Using data envelopment analysis and analytic hierarchy process for multiplicative aggregation of financial ratios

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Abstract—This research proposes a multiplicative aggregation approach using Data Envelopment Analysis (DEA) and Analytic Hierarchy Process (AHP) for financial ratios. The core logic of the proposed approach is to reflect the hierarchical structures and the priorities of financial ratios in performance assessment, simultaneously. According to this, we define a domain of efficiency losses based on two sets of weights of ratios. All ratios are treated as outputs without explicit inputs in a multiplicative two-level DEA framework. The first set represents the weights of output-ratios for each Decision Making Unit (DMU) with the minimal efficiency loss and the second set represents the weights of output-ratios bounded by AHP with the maximal efficiency loss. Using a parametric distance model, we explore various ranking positions for DMUs while the weights of output-ratios obtained from a multiplicative two-level DEA model shift towards the corresponding weights bounded by AHP. A case study of financial performance assessment of 21 commercial banks in Turkey highlights the usefulness of the proposed approach.

Introduction

Financial ratio analysis is a commonly used analytical tool for assessing the relative efficiency of Decision Making Units (DMUs). In financial ratio analysis, the ratio of a single output to a single input is used to measure the relative efficiency of each DMU relative to the maximum ratio across all DMUs, i.e., the ratio of ratios (Song et al., 2011). In practical applications, some aggregation methods are often employed in order to produce a single measure of performance from multiple ratios. Two popular methods to arrive at an aggregate measure of performance in the context of Multi-Criteria Decision Analysis (MCDA) are the Weighted Sum (WS) method and the Weighted Product (WP) method (San Cristobal Mateo, 2012). Several recent studies show that the WP method is theoretically superior to the WS method as an aggregate measure of performance (Emrouznejad and Cabanda, 2010; Ebert and Welsch, 2004; Zhou and Ang, 2009; Munda and Nardo, 2009). However, a problem in applying the WP method is how to properly select the weights for multiple ratios. From the viewpoint of Operations Research and Management Science (OR/MS) two major techniques, namely Data Envelopment Analysis (DEA) and Analytic Hierarchy Process (AHP) have received much attention as weighting and aggregation tools in performance assessment. Data Envelopment Analysis (DEA) is a data-oriented approach for assessing the relative efficiency of Decision Making Units (DMUs) with multiple inputs and outputs (Cooper et al., 2004). Recently, the standard DEA methodology has been extended to ratio analysis in order to generate a single measure of performance for ratios (see for example, Wu et al. (2005); Despic et al. (2007); Wei et al. (2011); Emrouznejad and Amin (2009)).

The use of DEA for ratio analysis can be categorized into two groups. The first one uses a mix of input-ratios and output-ratios to generate a single measure of performance (see for example, Edirisinghe and Zhang (2008); Mercan et al. (2003); Avkiran (2011)). The second one uses output-ratios without explicit inputs (see for example, Hakos and Salamouris (2004); Liu (2008); Ablanedo-Rosas et al. (2010)). On the other hand, AHP is a popular multi-criteria decision-making method that provides a priori information about the relative priority of (ratio) data in efficiency assessment. AHP usually involves three stages for generating weights. These stages are the decomposition into a hierarchy, comparative judgments, and synthesis of priorities (Saaty, 1980). AHP, however, seldom is used as a stand-alone tool in recent years. Rather, it is
combined with other OR/MS models (Vaidya and Kumar, 2006). The most common approach is the imposition of weight restrictions in DEA models. Referring to the literature, AHP can estimate the bounds of the following restrictions in DEA:

- Absolute weight restrictions. These restrictions directly impose upper and (or) lower bounds on the weights of inputs (outputs) using AHP (Entani et al., 2004).
- Relative weight restrictions. These restrictions limit the relationship between the weights of inputs (outputs) using AHP (Lee et al., 2012; Liu et al., 2005; Takamura and Tone, 2003; Tseng et al., 2009; Kong and Fu, 2012).
- Virtual weight restrictions. A single virtual input (output) is defined as the weighted sum of all inputs (outputs). We refer to the proportion of each component of such sum as the “virtual weight” of an input (output). These restrictions limit virtual weights using AHP (Premachandra, 2001; Shang and Sueoshi, 1995).
- Restrictions on input (output) units. These restrictions impose bounds on changes of inputs (outputs) while the relative importance of such changes is computed using AHP (Lozano and Villa, 2009).

