A mixed integer linear programming model for optimal delivery of fattened pigs to the abattoir

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Abstract. The structure of the pork sector in world economy is changing. In the last years, pig farms are becoming more and more specialized, while the size of their facilities is increasing. Moreover, they have tended to integrate and coordinate their operations into pork supply chains by using tighter vertical coordination linkages. This paper presents a mixed integer linear programming model describing a fattening pig unit operating under all-in all-out strategy. It is assumed that all production is sold to an abattoir prescribed by the pork supply chain (PSC) in which the farm is integrated. Piglets, feedstuff and veterinary care are provided by the PSC. The interest of the model is to analyze the deliveries of pigs to the abattoir depending on growth, body composition, truck capacity and penalties on selling price. An important practical aspect is that pigs are selected by visual inspection and hence, heavier pigs of the herd are the first delivered to the abattoir. Our contribution confirms the findings of past studies but demonstrates the benefit of fattening pigs with homogeneous weights and the importance of individual measures to avoid extra costs derived from grouped animals.

Keywords: optimal delivery to the abattoir; pig production; pig supply chain management

Introduction

Pig production is very important in many countries all around the world because pork is the most produced and consumed meat. Statistics of the Food and Agriculture Organization of the United Nations (FaoStat 2013) has outlined the European Union (EU-28 members) as the second largest producer in the world, after China. In recent years, the modern pork industry has greatly evolved as a result of the globalization process, advances in technology, scientific developments and changes in social and cultural attitudes (Trienekens et al. 2009). The trend observed today is a reduction in the number of farms although their sizes are increasing (Pérez et al., 2010; Kamhan et al., 2013). Additionally, consumer concerns about environment, animal welfare, food safety and food quality are becoming new challenges for the pork industry (Backus and Dijkhuizen 2002; Rodriguez et al., 2013). On the other hand, the adoption of new production systems like climate control, automatic feeding systems and animal identification devices has been accompanied by an increase in the scale and degree of technical specialization of farm operations. Hence, pig production has been evolving towards a progressive concentration in larger and more specialized and efficient production units (Taylor 2006). As a result, the profile of the typical farm is changing from a family-based, small-scale, independent firm to one in which larger firms are more tightly aligned along the pig production and distribution processes (Pérez et al., 2010; Rodriguez 2010).

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Nowadays, the new competitive strategy in pig farming is no longer based on individual farm units competing as solely autonomous entities, but rather integrated into a supply chain (Christopher 1998; Min and Zhou 2002). The PSC is a network of agents, e.g. specialized farm units at different stages in the pork production, integrated and coordinated by tighter vertical coordination linkages. Economies of scale have continued to accelerate changes in the pork production industry (Pérez et al., 2009; Ohlmann and Jones, 2011). Traditionally, judgement based on experience had been the basis for the production planning on individual farm units, but the increasing complexity of the pork supply chain made the development of more formal planning methods necessary (Boland et al., 1993; Plà, 2007; Rodriguez et al., 2013). The usual planning problem in fattening pig units is related with the optimal delivery to the abattoir of fattened pigs. Visual inspection is used for selecting pigs to be delivered as the cheapest method, whilst novel information technologies solutions remain expensive for this purpose. Hence, the objective of this paper is to formulate a mixed integer linear programming model (MILP) describing a fattening pig unit operating under all-in all-out (AIAO) strategy and explore the duration of the marketing time window. It is assumed that all production is sold to an abattoir prescribed by the pork supply chain into which the fattening farm is integrated.

The organization of this paper is as follows. We present in Section 2 a brief description of the problem highlighting the role of the growing process. In Section 3, we describe the proposed model while in Section 4, we present and discuss main results. Finally, we present the conclusions in the last section.

**Fattening pigs and the delivery to the abattoir**

Pig production involves a set of operations that can all be developed in either one farm or a set of specialized farms. Accordingly, there exist sow farms, rearing farms and fattening farms. Some companies may own different mixtures of these basic production units. We are concerned with the last type of farms where pigs are procured by rearing units, fattened and later on, delivered to the abattoir to be slaughtered, typically at the live weight of around 115 kg (Whittemore and Kyriazakis 2006). Nevertheless, slaughtering weight can vary considerably depending on growth rate and disease.

