Increasing uncertainty in cash flow simulation-based volatility estimation for real options: Actual increase in volatility or symptom of excess unresolved ambiguity uncertainty

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Abstract. This paper investigates how cash flow simulation-based volatility estimation methods work in the case of decreasing or increasing volatility during a project. Whereas contingent claims analysis-based methods work correctly in both situations, this does not necessarily hold for cash flow simulation-based methods, because the underlying asset value may not be known accurately in the beginning. Misinterpreting ambiguity for volatility may cause decision makers to significantly overestimate volatility, and therefore overestimating a project value with options. Ambiguity may also cause erroneous investment outlays as the decision maker cannot follow precisely the underlying asset value, because it is not an observable market price. This is problematic especially in the case of increasing volatility. This paper describes the dilemma and suggests some managerial advice on how to detect, understand, and mitigate the problem.

Keywords: real options; cash flow simulation; ambiguity; uncertainty; valuation

Introduction

Real options analysis (ROA) is a fairly new approach for valuing managerial flexibility in investment decisions. It has been suggested as the alternative valuation approach for traditional net present value calculation for situations where managerial
flexibility and decisions under uncertainty can increase the project value in 
comparison with the static expected value assumed by net present value (NPV). 
These decisions can be related for example to deferring, extending or abandoning 
the project. According to real options thinking, investments are characterized by 
sequential and irreversible investments made under conditions of uncertainty 
(Dixit and Pindyck 1995).

There are several deficiencies related to the real options analysis that have hindered 
the adoption of the valuation approach. Mathematical complexity, parameter 
estimations, lack of managerial skills to do the corrective actions during the project 
when needed, and the problems derived from the fact that models suiting well for 
financial options do not necessarily fit well to real investments valuation (Borison 
2005, Triantis 2005). Especially the volatility estimation has been considered very 
difficult (Mun 2003) because, with the exception of certain natural resource 
investments, there is often no clearly observable underlying asset, relevant past 
historical data, or future price information. Then, cash flow simulation based 
methods are one viable alternative or volatility estimation.

Majority of the existing methods provide reliable estimates under optimal 
conditions. However, one aspect that still has not been emphasized enough in previous 
research is that when doing a cash flow simulation for volatility estimation, the 
underlying asset value in the beginning may also have uncertainty in form of 
ambiguity. As a result, cash flow simulation based methods may provide very 
inaccurate estimations if they assume that the underlying asset value in the beginning 
is known. This is not a problem in cases where the classical and optimal conditions of 
real options analysis hold, i.e. when the principles of contingent claims analysis 
hold and the underlying asset value is known and observable. Especially the cases 
where ambiguity does not resolve in the beginning are difficult for simulation 
based methods. This paper discusses that problem and explains why this happens.

The structure of the paper is as follows. The following section presents the idea 
of cash flow simulation based volatility estimation methods. Then one of them, 
regression sum of squares error method, is presented in more detail because of its 
intuition of separating uncertainty to solved and unsolved uncertainty, and therefore 
illustrates how the simulation based methods technically work. After that, two 
different uncertainty patterns for a project – increasing or decreasing volatility 
over time - are presented, and the topic of volatility estimation under the different 
assumptions of real option valuation. These are 1) whether the valuation is based 
on contingent claims analysis with observable underlying asset or cash flow simulation 
without observable underlying asset, and 2) whether there are investment outlays 
during the investment period. Finally discussion and conclusions are presented.

Cash flow simulation based volatility estimation

If there is no observable underlying asset available, several authors have suggested 
that the present value of the project’s cash flows (less investments) could be used 
as the underlying asset, and its volatility could be estimated with Monte Carlo
In this approach, cash flow calculation with its uncertain parameters is modeled with distributions and correlations. Then this cash flow calculation is simulated to consolidate a high-dimensional stochastic process of several correlated variables into a low-dimension process. In practice this often means that the underlying asset is assumed to follow a univariate geometric Brownian motion where volatility parameter defines how much there is dispersion in the underlying asset value over time.