There are a number of other methods that do not necessarily apply additional restrictions to a DEA model. Such as converting the qualitative data in DEA to the quantitative data using AHP (Azadeh et al., 2008; Ertay et al., 2006; Jyoti et al., 2008; Korpela et al., 2007; Lin et al., 2011; Ramanathan, 2007; Yang and Kuo, 2003; Raut, 2011), ranking the efficient/inefficient units in DEA models using AHP in a two stage process (Ho and Oh, 2010; Jablonsky, 2007; Sinuany-Stern et al., 2000), weighting the efficiency scores obtained from DEA using AHP (Chen, 2002), weighting the inputs and outputs in the DEA structure (Pakkar, 2014a; Cai and Wu, 2001; Feng et al., 2004; Kim, 2000), constructing a convex combination of weights using AHP and DEA (Liu and Chen, 2004) and estimating the missing data in DEA using AHP (Saen et al., 2005).

Recent studies by the author (Pakkar, 2014b,c; Pakkar, 2015) demonstrate the effects of incorporating absolute weight bounds using AHP in a ratio-based DEA model, a multiplicative DEA model and a two-level DEA model. Using a similar approach, this research applies AHP in a multiplicative two-level DEA model to reflect the hierarchical structures and the priorities of financial ratios in performance assessment, simultaneously. The author believes that combining these previous works provides more reasonable and encompassing results for assessing the financial performance of DMUs.

Methodology

This research has been organized to proceed along the following stages (Figure 1):

1. Computing the efficiency of each DMU using ratio-based one-level DEA model (2) without explicit inputs. The computed efficiencies are applied in ratio-based two-level DEA model (4).
2. Computing the priority weights of output-ratios for all DMUs using AHP, which impose weight bounds into model (4).
3. Obtaining an optimal set of weights for each DMU using ratio-based two-level DEA model (4) (minimum efficiency loss η).
4. Obtaining an optimal set of weights for each DMU using model (4) bounded by AHP (maximum efficiency loss κ). Note that if the AHP weights are added to (4), we obtain model (9).
5. Measuring the performance of each DMU in terms of the relative closeness to the priority weights of output-ratios. For this purpose, we develop parameter-distance model (10). Increasing a parameter in a defined range of efficiency loss we explore how much a DM can achieve its goals. This may result in various ranking positions for a DMU in comparison to the other DMUs.

Fig. 1. A multiplicative aggregation approach for ratios using a two-level DEA and AHP
A ratio-based one-level DEA model

A ratio-based one-level DEA model can be formulated similar to a multiplicative or log-linear DEA model in which all ratios are treated as outputs without explicit inputs (Zhou et al., 2010). Suppose \( y_{ij} \) is output-ratio \( r (r = 1, 2, \ldots, s) \) for DMU \( j (j = 1, 2, \ldots, n) \) with unit input \( i (i = 1, 2, \ldots, m) \). Then the multiplicative DEA model can be developed as follows:

\[
Max \ E_k = \frac{\prod_{r=1}^{s} y_{rk}^{u_r}}{\prod_{i=1}^{m} 1^{v_i}}
\]

Subject to

\[
\frac{\prod_{r=1}^{s} y_{rj}^{u_r}}{\prod_{i=1}^{m} 1^{v_i}} \leq e, \quad j = 1, 2, \ldots, n
\]

\[
u_r, v_i \geq 0, \quad r = 1, \ldots, s, i = 1, \ldots, m
\]

where \( E_k \) is the efficiency value of DMU under assessment. \( k \) is the index for the DMU under assessment where \( k \) ranges over 1, 2, ..., \( n \). \( u_r \) and \( v_i \) are the weights of input \( i (i = 1, 2, \ldots, m) \) and output-ratio \( r (r = 1, 2, \ldots, s) \). The first set of constraints assures that if the computed weights are applied to a group of \( n \) DMUs, \( (j = 1, 2, \ldots, n) \), they do not attain an efficiency value of larger than \( e \) (the base of the natural logarithm). The second sets of constraints indicate the non-negative conditions for the model variables. Taking logarithms with base \( e \), model (1) can be converted to the following linear programming model:

\[
Max \ \hat{E}_k = \sum_{r=1}^{s} u_r \hat{y}_{rk}
\]