The most extended management of fattening farms is the all-in all-out (AIAO) management. AIAO management, practiced by the majority of producers with large facilities or as a part of a supply chain, empties the entire facility before replacing the animals with a new herd or batch of pigs (Ohlmann and Jones, 2011). Advantages of AIAO management are the easier cleaning and disinfection of facilities between batches, making difficult the spread of illness and disease. This means all young pigs are entered at the same time in the farm and sold also at the same time within a narrow time window. Once the farm is empty a cleaning and drying period of a week follows before a new batch of young pigs arrive. Fattening farms must determine when to send pigs and deliver them to the abattoir because not all of them reach marketable weight at the same time even though they have the same age.

Spanish pig production is highly specialised and mostly controlled by big companies or cooperatives, namely integrators, organised as supply chains. Then, optimal delivering policies of pigs for individual farms are different when operating in a supply chain context. The supply chain may determine the composition of the batch of pigs in terms of number of animals, suppliers, weight distribution at the beginning, feedstuffs, maximum duration of fattening, etc. Differences in marketing strategies are mainly due to different weight distributions at the beginning of the marketable time window. In view of the individual weight and value of pigs change differently over time depending on growth rates and specific pricing systems, they cannot be sold at the same time. Hence, the determination of when starting to deliver pigs of a batch to the abattoir and which and how many pigs to sell is the central problem to the pig manager.

We model the decision-making problem of delivering pigs to the abattoir as a MILP that determines the marketing strategy that maximizes expected profit. By discretizing the population into appropriate growth, meat quality and carcass weight categories, we formulate a mixed-integer program. We solve the problem individually for one farm considering the full fattening process. Later on, we test the impact of different marketing windows, transport costs and homogeneity in the growth of the batch of pigs.
Modelling approach

Decision variables

The decision variables are related to number of pigs: inventory on farm \((N)\) or sold to the abattoir \((X)\); number of trucks to deliver pigs \((Y)\). Other variables \((h \text{ and } d)\) are defined to represent the logical operation of pig deliveries to the abattoir. For instance, weight distribution and visual inspection of pigs lead to consider the selection of heavier pigs to be delivered first to the abattoir. The nature of decision variables is for some integers \((X, N \text{ and } Y)\) and others binaries \((h \text{ and } d)\). However, to make easier the resolution of the problem the integrality condition of \(X\) and \(N\) related with (a big) number of pigs is relaxed and defined as continuous. Description for each one follows:

\[
X_{it} : \text{number of pigs at growth stage } i, \text{ sent to the abattoir at time } t.
\]

\[
N_{it} : \text{inventory of pigs at growth stage } i \text{ and time } t.
\]

\[
h_{it} \in \{0,1\} : \text{binary variable representing two consecutive growth stages } (i-1 \text{ and } i) \text{ sending pigs to the abattoir at time } t \text{ when it takes the value 1, or 0 otherwise.}
\]

\[
d_{it} \in \{0,1\} : \text{binary variable representing whether pigs at growth stages } i \text{ and time } t \text{ are sent to the abattoir when it takes the value 1, or 0 otherwise.}
\]

\[
Y_t : \text{integer variable representing the number of trucks needed to transfer pigs from the farm to the abattoir at time } t.
\]

The subscripts of decision variables refer to two main qualities:

\[i \in P\] representing the partition applied to the population of animals. Each element of the partition is assumed that will grow according to similar parameters. Thus, this subscript represents a growth category preserved along the fattening period.

\[t \in T\] representing a period of time in which the fattening stage is divided, i.e. number of weeks.

Constraints

The constraints of the mathematical model are classified in several groups:

- Inventory of animals entered to the fattening unit at week \(t = 1\).
- Number of pigs that can be sent to the abattoir.
- Number of pigs remaining in the farm from one period to the next one.
- Inventory of animals at the end of the fattening process (it must be 0 under all-in all-out management strategy).
- Number of trucks needed to deliver pigs to the abattoir.
- Constraints related to the pigs selected for being transported to the abattoir. This group includes binary variables in the formulation. Therefore, extra constraints related with binary variables are needed:
  - Detecting if pigs of a growth category are sent to the abattoir or not
  - Only heaviest animals can be sent to the abattoir

