Efficient multivariate modeling of cross border effects in European bond volatility spillover: A multiple objective artificial neural network approach

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Abstract. In this paper we extend prior efforts to engineer an efficient mapping of volatility transmission across various western- and central-European government bond markets. The univariate Bayesian-enhanced multiple-objective K4-RANN has been a standard to produce an efficient minimization of the ill-effects of multicollinearity while attaining maximum smoothness in nonparametric time series analysis. This research introduces a multivariate extension to the K4 algorithm; an extension which permits multiple target variables to be specified in the estimating equation. The new K7-MRANN is employed to re-examine bond volatility spillover effects previously obtained from univariate parametric- and artificial neural network based conditional volatility investigations. A prior K4-RANN estimation produced residuals that were nearly devoid of latent economic effects along with network weights that both corroborated and extended prior parametric findings of a weak US spillover effect into established European bond markets. The K7-MRANN findings presented in this research report model residuals that are clearly linearly independent and devoid of any latent interpretation. The signed model weights produced by a simultaneous multivariate RANN provide convincing evidence of the uniform effect of a negative U.S. bond market spillover into the aggregate Europe bond markets as well into specific Euro-domestic bond markets. The multivariate modelling efforts also provide a measurable view into the intra-Europe sovereign spillover effects.

Keywords: forecasting; financial visualization; volatility spillovers; artificial neural networks
Introduction

The recent convergence of two analogous disciplines, financial and computational engineering, has created a new mode of scientific inquiry. The specific aim of this research is to re-examine reported findings by Dash and Kajiji (2008) from the economic period of mid-May, 2003 through January, 2005 (the DK-1 model). The analytical examination presented here is developed in two stages. The initial stage focuses on the process of engineering a complex multivariate nonlinear artificial neural network (ANN) mapping of government bond excess returns (i.e., the K7-MRANN). The algorithm is a multivariate extension of the univariate K4-RANN. The second stage of the research establishes a theoretical foundation for the K7-MRANN as an effective nonlinear regression-based modelling procedure.

Univariate volatility spillover modelling

The extraction of time-varying volatility of financial time series has largely been linked to the ARCH model process of Engle (1982) and Bollerslev (1986). Extant literature has established that volatility leverage effects in financial time series is also a well-known phenomenon (e.g. Koutmos and Booth (1995) and Booth, et al. (1997)). The EGARCH model extension of Nelson (1991) and Nelson and Cao (1992) has proven to be nearly ideal for capturing the leverage effects that define the overall market behaviour of financial instruments. Recently, Christiansen (2007) presented important evidence that own bond-market effects are significant and exhibit asymmetric impacts in the volatility generating process. Lastly, and more recently, findings provided by Le and Kakinaka (2010) employ a two-stage GARCH methodology to provide evidence of how mean return and volatility spillover effects impact the stock markets of the U.S., Japan and China.

The univariate excess return RBF ANN bond spillover model

Within the context of the Dash and Kajiji methodology, the univariate spillover model was defined by the conditional return on both the US and European sovereign government bond indexes as an AR(1) process:

\[ R_{US,t} = b_{US} + b_{US,i} R_{US,i-4} + e_{US,t} \]  

In this model, \( t = 1, \ldots, T \) and the idiosyncratic shock \( e_{i,t} \) is expected to be normally distributed with a mean of zero \( (E[e_i,t] = 0) \), and is uncorrelated \( (E[e_i,t,e_j,t] = 0; \forall i \neq j) \).
The univariate RBF ANN estimation of bond stochastic volatility

Volatility of aggregate European bond market excess returns were captured by the application of the K4-RANN to the econometric model where the conditional excess return on the European total return government bond index is assumed to be a multi-factor AR(1). The DK-1 model is specified as:

\[ R_{E,t} = b_{0,E} + b_{1,E} R_{E,t-1} + \gamma_{E} R_{US,t-1} + \phi_{E} e_{US,t} + e_{E,t} \]  

(2)

In this specification, the conditional mean of the European bond excess return depends on its own lagged return as well as the spillover effects introduced by the lagged US excess return, \( R_{US,t-1} \), and the US idiosyncratic risk shock, \( e_{US,t} \). In this model, the idiosyncratic shock \( (e_{US,t} \text{ and } e_{E,t}) \) are normally distributed with a mean of zero \( (E[e_{US,t}] = 0) \), is uncorrelated \( (E[e_{US,t}e_{E,j}] = 0; \forall i \neq j) \), and the conditional variance follows an asymmetric EGARCH(1,1).