Subject to

\[
\sum_{r=1}^{s} u_r \hat{y}_{rj} \leq 1, \quad j = 1, 2, \ldots, n
\]

\[u_r \geq 0, \quad r = 1, \ldots, s,
\]

where the caret sign (') denotes natural logarithms. Model (2) looks like an output-oriented DEA model without explicit inputs that theoretically is similar to the log-linear DEA model for efficiency analysis introduced in Charnes et al. (1982). Obviously, under a log-linear model the values of output-ratios must always be larger than 1, otherwise data transformations will be necessary. Although the efficiency value in the multiplicative model used in this paper is not invariant to data transformations, it is very robust against changes in data transformations (Zhou et al., 2010). Therefore, this would only slightly change the efficiency values without making a significant change in DMUs’ rankings.

Ratio-based two-level DEA models

We develop a log-linear two-level DEA framework to aggregate the performance of output-ratios under the category they belong to by a weighted-average method (Figure 2). Let \( \hat{y}_{rj} = \ln(y_{rj}) \) be output-ratio \( r (r = 1, 2, \ldots, s) \) under output-ratio category \( l (l = 1, 2, \ldots, S) \) for DMU \( j (j = 1, 2, \ldots, n) \). Let \( u_{rl} \) be the internal weight of output-ratio \( r \) under output-ratio category \( l \). Then the sum of \( u_{rl} \hat{y}_{rj} \) are defined as the values of output-ratio category \( l \) for the DMU \( j \) while \( \sum_{r=1}^{s} u_{rl} = 1 \). Let \( p_l \) be the weight of output-ratio category \( l \). Let \( u'_{rl} \) be the new multiplier of output-ratio \( r \) under output-ratio category \( l \) that is defined as: \( u'_{rl} = p_l u_{rl} \). Then a log-linear two-level DEA model for output-ratios can be developed as follows:
Max $\hat{E}'_k = \sum_{l=1}^{S} \sum_{r=1}^{s} u'_r \hat{y}_{lrk}$

Subject to
$$\sum_{l=1}^{S} \sum_{r=1}^{s} u'_r \hat{y}_{trj} \leq 1, \quad j = 1, \ldots, n$$
$$\sum_{r=1}^{s} u'_r = p_l, \quad l = 1, 2, \ldots, S$$
$$u'_r, p_l > 0, \quad l = 1, \ldots, S, r = 1, \ldots, s$$

Model (3) looks like a two-level DEA model without inputs that combines the two-level DEA model introduced by Kao (2008) and ratio analysis in a log-linear context. Note that the efficiency level obtained from model (3) can be less than or equal to that of obtaining from model (2).

Fig. 2. A two-level DEA framework for financial ratios

We continue developing our formulations based on a generalized distance model (see for example Mavi et al. (2012) and Hashimoto and Wu (2004)). Let $\hat{E}'_k$ (k = 1, 2, ..., n) be the best attainable efficiency level for the DMU under assessment, calculated from model (2). We want the efficiency $\hat{E}'_k(u'_l)$, calculated from the vector of weights $(u'_l)$, to be closest to $\hat{E}_k$. Our definition of closest is that the largest distance is at its minimum. Hence, we choose the form of the minimax model: $\min_{u'_l} \max_k \{\hat{E}'_k - \hat{E}'_k(u'_l)\}$ to minimize a single deviation which is equivalent to the following log-linear model:

(4)

Min $\eta$

Subject to
$$\hat{E}'_k - \sum_{l=1}^{S} \sum_{r=1}^{s} u'_r \hat{y}_{brk} \leq \eta,$$
$$\sum_{r=1}^{s} \sum_{l=1}^{S} u'_r \hat{y}_{trj} \leq \hat{E}'_j, \quad j = 1, \ldots, n$$
$$\sum_{r=1}^{s} u'_r = p_l, \quad l = 1, 2, \ldots, S$$
$$u'_r, p_l > 0, \quad l = 1, \ldots, S, r = 1, \ldots, s$$
$$\eta \geq 0.$$
Model (4) identifies the minimum efficiency loss $\eta$ (eta) needed to arrive at an optimal set of weights of output-ratios. The first constraint ensures that each DMU loses no more than $\eta$ of its best attainable efficiency, $\hat{E}_k^*$, obtained from model (2). The second set of constraints satisfies that the efficiencies of all DMUs are less than or equal to their upper bound of $\hat{E}_j$.