**Inventory of animals entered to the fattening unit at week \(t = 1\)**

It is assumed that a batch of \(K\) pigs is introduced in the fattening farm. Animals at the beginning of the growing process, \(t=1\), have a weight following a normal distribution, \(w_{1}\sim N(\mu_{1},\sigma_{1})\). This distribution is partitioned into \(|P|\) percentiles representing each one different growing categories \((i\in P)\):

\[
N_{i1} = (K/|P|) \forall i \in P
\]
Number of pigs that can be sent to the abattoir

The model permits pigs to be sent to the abattoir at any time. However, the grid of bonus and penalties paid by the abattoir affects internally the opening of the marketing window for fattened pigs. This way the abattoir tries to stimulate the delivery of pigs in a range of weight suitable for an easier processing. This procedure has been proven effective for abattoirs as Boland et al. (1993) remarked.

The number of pigs available to be sent to the abattoir, $X_{it}$, is bounded by $N_{it}$, i.e. the inventory of pigs in $i$-growth category at time $t$:

$$X_{it} \leq N_{it} \quad \forall \ i \in P, \ t \in T$$

The binary variable $h_{it}$ is explained later, but it is introduced as a LP modeling trick to detect when pigs in $i$ and ($i$-1)-growth category at time $t$ are sent to the abattoir, then $h_{it}=1$, otherwise $h_{it}=0$. Hence a complementary constraint is necessary (note that $K$ could be replaced by any arbitrary bigger number without affecting the functionality of these constraints):

$$X_{it} \geq N_{it} - K (1-h_{it}) \quad \forall \ i \in P \setminus \{1\}, \ t \in T$$

Let’s note that $h_{it}=1$ forces the equality $X_{it}=N_{it}$. This way, we can represent the rational behavior of farmer: all the pigs remaining in the $i$-growth category at week $t$ has to be sold before pigs of the growth category ($i$-1) can be also sent to the abattoir.

Number of pigs remaining in the farm from one period to the next one

The model represents the flow of pigs over time. The model assumes that groups of growing categories (initial partition of the population) do not change over time. In principle, pigs growing to the next week (i.e. $N_{it}$) are those present in the current week minus sales to the abattoir (i.e. $N_{it} - X_{it}$). The equality constraint can be relaxed if casualties are considered from week to week.

$$N_{i(t+1)} = (N_{it} - X_{it}) \quad \forall \ i \in P, \ t \in T \setminus \{T\}$$

Inventory of animals at the end of the fattening process

Furthermore, as AIAO strategy is considered an additional constraint for the last week of the process has to be set:

$$N_{i(T)} - X_{i(T)} \leq 0 \quad \forall \ i \in P$$

Note that for a good modeling of the system this constrain makes necessary to consider a set of weeks, $T$, big enough to represent the duration of the whole fattening process, unless $|T|$ is fixed for management reasons.

Number of trucks needed to deliver pigs to the abattoir

Pigs sold to the abattoir in the $i$-growth category at time $t$, $X_{it}$, must be loaded in trucks, $Y_t$, with a limited capacity, $Cct$, and transported to the abattoir. It is usual to consider capacities between 200 and 240 pigs depending on individual live weight of the load.

$$\sum_{i=1}^{P} X_{it} \leq Cct \cdot Y_t \quad \forall \ t \in T$$

A complementary constraint can be considered regarding the load weight capacity of trucks ($Lw$). Then, the mean live weight of pigs ($\bar{W}_{it}$) in the category growth $i$ at week $t$ has to be considered:

$$\sum_{i=1}^{P} X_{it} \cdot \bar{W}_{it} \leq Cct \cdot Lw \cdot Y_t \quad \forall \ t \in T$$
Let’s note that this constraint is not necessary in regular fattening unit when only fattened pigs are delivered to the abattoir because load capacity of trucks is far enough. A different case is met when trucks have to transport adults culled from breeding farms. They are much heavier and this constraint can be relevant, but not for the case presented here.

Delivering pigs to the abattoir

The delivery of pigs has to follow several rational rules imposed by the fact that no individual measures of weight are available. In that case, the heaviest pigs are the first to be selected for loading a truck and being transported to the abattoir. Remaining pigs have the chance of growing further and being selected later for the next delivery and so on till the farm is emptied.