Copeland and Antikarov (2001) present the three assumptions that are commonly used to justify those simulation approaches. Firstly, according to Samuelson’s proof, correctly estimated rate of return of any asset follows random walk regardless of the pattern of the cash flows. Secondly, an investment with real options should be valued as if it were a traded asset in markets even though it would not be publicly listed. Thirdly, the present value of the cash flows of the project without flexibility is the best unbiased estimate of the market value of the project were it a traded asset. This is called the marketed asset disclaimer assumption. These three assumptions are not that strict as those that are assumed in financial option valuation. Therefore, the accuracy of calculations is significantly lower than the reliability of financial option valuation results. Also real option valuation related to an observable underlying asset (e.g. commodities) has often much better accuracy. The simulation approach, however, is often best that can be done. The accuracy in such case is then more based on the reliability of the assumptions and modeling of the cash flow calculation. The higher the subjectivity and uncertainty, the less is also the accuracy of the simulation based volatility estimations.

The existing cash flow simulation-based volatility estimation methods are the logarithmic present value approach of Copeland and Antikarov (2001) and Herath and Park (2002), the conditional logarithmic present value approach of Brandão, Dyer and Hahn (2005a, 2005b), the two-level simulation and the least-squares regression methods of Godinho (2006), generalized risk-neutral volatility estimation over different time periods (Haahtela 2007), and regression sum of squares error method (Haahtela 2011). All these methods are based on the same basic idea of using Monte Carlo simulation technique to develop a probability distribution for the rate of return, and then the volatility parameter $\sigma$ of the underlying asset is estimated by calculating the standard deviation of the rate of return. While Copeland and Antikarov approach, similarly to that of Herath and Park, over-estimates the volatility (see e.g. Smith 2005), other approaches provide the unbiased estimate of the volatility.

All the methods mentioned above and especially the reliability of their results are similarly affected by several issues. The applicability of these approaches is usually case dependent (Brandao et al 2005, Smith 2005). Also, the high-dimensional process is based on a simulated cash flow, not on a real observable process. As such, it may have highly subjective estimates, especially when estimating the correlations between different cash flow variables. Furthermore, the whole valuation may appear as a black box for the decision-maker after the consolidation, without possibility to monitor and understand how different parameters and their changes affect the consolidated process.
Regression sum of squares error method

The regression sum of squares error method allows estimating changing volatility during the investment. The approach can be considered to be an enhanced mixture of the previously mentioned methods. It is computationally very efficient and requires only one pass of simulation runs regardless of the number of time periods. The method also allows parameterization of a changing displaced diffusion process, which is a more flexible alternative for real option valuation, allowing also negative underlying asset values and other distribution shapes than a lognormal. However, the reason why the approach is shortly described here is the intuition and logic of the approach: ordinary regression equations are used each time period for forecasting the present value of the forthcoming cash flows, and then by comparing the basic regression statistics of $R^2$ and standard error for the equations, we can easily see how much of the uncertainty is solved and is not yet solved during each time period. This information is enough for calculating the changing volatility for each time period.

Many projects have significant ambiguity in the beginning. Gradually more information comes available and future estimates become more reliable. Also, proportional uncertainty is often assumed to decline over time. Therefore, volatility is often assumed to be time changing and declining. However, whereas volatility resolves by itself with the passage of time, ambiguity resolves usually as a result of own work, learning, and information gathering. As a result, subjective probability distributions will converge towards more objective probability distributions. After that, major source of uncertainty can be regarded to be caused by volatility. Haahrta (2008) discusses the topic and suggests an approach that separates the two different forms of uncertainty, volatility and ambiguity, from each other in the underlying asset value in the beginning. However, there are many real life cases, especially in the field of R&D, where this assumption does not hold and the ambiguity does not necessarily resolve in the early stages. Secondly, the proportional volatility does not necessarily decline over time as often assumed: it may also increase.

In the beginning, the whole uncertainty of the investment project in terms of variance is estimated. At that point, none of the uncertainty has revealed while at the end of the project all the uncertainty has revealed. Then, during each time period of the investment, more and more of this uncertainty reveals. If we are able to construct good forward-looking value estimators for present values $PV_1,...,PV_n$ for all time periods, we can estimate the overall uncertainty solved (and unsolved) for each time point.

One way to forecast the future values is to use regression equations to estimate the expected value and standard deviation of $PV$, time period by time period, by linking the explaining cash flow calculation variables to the explanatory estimators of $PV$. Then, after the cash flow calculation simulation, the regression is done for forecasting the expected values and standard errors of $PV$s. As a reminder, regression divides the total variation, i.e. the sum of squares (SS) into two parts: the sum of squares predicted due regression (SSDR predicted due regression (SSDR) and the sum of squares error (SSE). SSDR is the sum of the squared deviations of the predicted scores from the mean predicted, and the SSE is the sum of the squared errors of
prediction. The SSDR therefore describes the variation that the regression model can explain and the SSE is the unexplained variation. Therefore, regression analysis does exactly what we need: it divides the uncertainty to resolved and unresolved uncertainty for each time period. This information, as explained in more detail in Haahela (2011), is enough for calculating the volatility for each time period. The procedure is so simple that it can be done even with a spreadsheet program.