In addition, a set of constraints is added to model (4): $\sum_{l=1}^{s} \hat{u}_{lr} = p_l$, where $\hat{u}_{lr}$ are output-ratio multipliers. This implies that the sum of output-ratio weights under each output-ratio category equals to the weight of that category. It should be noted that the original output-ratio weights used for calculating the weighted averages are obtained as: $u_{lr} = u_{lr}' / p_l$.

**Prioritizing output-ratio weights using AHP**

Model (4) identifies the minimum efficiency loss $\eta$ (eta) needed to arrive at a set of weights of output-ratios by the internal mechanism of DEA. On the other hand, the priority weights of output-ratios, and the corresponding categories are defined out of the internal mechanism of DEA by AHP (Figures 3).

![Fig. 3. The AHP hierarchical model for prioritizing output-ratios](image)

In order to more clearly demonstrate how AHP is integrated into the newly proposed DEA approach, this research presents an analytical process in which output-ratio weights are bounded by the AHP method. The AHP procedure for imposing weight bounds may be broken down into the following steps:

**Step 1**: A decision maker makes a pairwise comparison matrix of different criteria, denoted by $A$, with the entries of $a_{ij}$ ($l = q = 1, 2, ..., S$). The comparative importance of criteria is provided by the decision maker using a rating scale. Saaty (1980) recommends using a 1-9 scale. In a similar way, a pairwise comparison matrix can be made to compare the importance of each sub-criterion with respect to a criteria. This matrix is denoted by $B$ with the entries of $b_{rt}$ ($r = t = 1, 2, ..., s$).

**Step 2**: The AHP method obtains the priority weights of criteria by computing the eigenvector of matrix $A$ (Eq. 5), $w = (w_1, w_2, ..., w_S)^T$, which is related to the largest eigenvalue, $\lambda_{\text{max}}$.

$$Aw = \lambda_{\text{max}}w$$

In a similar way, the priority weights of sub-criteria under each criterion are obtained by computing the eigenvector of matrix $B$ (Eq. 6), $e_r = (e_{r1}, e_{r2}, ..., e_{rs})^T$.

$$Be_r = \lambda_{\text{max}} e_r$$

To determine whether or not the inconsistency in a comparison matrix is reasonable the random consistency ratio, $C.R.$, can be computed by the following equation:

$$C.R. = \frac{\lambda_{\text{max}} - N}{(N-1)RI}.$$
where $R.I.$ is the average random consistency index and $N$ is the size of a comparison matrix. To obtain the weight bounds for output-ratio weights in the two-level DEA model, this study aggregates the priority weights of two different levels in AHP as follows:

$$
\bar{w}_{lr} = w_r e_{lr}, \quad \sum_{l=1}^{S} w_r = 1 \quad \text{and} \quad \sum_{r=1}^{s} e_{lr} = 1,
$$

where $w_r$ is the priority weight of criterion $l$ ($l = 1, \ldots, S$) in AHP and $e_{lr}$ is the priority weight of sub-criterion $r$ ($r = 1, \ldots, s$) under criterion $l$. In order to estimate the maximum efficiency loss $\kappa$ (kappa) necessary to achieve the priority weights of output-ratios for each DMU the following log-linear program is proposed:

$$
\min \kappa
$$

Subject to

$$
\bar{u}'_{lr} = \alpha \bar{u}_{lr}, \quad l = 1, \ldots, S, r = 1, \ldots, s
$$

$$
\bar{E}_k^* - \sum_{l=1}^{S} \sum_{r=1}^{s} u'_{lr} \hat{y}_{lk} \leq \kappa,
$$

$$
\sum_{l=1}^{S} \sum_{r=1}^{s} u'_{lr} \hat{y}_{lj} \leq \bar{E}_j^*, \quad j = 1, \ldots, n
$$

$$
\sum_{r=1}^{s} u'_{lr} = p_l, \quad l = 1, 2, \ldots, S
$$

$$
u'_{lr}, p_l > 0, \quad l = 1, \ldots, S, r = 1, \ldots, s
$$

$$
\kappa < 1,
$$

$$\alpha, \kappa \geq 0
$$

The first sets of constraints change the AHP computed weights to weights for the new system by means of a scaling factor $\alpha$. The scaling factor $\alpha$ is added to avoid the possibility of contradicting constraints leading to infeasibility or underestimating the relative efficiencies of DMUs (Podinovski, 2004). The optimal solution to model (9) produces a set of weights for output-ratios that are used to compute the relative efficiencies of DMUs.