Previously, we had introduced the binary variable \( h_{it} \in \{0,1\} \) to detect if two consecutive groups of growth categories are sending pigs to the abattoir. Now, we introduce another auxiliary binary variable, \( d_{it} \in \{0,1\} \), intended to detect if a group of pigs in \( i \)-growth category at time \( t \) is sent or not to the abattoir:

\[
X_{it} \leq K \cdot d_{it} \quad \forall i \in P, \; t \in T
\]
\[
X_{it} \geq 1 - K \cdot (1- d_{it}) \quad \forall i \in P, \; t \in T
\]

As before, note that K could be replaced by any arbitrary bigger number without affecting the functionality of this constraint. We complement this constraint with two more giving full sense to the binary variable \( h_{it} \) as described:

\[
h_{it} \leq d_{it} \quad \forall i \in P \setminus \{1\}, \; t \in T
\]
\[
d_{it} + h_{i+1,t} \leq 1 + h_{i+1,t} \quad \forall i \in P \setminus \{|P|\}, \; t \in T
\]

As result, the introduction of these constraints provokes that deliveries of two consecutive categories are feasible only when all the pigs of the heavier category are sent. Furthermore, to ensure and reinforce an ordered delivering of pigs the following constraints are also considered:

\[
N_{i+1,t} \geq N_{it} \quad \forall i \in P \setminus \{|P|\}, \; t \in T \setminus \{|T|\}
\]

These constraints avoid the delivery of intermediate categories of pigs.

Objective function

The primary objective of the model is the maximisation of the profit of a batch of fattened pigs. The objective function accounts for incomes and costs during the fattening period. The only income is derived from pig sales to the abattoir while costs summarise main sources of production costs, these are piglets, feeding, transport and other cost.

The income function \( I \) is defined as:

\[
I(X) = \sum_{t=1}^{T} \sum_{i} X_{it} \cdot \bar{w}_{it} \cdot (Pv + \text{Bonus}(%\text{lean}_i, w\text{carcass}_i))
\]

where:

\( \bar{w}_{it} \): represents the mean live weight of a pig (kg) in the category growth \( i \) at week \( t \).

\( Pv \): is the base selling price in € per kg of pig alive.

\( \text{Bonus}(%\text{lean}_i, w\text{carcass}_i) \): is a bonus if positive, or a penalisation if negative, on the base selling price that the abattoir computes depending on carcass weight and the lean percentage.

\( %\text{lean}_i \): lean composition of a pig calculated from the live weight of pigs.

\( w\text{carcass}_i \): carcass weight calculated from the live weight of pigs.

The cost function \( C \) is defined as:
\[ C(K, X_t, Y_t) = \text{Purchase}(K) + \text{Transport}(Y_t) + \text{Feeding}(X_{it}) + \text{Others}(X_{it}) \]
\[ = p_0 \cdot K + \sum_{t=1}^{T_i} C_t \cdot Y_t + \sum_{t=1}^{T_i} \sum_l \left( \bar{C}_t \cdot X_{lt} \right) + \sum_{t=1}^{T_i} \sum_l \left( k_{it} \cdot X_{lt} \right) + M \]

where:

**Purchase**\((K) = p_0 \cdot K\) represents the purchase cost of \(K\) piglets to be fattened. The usual practice is to express the purchase price per standard average pig (e.g. pigs of 20 kg under Spanish conditions) regardless the exact live weight, being \(p_0\) that price.

**Transport**\((Y_t) = \sum_{t=1}^{T_i} C_t \cdot Y_t\) represents the cost of transports to the abattoir.

**\(C_t\)** is the unitary cost for one trip to the abattoir.

**Feeding**\((X_{it}) = \sum_{t=1}^{T_i} \sum_l \left( \bar{C}_t \cdot X_{lt} \right)\) represents the expected total cost of concentrates consumed by pigs during the fattening process.

**Others**\((X_{it}) = \sum_{t=1}^{T_i} \sum_l \left( k_{it} \cdot X_{lt} \right) + M\) represent other costs per sold pig integrated in the unitary cost \(k_{it}\) plus fixed costs accounted by \(M\).