Following the previous assumption, the conditional variance of the idiosyncratic risk shock \( (e_{E,t}) \) is assumed to follow an asymmetric EGARCH(1,1) specification. For simplicity we replace the specific country notation with the generalized term:

\[ \sigma^2_{E,t} = \omega + \alpha \sigma^2_{E,t-1} + \gamma \sigma^2_{E,t-1} \]  

(3)

Specifically, the generalized EGARCH model implemented for all excess bond return generating models presented in this section is represented by:

\[ \ln(\sigma^2_{E,t}) = \omega + \sum_{k=1}^{q} \alpha_k g(z_{t-k}) + \sum_{j=1}^{p} \gamma_j \ln(h^2_{t-j}) \]  

(4)

where,

\[ g(z_t) = \theta z_t + \gamma \left[ |z_t| - E[|z_t|] \right] \]  

(5)

and

\[ z_t = \frac{e_t}{\sqrt{h_t}} \]  

(6)

and

\[ E[z_t] = \left( \frac{2}{\pi} \right)^{0.5}; \quad z_t \sim N(0,1) \]  

(7)

For the expressed modelling purpose of the DK-1 the parameter \( \gamma \) was set to 1.
**Multivariate RBF ANN estimation of stochastic volatility**

The econometric specification developed in this section extends the DK-1 bond volatility spillover model to a multivariate framework. The univariate approach ignores the multivariate probability density that describes the joint interaction of the excess returns of the $i$-th country with the aggregate European bond market.

Consider the following multivariate model specification. For country $i$, the conditional excess return is determined not only by spillover effects but also from simultaneous inter-country risk:

$$
(R_{t,1},...,R_{t,k}) = b_0 + b_1 \sum_{i} R_{t,1,i} + \gamma R_{t,1,1} + \delta R_{t,1,1} + \psi e_{t,1} + \varepsilon
$$

(8)

Under this formulation we are able to examine the vector $\mathbf{v}$ for multivariate GARCH (MGARCH) effects. The MGARCH application permits an interrogation of the relations between the volatilities and co-volatilities of the respective idiosyncratic terms (for a survey of comparable MGARCH models, see Bauwens (2006), et. al.

**Regression by linear combination of basis functions**

Kondor (2004) provides a synthesis of univariate regression by using a linear combination of basis functions. We extend this methodology of the multivariate regression process to attain multivariate nonlinear regression using the K7-MRANN algorithm.

**The enhanced univariate RANN regression model**

The objective of the univariate multiple-objective K4-RANN algorithm is to perform the task of fitting the data to the data points by ANN mapping to identify the real parameters $\theta_1, \theta_2, ..., \theta_P$ given data points $(x_1, y_1), (x_2, y_2), ..., (x_N, y_N)$ where $x \in \mathbb{X}$ and $y \in \mathbb{Y}$ given a real valued function $f: \mathbb{X} \leftrightarrow \mathbb{Y}$. A practical way to accomplish this goal is to search for $f$ in a finite dimensional space of functions overlapping a given basis. This is equivalent to specifying a set of functions $q_1, q_2, ..., q_P$ from $\mathbb{X}$ to $\mathbb{Y}$ while searching for $f$ in the form of a linear combination:

$$
f(x) = \sum_{i=0}^{P} \theta_i q_i(x)
$$

(9)

Performing the regression by use of a Gaussian radial basis functions is equivalent to centering around the data points with a pre-set variance, $\sigma$, such that $q_i = e^{-(x-x_i)^2/(2\sigma^2)}$ (e.g., see Kajiji (2001) for a discussion on SSE optimization for
the dual objectives of smoothness and accuracy by Tikhonov (1977) regularization. The modified SSE is restated as the following cost function:

\[
C = \sum_{i=1}^{m} \left( \hat{y}_i - f(x_i) \right)^2 + \sum_{j=1}^{n} k_j w_j^2
\]  

(10)

where \( k_j \) are regularization parameters or weight decay parameters. Under this specification the function to be minimized is stated as:

\[
C = \arg\min_k \left( \sum_{i=1}^{m} \left( y_i - f(x_i | k) \right)^2 \right)
\]  

(11)

The multivariate regression model

In this research we consider a multivariate extension to a multiple linear regression model. Specifically, we consider the model relationship between \( q \) responses \( y_1, \ldots, y_q \) and a single set of \( p \) predictor variables \( x_1, \ldots, x_p \). Each of the \( q \) responses is assumed to follow its own regression model, i.e.

\[
y_1 = \beta_{01} + \beta_{11} x_1 + \beta_{21} x_2 + \cdots + \beta_{p1} x_p + \epsilon_1
\]  

(12)

\[
y_2 = \beta_{02} + \beta_{12} x_1 + \beta_{22} x_2 + \cdots + \beta_{p2} x_p + \epsilon_2
\]

\[
\vdots
\]

\[
y_q = \beta_{0q} + \beta_{1q} x_1 + \beta_{2q} x_2 + \cdots + \beta_{pq} x_p + \epsilon_q
\]

where, \( \epsilon \) is a zero mean error term with \( \text{var}(\epsilon) = \sum \epsilon \).