A parametric distance model

We can now develop a parametric distance model for various discrete values of parameter $\theta$ such that $\eta \leq \theta \leq \kappa$. Let $u'_l(\theta)$ be a vector of output-ratio weights for a given value of parameter $\theta$, where the output-ratios are under output-ratio category $l$ ($l = 2, \ldots, S$). Let $u'_r(\kappa)$ be the vector of priority weights of output-ratios under output-ratio category $l$, obtained from model (9). Our objective is to minimize the total deviations between $u'_l(\theta)$ and $u'_r(\kappa)$ with the shortest Euclidian distance measure subject to the following constraints:

$$
\min Z_\kappa(\theta) = \left[ \sum_{l=1}^{S} \sum_{r=1}^{s} (u'_{lr} - u'_r(\kappa))^2 \right]^{1/2}
$$

Subject to

$$
\bar{E}_k^* - \sum_{l=1}^{S} \sum_{r=1}^{s} u'_{lr} \hat{y}_{lk} \leq \theta,
$$

$$
\sum_{l=1}^{S} \sum_{r=1}^{s} u'_{lr} \hat{y}_{lj} \leq \bar{E}_j^*, \quad j = 1, \ldots, n
$$

$$
\sum_{r=1}^{s} u'_{lr} = p_l, \quad l = 1, 2, \ldots, S
$$

$$
u'_{lr}, p_l > 0, \quad l = 1, \ldots, S, r = 1, \ldots, s
$$
Because the range of deviations computed by the objective function is different for each DMU, it is necessary to normalize it by using relative deviations rather than absolute ones (see for example Romero (2003)). Hence, the normalized deviations can be computed by:

$$\Delta_k(\theta) = \frac{Z_k^*(\eta) - Z_k^*(\theta)}{Z_k^*(\eta)},$$

where $Z_k^*(\theta)$ is the optimal value of the objective function for $\eta \leq \theta \leq \kappa$. We define $\Delta_k(\theta)$ as a measure of closeness which represents the relative closeness of each DMU to the weights obtained from model (10) in the range $[0, 1]$. Increasing the parameter $\theta$, we improve the deviations between the two systems of weights obtained from models (4) and (9) which may lead to different ranking positions for each DMU in comparison to the other DMUs. It should be noted that in a special case where the parameter $\theta = \kappa = 0$, we assume $\Delta_k(\theta) = 1$.

A Case study: The financial performance assessment

In this section we present the application of the proposed approach to assess the financial performance of 21 commercial banks in Turkey. The banks’ financial ratios, adopted from Özdemir (2013), are presented in Table 1. Since in a multiplicative aggregation, (ratio) data must always be larger than 1, we add a positive constant to each ratio in the last three columns of Table 1 prior to applying the log transform. The transformation is therefore $\ln(y_{r_{ij}} + x)$, where $x$ is a constant. We choose $x$ so that $\ln(y_{r_{ij}} + x)$ turns to the small number of 0.01 while $y_{r_{ij}}$ is the minimum output-ratio $r$ under category $l$ for all banks.