To summarize, the objective function representing the profit of the fattening process of a batch of pigs is:

\[ \text{O.F.:} \quad \text{Maximise } R = \text{I}(X_{it}) - C(X_{it}) = \sum_{t=1}^{T_i} X_{it} \cdot \bar{w}_{it} \cdot (P_v + \text{Bonus}(\%\text{lean}_{it}, \text{wecarcass}_{it})) - \left( p_0 \cdot K + \sum_{t=1}^{T_i} C_t \cdot Y_t + \sum_{t=1}^{T_i} \sum_l \left( \bar{C}_t \cdot X_{lt} \right) + \sum_{t=1}^{T_i} \sum_l \left( k_{it} \cdot X_{lt} \right) + M \right) \]

Where the relevant decision variables are \(X_{it}\) (how many pigs in the category growth \(i\) at week \(t\) have to be sent to the abattoir) and \(Y_t\) representing the number of trucks needed the week \(t\) to perform the transportation.

### A case study

The algebraic modelling language ILOG OPL Studio was used with CPLEX 12.4 as the linear optimization solver for implementing and solving the different instances developed for this case study. All the cases were solved in few seconds and results reported in a spread sheet of easy inspection. The case presented here is based on a typical Spanish fattening farm integrated into a private pig company or integrator who provides piglets, feedstuff, veterinary care and medicines, technical advice during the fattening process and regular control over the growth of animals. The farm operates under AIAO management. The company owns the abattoir where pigs are slaughtered and determines the pricing grid to reward integrated farmers. Furthermore, the integrator fixes also the maximum duration of fattening according to the production plans of the company, including the procurement to the abattoir and the supply of piglets to conform a new batch to be fattened. Biological parameters related basically with growth and consumption are referred to a cross breed of (Large White x Landrace) x Pietrain.

### Parameters of the model

As pigs reach marketable weights near the end of the fattening phase, a decision of when to sell them, which and how many pigs to sell has to be taken. The objective of our model is to determine the delivering policy, i.e. marketing strategy, for a fattening farm that maximizes the profit of a batch of fattened pigs. In our case we consider a fattening farm with a housing capacity of \(K=1000\) pigs. Ten groups of 100 pigs (\(K/|P|\)) are considered to split the total population (\(|P|=10\)).

A batch of pigs are fattened on farm during a fattening period under AIAO management, where that period ranges from the arrival of the first young pigs to the farm (with pigs weighting around 20 kilograms) till the week the last pig of the batch is delivered to the abattoir. Thereafter, the farm can be cleaned for a week and prepared for another incoming batch of pigs. The duration of the fattening phase typically ranges from 14 weeks to 20 weeks resulting in approximately two or three cycles per year, being \(T=17\) the maximum range selected for this instance. This range varies also according to the breed selected for fattening and specific growing traits.
In view of producing uniformly-sized lean products, abattoirs specify a bonus or penalisation of €/kg based on its carcass weight and the percentage of lean as quality indicator (Table 1). Hence, bonus or penalisations are applied in order to motivate producers to deliver pigs in homogeneous groups. The appropriate delivering strategy and the corresponding revenue will depend on the price grids of the abattoirs. Base prices are agreed in auctions markets weekly (Figure 1), however the annual mean is calculated for this instance ($P_0= 1.097$ €/kg pig alive).

Table 1. Bonus and penalisation of €/kg of carcass is a function of weight and percentage carcass lean (confidential data from an integrator belonging to the Official Spanish databank: BDPorc)

<table>
<thead>
<tr>
<th>%Lean</th>
<th>45 – 67.9</th>
<th>68 – 71.9</th>
<th>72 – 74.9</th>
<th>75 – 100</th>
<th>100.1 – 105</th>
<th>105.1 – 120</th>
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<tbody>
<tr>
<td>62 &amp; more</td>
<td>-0.1143</td>
<td>-0.0381</td>
<td>0.09906</td>
<td>0.1905</td>
<td>0.14478</td>
<td>0.0381</td>
</tr>
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<td>0.09144</td>
<td>0.18288</td>
<td>0.13716</td>
<td>0.03048</td>
</tr>
<tr>
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<td>0.0762</td>
<td>0.16764</td>
<td>0.12192</td>
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</tr>
<tr>
<td>59</td>
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<td>-0.08382</td>
<td>0.05334</td>
<td>0.14478</td>
<td>0.09906</td>
<td>-0.00762</td>
</tr>
<tr>
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<td>0.01524</td>
<td>0.10668</td>
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<td>-0.5334</td>
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</table>