**Test scenarios for inspecting increasing volatility patterns**

We investigate the issue of changing uncertainty and volatility with two different uncertainty pattern examples on five different scenarios. The first stochastic pattern illustrates a common case where uncertainty (volatility) reduces over time. This is illustrated on the left in Figure 1. The second pattern of uncertainty is such that the uncertainty actually increases over time before maturity (right picture in Figure 1).

![Fig. 1. Two different patterns for volatility and uncertainty. The left picture illustrates a case where uncertainty reveals faster in the beginning and the volatility decreases between the time periods. The right picture illustrates a case where uncertainty reveals faster at the end stages and the volatility increases between the time periods. The gray area inside the cone shows how much of the total uncertainty has resolved in proportion to the overall uncertainty at the end.](image)

The first of the different scenarios presents a plain vanilla European option valuation. Then, we present four other real option valuation (ROV) scenarios based on two different dimensions. The first dimension is whether the real option valuation is based either on i) contingent claims analysis with observable underlying asset or ii) cash flow simulation based estimation without clearly observable underlying asset. The second dimension is whether there are investment outlays (yes/no) during the project at Tₜ. As a result, we have the following five different scenarios for both stochastic patterns:

1. Plain vanilla financial European option expiring at maturity T₄.
2. ROV based on contingent claims analysis without investment outlays
3. ROV based on contingent claims analysis with investment outlays
4. ROV based on cash flow simulation volatility estimation without investment outlays
5. ROV based on cash flow simulation volatility estimation with investment outlays

Scenario 1: Plain vanilla financial European option expiring at maturity $T_4$

In case of a financial European option, both cases are of the same value to us: at the end, we get the payoff of the option which is the same regardless of the price path. Even American option without convenience yield, dividends or any cash outlays has the same property: the value of the option is the same regardless of the price path. The only thing that matters is the average volatility before option maturity. The only difference is that in the left case we can estimate earlier on whether we are going to exercise the option and what is likely to be its worth to us.

Scenario 2: ROV based on contingent claims analysis without investment outlays

If the real option valuation is based on contingent claims analysis and clearly observable underlying asset value, the analogy to financial option valuation is straight. The value of the project (basic value of the project plus the option value) is the same for both cases regardless of the stochastic pattern. However, this scenario is mostly theoretical and artificial by nature, yet it shows that there are some (yet rare) cases where real option valuation is analogous to financial option valuation.

Scenario 3: ROV based on contingent claims analysis with investment outlays

In this scenario, we know the price of the underlying asset in the beginning, and we also how it is likely to change over time in both cases of decreasing volatility and increasing volatility. The price and volatility structure over time, i.e. the volatility term structure, describes for both cases how the price is expected to change over time. This information is available as clearly observable market data for expected future prices, which can be used to calculate implied volatilities for different time periods and underlying asset price levels. Furthermore, this information can then be used for valuing options with implied binomial or trinomial trees (or with certain other ways). The valuation of the project can be regarded as sufficiently reliable with this way.

This time, however, we need to pay something at $T_1, \ldots, T_4$ to continue the project. This is very typical for real cases, where we need to decide at certain time points if we want to continue the project or not. Usually projects have labor costs related to development stages and also possibly some costs related to required investments in later stages. As a result, this is a typical case for R&D which can be described
as a series of call options on call options: we only continue the project at each stage if the expected cash flows in future exceed the expected costs.

In this scenario, the value of the option is dependent of the path of the underlying asset price movements. The earlier the uncertainty reveals, the better is our understanding of whether we should continue the project or not. In the case of decreasing volatility (left picture), much more of the uncertainty is revealed already by time $T_2$. Therefore, at that time point, decision maker knows whether to continue the project and invest more or not. Also the expected value of the project is likely to stay within certain limits in later stages after $T_2$ as the uncertainty (volatility) has reduced significantly. The situation for the investor with increasing volatility (right picture) is different. The uncertainty only reveals on the later stages of the project. Therefore, the investor can also calculate the expected value of the project with options, but the project outcome is much more likely to change between $T_2$ and $T_4$ as there is still a lot of uncertainty left. There is much higher risk that rational investment outlays at $T_2$ and $T_3$ turn out later to be false decisions. As a result, project where the uncertainty reveals earlier is more valuable to the investor.