Table 1. Financial ratio data of 21 commercial banks in Turkey*

<table>
<thead>
<tr>
<th>Categories</th>
<th>Assets quality</th>
<th>Income and expenditure structure</th>
<th>Profitability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Banks</td>
<td>TC/TA</td>
<td>TC/TD</td>
<td>NII/TA</td>
</tr>
<tr>
<td>Bank 1</td>
<td>44.45</td>
<td>63.18</td>
<td>3.14</td>
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<tr>
<td>Bank 2</td>
<td>61.69</td>
<td>84.86</td>
<td>3.58</td>
</tr>
<tr>
<td>Bank 3</td>
<td>64.26</td>
<td>94.04</td>
<td>2.78</td>
</tr>
<tr>
<td>Bank 4</td>
<td>52.64</td>
<td>91.53</td>
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<td>Bank 5</td>
<td>67.28</td>
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<td>Bank 6</td>
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<td>101.82</td>
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<td>Bank 7</td>
<td>59.1</td>
<td>93.74</td>
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<td>Bank 8</td>
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<tr>
<td>Bank 9</td>
<td>29.36</td>
<td>47.92</td>
<td>2.55</td>
</tr>
<tr>
<td>Bank 10</td>
<td>67.34</td>
<td>112.08</td>
<td>3.48</td>
</tr>
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<td>Bank 11</td>
<td>57.15</td>
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<td>37.05</td>
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</tr>
<tr>
<td>Bank 15</td>
<td>62.31</td>
<td>111.74</td>
<td>3.86</td>
</tr>
<tr>
<td>Bank 16</td>
<td>47.3</td>
<td>102.42</td>
<td>2.04</td>
</tr>
<tr>
<td>Bank 17</td>
<td>84.72</td>
<td>102.74</td>
<td>2.72</td>
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</table>
Table 2. The AHP hierarchical model

<table>
<thead>
<tr>
<th>Objective level</th>
<th>Criteria level</th>
<th>Sub-criteria level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prioritizing the financial ratios</td>
<td>Asset quality</td>
<td>Total credits/Total assets $e_{11} = 0.875$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total credits/Total deposits $e_{12} = 0.125$</td>
</tr>
<tr>
<td></td>
<td>Income &amp; expenditure structure</td>
<td>Net interest income after provisions for credits and other receivables/Total assets $e_{21} = 0.889$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Non-interest income/Total assets $e_{22} = 0.111$</td>
</tr>
<tr>
<td></td>
<td>Profitability</td>
<td>Net profit/Equities $e_{31} = 0.875$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Net profit/Total assets $e_{32} = 0.125$</td>
</tr>
</tbody>
</table>

Solving model (4) for the bank under assessment, we obtain an optimal set of weights for the output-ratios under their own categories with minimum efficiency loss ($\eta$). In this case, the efficiency of all banks calculated from the ratio-based two-level DEA model is incidentally identical to that calculated from the ratio-based one-level DEA model, $\hat{E}_k' = \hat{E}_k^*$ for $k = 1, 2, ..., 21$. Therefore, the minimum efficiency loss for the bank under assessment is $\eta = 0$ (Table 5). This implies that the measure of relative closeness to the AHP weights for the bank under assessment is $\Delta_1(\eta) = 0$.

On the other hand, solving model (9) for the bank under assessment, we adjust the priority weights of financial ratios obtained from AHP in such a way that they become compatible with the weights’ structure in the DEA models. Table 4 presents the optimal weights of ratios as well as its scaling factor for all banks.

It should be noted that the priority weights of AHP used for incorporating weight bounds on output-ratio weights are obtained as $\bar{\alpha}_k = \bar{u}_k' / \alpha$. Similarly, the priority weights of AHP at criteria level can be obtained as $p_i/\alpha$ while $p_i = \sum_{r=1}^8 u_r'$. The maximum efficiency loss for the bank under assessment to achieve the corresponding weights in model (9) is equal to $\kappa$ (Table 5). As a result, the measure of relative closeness to the priority weights of output-ratios for the bank under assessment is $\Delta_1(\kappa) = 1$.

Table 2 depicts the hierarchical structure of financial ratios and the corresponding priority weights in the AHP model as constructed by Özdemir (2013). There are three categories of financial ratios at the criteria level. Each one includes two different ratios at the sub-criteria level. For the criteria and sub-criteria shown in Table 2, four comparison matrices need to be elicited from the DM—one for estimating the priority weights of the criteria with respect to the problem objective and three for computing the weights of sub-criteria with respect to each criterion. Table 3 shows the results of the pairwise comparison matrix at criteria level with respect to the objective.
Going one step further to the solution process of the parametric distance model in (10), we proceed to the estimation of total deviations from the AHP weights for each bank while the parameter $\theta$ is $0 \leq \theta \leq \kappa$ in (10). Table 6 represents the ranking position of each bank based on the minimum deviation from the AHP weights for $\theta = 0$. It should be noted that in a special case where the parameter $\theta = \kappa = 0$ we assume $\Delta_k(\theta) = 1$.