Fig. 1. Price (€/kg) for pigs in 2009 published in Mercolleida (main pig auction market in Spain)
A delay in the optimum delivering time of pigs implies an increment in the feeding cost while growth rate is getting worse, with the risk of getting penalisations if the weight of pigs overtakes specific ranges or it deposes too much fat (i.e. reducing the lean percentage of the meat). Furthermore, if the duration of the fattening phase is longer, the number of batches the farm produces annually decrease. As result, the annual profit per fattening place will decrease. Otherwise, an anticipatory delivery generates penalisations mainly in terms of carcass weight and a waste of better growing rates and good conversion efficiency of feed into meat.

In general, pigs are delivered over a period of few weeks (marketing time window) using trucks with a maximum capacity of 240 pigs ($C_{\text{tr}}$) and having a cost per trip of 600 € ($C_{\text{t}}$). It is of interest to determine the timing of these deliveries near the end of the fattening phase to maximize expected profit given the natural variability of growth of the batch. In this context, the growth model of pigs is essential for this problem. To this respect we follow the proposed model by Castro (2001) and Fernandez et al. (2011) derived from experimental data with individual controls for hybrid pigs selected for meat production, a crossbreed between (Large White x Landrace) x Pietrain. Therefore, we have discretised the growth model to consider means and standard deviations of weekly live weight. Besides that, standard formulae of truncated normal distribution considered ($\tilde{W}_o$).

Table 2. Mean and standard deviation of live weight and accumulated feed intake over time in weeks (valid for the range of 10 to 26 weeks of pig lifespan, according to Castro (2001) and Fernandez et al. (2011))

<table>
<thead>
<tr>
<th>week</th>
<th>Weight(kg) mean</th>
<th>Weight(kg) sd</th>
<th>Intake(kg) mean</th>
<th>Intake(kg) sd</th>
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<tbody>
<tr>
<td>10</td>
<td>29.7</td>
<td>3.9</td>
<td>5.1</td>
<td>5.5</td>
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<td>11</td>
<td>33.4</td>
<td>4.6</td>
<td>12.1</td>
<td>8.5</td>
</tr>
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<td>5.4</td>
<td>20.5</td>
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<td>42.6</td>
<td>6.3</td>
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<td>47.9</td>
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<td>110.1</td>
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<td>47.1</td>
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<tr>
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<td>94.8</td>
<td>16.3</td>
<td>157.6</td>
<td>51.0</td>
</tr>
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<td>23</td>
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<td>17.5</td>
<td>173.7</td>
<td>54.9</td>
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</tr>
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<td>19.8</td>
<td>205.3</td>
<td>62.8</td>
</tr>
<tr>
<td>26</td>
<td>112.8</td>
<td>21.0</td>
<td>220.6</td>
<td>66.7</td>
</tr>
</tbody>
</table>

Apart modelling growth, the total feed intake of pigs is also relevant to calculate feeding cost and the consumption over time, in particular for remaining animals in the farm when deliveries to the abattoir start. A unitary feeding cost of 0.4 €/kg is taken into account to calculate the total feeding cost of a pig sold the $t$-week ($\tilde{C}_{\tilde{f}_t}$). Table 2 summarises the normal distribution assumed for live weight: mean and corresponding standard deviation over time, and also for the accumulated feed intake.

The common practice is delivering pigs when there are enough in number and within an optimal weight range to fill a truck. Regular capacity ranges from 200 to 240 pigs. A shortcoming of the process is that no objective measure of weight is available from individual animals and the pig weight is assessed by eye before making a decision. This fact makes impossible to select animals with an intermediate weight because the risk of error is very big. Only after slaughtering, the abattoir provides the individual measures per pig of relevant traits (e.g. live weight, carcass weight, lean and fat composition); however the correlation between relevant variables as live weight, carcass weight and lean percentage is quite high and useful to estimate the percentage of lean of pigs (Table 3).
Table 3. Mean and standard deviation of percentage lean and percentage of carcass depending on live weight (Castro, 2001).