Restated, the multivariate linear regression model is: \( Y = \beta X + \epsilon \), with \( \beta_0 = 0 \), \( E(\epsilon) = 0 \) and \( \text{cov}(\epsilon_i, \epsilon_j) = \sigma^2 I \). We use the multivariate OLS model to estimate the linear regression of the double-log production theoretic model to extract multivariate education scale economies.

The multivariate multi-objective regression-based RANN

As in the univariate model, the multivariate version is generalized as a supervised least-squares method. The multivariate supervised learning function is stated as,

\[
Y = f(X)
\]  

(13)
where \( Y \), the target matrix with \( q \) number of outputs, is a function of the input matrix \( X \) with \( p \) number of inputs. The function can be restated as:

\[
y_j = f(x_i) = \sum_{j=1}^{m} w_{ij} h_j(x)
\]  

(14)

where, \( m \) is the number of basis functions, \( h \) is the number of hidden units, \( w \) is the weight vector, and \( i = 1..p \) where \( p \) is the number of input vectors; and \( l = 1..q \) where \( q \) is the number of output vectors.

The flexibility of \( f(x) \) and its ability to model many different functions across multiple targets is inherited from the freedom to choose different values for the weight matrix, \( w \). Within the RANN architecture, the multivariate weight matrix is found through optimization of an OLS objective function. This is equivalent to minimizing the multivariate sum of squared errors (SSE):

\[
SSE_i = \sum_{j=1}^{\hat{y}_i} (\hat{y}_i - f(x_i))^2
\]  

(15)

The K7-MRANN extends equation 11 to its multivariate multi-objective counterpart as stated in equation 16.

\[
\arg\min_{k_i} \left( \sum_{j=1}^{\hat{y}_i} (\hat{y}_i - f(x_i))^2 + \sum_{j=1}^{m} k_j w_{ij}^2 \right)
\]  

(16)

For each equation the computationally efficient Bayesian enhanced K7-MRANN algorithm assures that \( q \) individual functions are mapped for smoothness and accuracy. In summary, the multivariate K7-MRANN incorporates the algorithmic enhancements evidenced in the univariate predecessor algorithm (K4-RANN). Both the K4-RANN and K7-MRANN have the ability to reconcile the twin evils that deter efficient ANN modelling: data dimensionality and inflated residual sum of squares (see Kajiji and Dash (2011), for an extended discussion).

**Modelling European bond volatility spillover**

Weekly data for all government total return bond indices under study are obtained from Global Financial Data for the period May 2003 to January 2005 inclusive (a total of 90 observations). Non-synchronous data issues are partially reduced by the use of weekly data. The two EMU-member countries, Germany (REX government bond performance index) and Spain (Spain 10-year government bond total return index), and the two non-EMU countries, Sweden (government bond return index w/GFD extension and the Slovenia 10-year government bond yield index) define the European local market. The USA effect is sampled by the inclusion of the Merrill Lynch U.S. government bond return index. Lastly, the JP Morgan European total return government bond index samples the aggregate European government
bond market. Total return indices are preferred as they are derived under the assumption that all received coupons are invested back into the bond index.

**K7-MRANN data scaling and estimated performance**

For the multivariate bond volatility model the data were scaled over the interval [0,1.01] by *Normalized Method 1*. This method replaces the actual data point (D) with a normalized data input value (D_V) based on the following transformation:

\[ D_V = \frac{(D - D_L)}{(D_U - D_L)} \]

Here \( D_L \) and \( D_U \) are computed as:

\( D_L = D_{\text{min}} - (D_{\text{max}} - D_{\text{min}}) \times S_L / 100 \)
\( D_U = D_{\text{max}} + (D_{\text{max}} - D_{\text{min}}) \times S_U / 100 \)

The upper- (lower-) headroom percent, \( S_L \) and \( S_U \), are set separately at 0.00% and 1.01%, respectively during the model-building exercise. For additional definition and algorithmic details see hyperlinked tables B and C. The output performance measures generated by applying the K7-MRANN to equation 8 produced an R-square measure of 86.27%.

**Results**

In this section we present the detailed results and policy implications from solving equation 8. The analysis begins with an examination of the residual matrix. This is followed by an examination of the matrix of model weights and the resultant policy implications. A comparison with the findings presented in previous literature is also presented in this section.

**K7-MRANN residual diagnostics**

Table 1 reports the results of applying a Varimax rotation to the principal components extracted from the residuals of the K7-MRANN solution. Factor one is an EMU dimension which accounts for slightly more than 57 percent of the variability in the idiosyncratic risk terms while virtually all remaining variability is captured in factor 2. Factor 2 is dominated by non-EMU effects (Slovenia).

