Location DC3 will be served for customer groups 2, 5, 8, 10, 11 and factory will be served customer groups 1, 3, 6, 7.

Conclusions

In this research, an integrated location distribution and investment model for new DCs using multi-objective possibilistic mixed integer programming is proposed. Three objectives are considered in the proposed model; minimizing net present value of the total cost, maximizing customer service based on supplied quantities and maximizing customer service based on the ability to response customers. Possibilistic programming is used to deal with uncertainties of cost coefficients.
and ε-constraint method is applied to find the compromise solution. Location, distribution, investment problems are determined simultaneously with long term consideration while conventional approaches separately consider these problems. Investment plans for each year can be easily generated for each tentative location with determinations of net present value, service level based on quantity and ability to response customers. Types and sizes of investment are also included in the model. The model can create the complete solutions of new DCs’ investment under uncertain environments.

For the case study firm, the best compromise investment plan was selected with the appropriated net present value and service levels. Moreover, distribution and investment plans are also generated for each potential location. These solutions are useful for the firm to prepare resources for each year.

References