<table>
<thead>
<tr>
<th>Weight</th>
<th>%Lean mean</th>
<th>%Lean sd</th>
<th>Carcass</th>
</tr>
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<tbody>
<tr>
<td>50</td>
<td>61.0</td>
<td>5.9</td>
<td>78.79%</td>
</tr>
<tr>
<td>55</td>
<td>60.7</td>
<td>6.0</td>
<td>78.82%</td>
</tr>
<tr>
<td>60</td>
<td>60.4</td>
<td>6.2</td>
<td>78.84%</td>
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<td>65</td>
<td>60.1</td>
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<td>78.86%</td>
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<td>70</td>
<td>59.8</td>
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</tr>
<tr>
<td>75</td>
<td>59.5</td>
<td>6.9</td>
<td>78.90%</td>
</tr>
<tr>
<td>80</td>
<td>59.1</td>
<td>7.1</td>
<td>78.93%</td>
</tr>
<tr>
<td>85</td>
<td>58.8</td>
<td>7.4</td>
<td>78.95%</td>
</tr>
<tr>
<td>90</td>
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<td>7.7</td>
<td>78.97%</td>
</tr>
<tr>
<td>95</td>
<td>58.2</td>
<td>8.0</td>
<td>78.99%</td>
</tr>
<tr>
<td>100</td>
<td>57.9</td>
<td>8.4</td>
<td>79.02%</td>
</tr>
<tr>
<td>105</td>
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<td>10.6</td>
<td>79.15%</td>
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<td>135</td>
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<td>11.0</td>
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<td>55.5</td>
<td>11.4</td>
<td>79.19%</td>
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<td>79.22%</td>
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<tr>
<td>170</td>
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<td>14.0</td>
<td>79.33%</td>
</tr>
<tr>
<td>175</td>
<td>53.4</td>
<td>14.4</td>
<td>79.35%</td>
</tr>
</tbody>
</table>

Results and discussion

During the development of the model the implementation was performed in an incremental way, that is, with an increase in the degree of the difficulty. This procedure allows us to better appreciate the role of binary variables. A preliminary result was obtained avoiding the binary variables $h$ and $d$ just to assess their need. This was confirmed as show in Table 4. The figures show how to solve the problem but providing an unrealistic solution considering a truck capacity of 200 pigs per truck. For instance, without weighting measures it is impossible at week 22 to select the 6th and 9th growth category leaving the intermediate 7th category till week 25. We mention this relevant fact because recently, it is found results in literature from a similar model presenting this kind of practical inconsistencies (Kamhan et al., 2013) making the model inappropriate for the intended practical purposes.

The analysis of Table 4 provides another lesson. That’s the fact that if we could know and detect the slower growing pigs, we could sell or cull them beforehand to avoid economic losses (see sales in week 20th on Table 4). This is in agreement with several papers in literature considering the benefits of individual measures on live weight and consumption (Kure, 1997; Kristensen et al., 2012). Individual measures would allow the farmer to value more precisely the feed cost and the expected profit of pigs individually making eventually a more informed decision.
Table 4. Delivery of pigs \((X_t)\) when binary variables \(h\) and \(d\) are not included in the model.

\[
\begin{array}{ccccccc}
(i,t+9) & 20 & 21 & 22 & 23 & 24 & 25 \\
1 & 100 & 0 & 0 & 0 & 0 & 0 \\
2 & 100 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & 0 & 0 & 0 & 0 & 100 \\
4 & 0 & 0 & 0 & 0 & 100 & 0 \\
5 & 0 & 0 & 0 & 100 & 0 & 0 \\
6 & 0 & 0 & 100 & 0 & 0 & 0 \\
7 & 0 & 0 & 0 & 0 & 100 & 0 \\
8 & 0 & 100 & 0 & 0 & 0 & 0 \\
9 & 0 & 0 & 100 & 0 & 0 & 0 \\
10 & 0 & 100 & 0 & 0 & 0 & 0 \\
\end{array}
\]