<table>
<thead>
<tr>
<th></th>
<th>Factor 1</th>
<th>Factor 2</th>
<th>Factor 3</th>
<th>Factor 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sweden</td>
<td>0.804</td>
<td>-0.594</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Germany</td>
<td>0.790</td>
<td>-0.612</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Spain</td>
<td>0.788</td>
<td>-0.616</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Slovenia</td>
<td>0.614</td>
<td>-0.789</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Eigenvalue</td>
<td>2.268</td>
<td>1.729</td>
<td>0.002</td>
<td>0.001</td>
</tr>
<tr>
<td>Cumulative %</td>
<td>0.567</td>
<td>0.999</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>
Model weights and policy implications

The weight matrix generated by applying the K7-MRANN to the multivariate spillover model is presented in Table 2. The weights are the estimated parameters attached to each predictor variable; or, the quasi nonlinear regression parameter estimates.

Table 2. K7-MRANN Spillover Model Weights.

<table>
<thead>
<tr>
<th>Return Generating Model</th>
<th>Lagged Germany $b_{1i}$</th>
<th>Lagged Sweden $b_{2i}$</th>
<th>Lagged Spain $b_{3i}$</th>
<th>Lagged Slovenia $b_{4i}$</th>
<th>Lagged Euro $\delta_i$</th>
<th>Lagged USA $\gamma_i$</th>
<th>Euro Residual $\psi_i$</th>
<th>USA Residual $\phi_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>-0.7707</td>
<td>1.6275</td>
<td>1.7139</td>
<td>0.3434</td>
<td>1.5684</td>
<td>1.5771</td>
<td>-0.9211</td>
<td>-4.4926</td>
</tr>
<tr>
<td>Sweden</td>
<td>-1.2523</td>
<td>1.6448</td>
<td>1.6474</td>
<td>0.2934</td>
<td>1.2423</td>
<td>1.6652</td>
<td>-0.2510</td>
<td>-4.3313</td>
</tr>
<tr>
<td>Spain</td>
<td>-0.8412</td>
<td>1.9424</td>
<td>1.3946</td>
<td>0.2902</td>
<td>1.6484</td>
<td>1.5534</td>
<td>-0.9182</td>
<td>-4.4182</td>
</tr>
<tr>
<td>Slovenia</td>
<td>-0.5612</td>
<td>2.4429</td>
<td>1.8034</td>
<td>-0.0566</td>
<td>2.0115</td>
<td>1.3632</td>
<td>-1.6462</td>
<td>-4.7723</td>
</tr>
</tbody>
</table>

Comparative univariate and multivariate signed model weights

One of the interesting comparisons between the univariate K4-RANN and K7-MRANN solutions occurs with a change in signed weights. Table 3 presents a summary of the signed weights across the two models. This table highlights the important differences across model. Except for Germany, the U.S. stochastic volatility contribution is fairly consistent. The difference in signed modelling for the Euro-stochastic volatility effect is now consistent across all countries. The U.S. signed contribution in lagged return is also consistent across Europe.

Table 3. Comparative K4 / K7 Signed Spillover Model Weights.

<table>
<thead>
<tr>
<th>Return Generating Model</th>
<th>K4 / K7 Lagged Country $b_{1i}$</th>
<th>K4 / K7 Lagged Euro $\delta_i$</th>
<th>K4 / K7 Lagged USA $\gamma_i$</th>
<th>K4/K7 Euro Residual $\psi_i$</th>
<th>K4/K7 USA Residual $\phi_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>+ / -</td>
<td>- / +</td>
<td>+ / +</td>
<td>- / -</td>
<td>+ / -</td>
</tr>
<tr>
<td>Sweden</td>
<td>+ / +</td>
<td>+ / -</td>
<td>+ / +</td>
<td>+ / -</td>
<td>- / -</td>
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<tr>
<td>Spain</td>
<td>+ / +</td>
<td>+ / +</td>
<td>- / +</td>
<td>+ / -</td>
<td>- / -</td>
</tr>
<tr>
<td>Slovenia</td>
<td>+ / -</td>
<td>+ / +</td>
<td>- / +</td>
<td>- / -</td>
<td>- / -</td>
</tr>
</tbody>
</table>

Summary and conclusions

This paper applied computational financial engineering methods to investigate the structure of volatility spillover effects across EMU and non-EMU European government bond markets. The K7-MRANN computational method produced an efficient separation of global, regional and local bond volatility effects that support and
extend recent studies on market integration (e.g., see Abad et. al (2009)). As demonstrated by the comparative signed results, the K7-RANN produced a more plausible explanation of negative spillover effects for Germany, Sweden and Slovenia.

References

Kondor R. (2004). Regression by Linear Combination of Basis Functions. Thesis, School of Computer Science, University of St. Andrews