**Scenario 4: ROV based on cash flow simulation volatility estimation without investment outlays**

In this, the volatility estimation is based on simulated cash flows. As a result, the underlying asset value in the beginning is not necessarily well known. However, in the case on the left, the uncertainty reduces fast, and therefore by the time $T_2$ the expected value of the project is well known. However, in the case where the uncertainty reveals only in the later stages, it very difficult to estimate the value of the project at time $T_2$. However, because there are no investment outlays during the project, both cases are still of the same value to investor.

**Scenario 5: ROV based on cash flow simulation volatility estimation with investment outlays**

This scenario is by far the most interesting, because this represents mostly the real life investment situations. In this this, the volatility estimation is based on simulated cash flows, and therefore the underlying asset value in the beginning is not necessarily well known. However, in the case on the left, the uncertainty reduces fast, and therefore by time $T_2$ the expected value of the project is well known. Therefore, the only ‘risk’ for the decision maker is to make a significantly false investment is at time $T_1$ when the investor cannot yet estimate well what is the expected value of the forthcoming cash flows. Otherwise, the later investment decision can be considered to be sufficiently reliable, even if not of the same level as in the scenario 3 with contingent claims analysis.

However, the situation is completely different with the increasing uncertainty on the right hand side. While in case of contingent claims analysis with observable underlying asset (scenarios 2 and 3) we know the underlying asset value in the
beginning, in case of cash simulation based solution we do not necessarily not know the underlying asset value in the beginning. The problem is that because we do not know what the expected value is in the beginning, or at \( T_1, T_2 \) or not yet even \( T_3 \), the simulation and regression based methods cannot resolve much of the uncertainty. That is, proportion of explained uncertainty vs. unexplained uncertainty does not change nearly at all between the time periods. Therefore it looks like there is only little uncertainty and volatility in the beginning. This is true, partly, because there is actually very little volatility kind of uncertainty while there may be a lot of ambiguity type of uncertainty.

Looking purely from the volatility estimation perspective, the situation is very awkward: the volatility estimation would provide exactly the same numbers in scenarios 3 and 5, and from that viewpoint, naïve interpretation would be that the both should have the same value. This is of course not true, because in scenario 3 we can base our decision on knowledge of the underlying asset value when deciding whether to continue the project at times \( T_x \), while in scenario 5 those decisions, especially at \( T_1 \) and \( T_2 \), are more based on guessing, because the decision maker has only limited knowledge or insight of the true underlying asset value. As a result, a project with real options would be of lower value to the investor in scenario 5.

As a solution for this dilemma of scenario 5, the decision maker should take a rather cautious stand and probably assume already in the beginning that the costs related to some later stages are actually already committed: there will not be anyway enough information for the right decision at times \( T_1 \) or \( T_2 \). This provides more accurate picture of the reality and correctly also lowers the value of the project.

**Discussion and conclusions**

This paper investigated how the cash flow simulation based volatility estimation methods work in case of decreasing or increasing volatility during the project. While contingent claims analysis based methods work correctly in both situations, this does not necessarily hold for cash flow simulation based methods, because the underlying asset value may not be known accurately in the beginning. This ambiguity in the underlying asset stems from the fact that both the expected value and the volatility estimation are based on the same simulated cash flow calculation. As such, this ambiguity is a natural phenomenon in investments. The dilemma is that we cannot isolate it from the volatility type of uncertainty, and therefore it makes volatility estimation and real option valuation difficult.

If the decision maker is using cash flow simulation based method for real option valuation, it is essential to investigate how the uncertainty reveals over time. If the pattern is decreasing, as is the case on the left in Figure 1, the method provides quite reliable results. However, in case of increasing volatility according to the estimation method, there is a significant risk for the underlying asset value ambiguity in the early stages. Then decision maker should be careful and take a rather conservative stand to the valuation and consider some future investment outlays committed already in the earlier stages.
The results also provide us another insight for real option valuation. While many methods based on contingent claims analysis provide correct and reliable results under optimal conditions in theory, their applicability to real life cases is very scarce. On the other hand, some other methods with looser assumptions and less strict theoretical connection may still provide reliable results. However, when we go further from the original assumptions and analogy of financial options, the more careful we have to be when using the models and interpreting the results. This also indicates that there is still significant need for new real option valuation research that could solve or mitigate these peculiarities.

References

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