Table 3. Pairwise comparison matrix at criteria level with respect to the objective C.R = 0.077, $c_i$ = Criterion $l$ ($l = 1, 2, 3$)

<table>
<thead>
<tr>
<th></th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$w_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>1</td>
<td>1/7</td>
<td>1/9</td>
<td>0.055</td>
</tr>
<tr>
<td>$c_2$</td>
<td>7</td>
<td>1</td>
<td>1/3</td>
<td>0.290</td>
</tr>
<tr>
<td>$c_3$</td>
<td>9</td>
<td>3</td>
<td>1</td>
<td>0.655</td>
</tr>
</tbody>
</table>

Table 4. Optimal weights of output-ratios of model (9) for all banks

<table>
<thead>
<tr>
<th></th>
<th>$u^i_{11}$</th>
<th>$u^i_{12}$</th>
<th>$u^i_{21}$</th>
<th>$u^i_{22}$</th>
<th>$u^i_{31}$</th>
<th>$u^i_{32}$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.030914</td>
<td>0.004416</td>
<td>0.165744</td>
<td>0.020716</td>
<td>0.369032</td>
<td>0.052719</td>
<td>0.643541</td>
</tr>
</tbody>
</table>

Table 5. Minimum and maximum efficiency losses for each bank

<table>
<thead>
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<th>Banks</th>
<th>$\hat{E}_k^\tau$</th>
<th>$\eta$</th>
<th>$\kappa$</th>
<th>Banks</th>
<th>$\hat{E}_k^\tau$</th>
<th>$\eta$</th>
<th>$\kappa$</th>
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<tbody>
<tr>
<td>Bank 1</td>
<td>0.9163</td>
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<td>0.1448</td>
<td>Bank 12</td>
<td>0.9755</td>
<td>0</td>
<td>0.1677</td>
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<td>0</td>
<td>0.0317</td>
<td>Bank 13</td>
<td>1</td>
<td>0</td>
<td>0.1623</td>
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<td>0</td>
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<td>Bank 14</td>
<td>0.9876</td>
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<td>0</td>
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<td>Bank 21</td>
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</table>

Table 6 shows that Bank15 is the best performer in terms of efficiency and the relative closeness to AHP weights in comparing to the other banks. Nevertheless, increasing the value of $\theta$ from $0$ to $\kappa$ has two main effects on the performance of the other banks: improving the degree of deviations and reducing the efficiency. This, of course, is a phenomenon one expects to observe frequently. The graph of $\Delta(\theta)$ versus $\theta$, as shown in Figure 4, is used to describe the relation between the relative closeness to AHP weights and efficiency loss in DEA for each bank. This may result in different ranking positions for each bank in comparison to the other banks.

Table 6. The ranking position of each bank based on the minimum distance to priority weights of output-ratios

<table>
<thead>
<tr>
<th>Banks</th>
<th>$Z^\tau(\eta)$</th>
<th>Rank</th>
<th>Banks</th>
<th>$Z^\tau(\eta)$</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
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<td>Bank 12</td>
<td>0.4457</td>
<td>13</td>
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<tr>
<td>Bank 2</td>
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<td>13</td>
<td>Bank 13</td>
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<td>0.4469</td>
<td>16</td>
<td>Bank 14</td>
<td>0.6015</td>
<td>21</td>
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</tbody>
</table>
We develop a multiplicative aggregation approach based on DEA and AHP methodologies to assess the performance of DMUs under hierarchical structures of ratio data in which output-ratios are organized into different categories. We define two sets of weights in a multiplicative two-level DEA framework. The first set represents the weights of output-ratios with minimum efficiency loss. The second set represents the corresponding priority weights using AHP with maximum efficiency loss. We assess the performance of each DMU in comparison to the other DMUs based on the relative closeness of the first set of weights to the second set of weights. Improving the measure of relative closeness in a defined range of efficiency loss, we explore the various ranking positions for the DMU under assessment in comparison to the other DMUs. Finally, the results obtained from the case study of Turkish commercial banks indicate that the proposed approach can be effectively applied for assessing the financial performance of companies.
References