Afterwards, the full model was completed with binary variables and solved. Reasonable results were obtained under practical point of view as shown in Table 5. The optimal profit was of 16305 € or 16.31 €/pig or 0.166 €/kg of live weight sold. This result was a 81% better compared with the delivery of all the pigs the last week of the marketing window (8.98 €/pig). Beyond the optimal economic value, several changes in the quality of the optimal solution are observed in Table 5. For instance, always the pigs sent to the abattoir are the heaviest present on farm. The selection of these animals by eye is feasible. However, the individual selection of animals of different ranges of weight is impossible without additional measures. The number of trucks with a capacity of 240 pigs is also derived, more exactly, the number of trips to cover the transportation to the abattoir: these are 5 in total, exiting the farm in four different weeks. Hence, the optimal marketing time window is of six weeks what is in agreement with practical recommendations of integrators. Furthermore, the duration of the fattening stage is of 16 weeks as maximum (from week nine to 25) plus an additional week for cleaning and drying what is also in agreement with results reported by Ohlmann and Jones (2011). However the length of the marketing window differs likely because of different objective function, growth model and grid of payment. For instance, Ohlmann and Jones (2011) used transition probabilities to represent the growth of animals while in this paper a specific growth model and its discretisation is proposed to better estimate herd distribution of live weight over time and make decisions accordingly.

Table 5. Delivery of pigs \((X_t)\) when binary variables \(h\) and \(d\) are included in the model.

\[
\begin{array}{ccccccc}
(i,t+9) & 20 & 21 & 22 & 23 & 24 & 25 \\
1 & 0 & 0 & 0 & 0 & 0 & 100 \\
2 & 0 & 0 & 0 & 0 & 0 & 100 \\
3 & 0 & 0 & 0 & 0 & 0 & 100 \\
4 & 0 & 0 & 0 & 100 & 0 & 0 \\
5 & 0 & 0 & 100 & 0 & 0 & 0 \\
6 & 0 & 40 & 60 & 0 & 0 & 0 \\
7 & 0 & 100 & 0 & 0 & 0 & 0 \\
8 & 0 & 100 & 0 & 0 & 0 & 0 \\
\end{array}
\]

Other complementary results were obtained for extreme situations. For example, the optimal solution of the problem when the cost of transport was extremely high was the same as that shown in Table 5, this situation means that the model already provides the maximum income from the fattened pigs even though when total cost arises. However, a change is observed if the unitary cost of transport is reduced sending pigs to the abattoir each week during the optimal marketing time window ranging from 20 to 25 weeks of pigs lifespan. In all the cases computed, the mean weight of pigs sent to the abattoir is around one hundred kg, what is a regular practice in Spain.
The abattoir prefers uniformly-sized fattened pigs and establishes corresponding marketing system that penalises carcasses with weights and lean composition outside a desired range in order to motivate farmers to deliver pigs in homogeneous batches (Boland et al. 1993). Thus, an additional exploratory analysis was performed to observe the impact of variations on the weight distribution. A reduction was applied to the deviation of the mean weight at each growth category. Again, the delivery of pigs to the abattoir was the same shown in Table 5. However, the optimal profit, and the average weight of animals sent to the abattoir was higher when the batch of animals had a more homogeneous growth. That is, when weight distribution was more concentrated around the mean. A different result was obtained if no bonus was considered on pig sales. The optimal policy in that case was sell all pigs at week 21 with a lower averaged weight (89 kg) and a lower profit per batch, only 2.91 € per pig. Again, this result is in agreement with Boland et al. (1993) and Ohlmann and Jones (2011) who stated that prices grids provoke changes in marketing policies.

Conclusions

In the last years, pork sector is changing and pig farms are becoming more and more specialized, while the size of their facilities is increasing. Pig farms have tended to integrate and coordinate their operations into pork supply chains. This integration and coordination affects the decision making process at each pig production unit, in particular on fattening farms. So, this paper presents a mixed integer linear programming model description a fattening pig unit operating under all-in all-out strategy. The growth model approach presented is flexible assuming pre-stated percentiles of herd distribution as growing categories. The results presented confirm preliminary outcomes found in the literature advocating for the benefit of fattening homogeneous batches of animals. The reduction on variability at the entry of the process permits to reduce the marketing window of pigs and rises the efficiency of the process. It is also shown how a time window of five weeks delivering animals to the abattoir suffices to empty the farm and prepare it for the next batch of animals. Another interesting conclusion is that in case of individual measures of weight and consumption, better strategies could be implemented because worst animals could be removed from the herd avoiding further losses. In the end, it is highlighted the importance of credible solutions for practical purposes, this fact is sometimes neglected in literature as shown. Summarising, our contribution confirms the findings of past studies but demonstrates the importance of future trends relying on individual measures to avoid extra costs derived from grouped animals.

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