Coordination single buyer-supplier with vendor managed inventory from the net present value perspective

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Abstract. It This paper developed the coordination inventory models with vendor-managed inventory (VMI). The mathematical models are derived from Net Present Value (NPV) functions for the single-buyer single-supplier under deterministic conditions. The main objective is to develop a model to find optimal lot-size and profit function in supply chain. The model is analyzed under two different scenarios consisted of without-coordination and coordination inventory policy. In the case of without coordination the buyer and the supplier act independently to optimise their profits or costs; while in the coordination scenario both actors are agree to joint decision policy to determine the optimal variables to achieve profit (or cost) functions. The classical inventory model is applied for non-coordination scenario as the based case to explore the effects of NPV to optimal lot-size and profit function. In the proposed model, four different VMI strategies which derived from NPV are applied in the coordination models where the supplier is authorized to manage the buyer’s inventory and make all the replenishment decisions. The experimental results have shown that the profit function derived from NPV is given the total profit in supply chain better than classical inventory model. Furthermore, coordinated VMI approaches derived from NPV have shown that the VMI+ can achieve a very good coordination. This model guarantees that the profit function for both players remains the same without coordination and increases as a result of coordination.

Keywords: vendor managed inventory; net present value; buyer-supplier coordination

Introduction

The supply chain management (SCM) is a term used to describe the management of materials and information across the entire supply chain, from suppliers to buyers (retailers), and final customer. Inventory costs can be classified into two parts concluded of the cost of capital (or investment) and physical storage costs. The capital costs are the money expended due to time delays between in and out payment streams in the companies including of depreciation and interest charges (Grubbstrom et al., 1986). The cost of holding inventory included of the money invested, expense for managing warehouse, handling and other variable costs, insurance and taxes, and loses from deterioration, damage, theft, and obsolescence (Silver et al., 1998).

In order to minimise a total costs, or alternatively maximise a total profits in a supply chain the cooperative decisions among supplier and buyer would be applied which this method lead to achieve benefit for all parties.

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(Jaber and Zolfaghari., 2008). When there is no cooperation between the firms, each firm in the supply chain will focus on optimizing its own cost or profit function based on the variables. In this case, the decisions concerning production, purchase and shipment are made separately and often sequentially by each of the member of the chain. Although each member in the supply chain has different operational goals, the performance of all members depends not only on how well each member manages its operational processes, but also on how well the members coordinate their decisions. Buyer and supplier act independently is thus clearly not guaranteeing that the best overall optimal result is achieved that would minimise (maximise) the total cost (profit) function of the supply chain. It is thus very important on how decisions in a supply chain can be coordinated in order to get closer to the joint optimal solution. Several researchers proposed the idea of optimizing the total cost to analyze the integration in a single supplier-buyer model was considered early on by Goyal (1976). Banerjee (1986) developed the model by incorporating a finite production rate and following a lot-for-lot policy for the vendor. By relaxing Banerjee’s lot-for-lot assumption, Goyal (1988) proposed a more general joint economic lot sizing model. Lu (1995) specified the optimal production and shipment policies when the shipment sizes are equal. He relaxed the assumption of Goyal (1988) about completing a whole batch before starting shipments. Goyal (1995) then developed a model where successive shipment sizes increase by a ratio equal to the production rate divided by the demand rate. He found an expression for the optimal first shipment size as a function of the number of shipments. Later, Hill (1997) took this idea one step further by considering a more general type of policy, the single-buyer, single-supplier integrated production inventory model based on successive shipments to buyer, within a single production batch increasing by a fixed factor. In addition, the coordinated inventory models have been argued in the following literature. According to Goyal and Gupta (1989) applying the inventory decision models of players in a particular supply chain is a common method of obtaining good coordination. Ben-Daya et al. (2008) have contributed to improve literature on cooperated inventory models. The coordination orders in a two-level (vendor-buyers) supply chain have become the main issue discussed by various researchers in their studies (Weng et al. 1995; Gurnani, 2001; Boyaci and Gallego, 2002; Viswanathan and Wang, 2003; Jaber and Osman, 2006).

In the net present value (NPV) approach is widely accepted to use the right framework for studying production and inventory control systems it see in Grubbstrom (1980) and Van der Lann and Teunter (2002). Grubbstrom (1980) presented the inventory control models that can fully capture the impact of interactions of various Cash-Flow Choice Problem or CCP implicitly present in the system which this approach is generally considered to be the right approach in financial decision making, it focuses directly on cash flows rather than derivative and profits. Annuity stream principle is proposed by Grubbstrom and Thorstenson (1986), this method is the transformation of a set of discrete and/or continuous cash flows to compute one continuous stream of cash flow, such that latter has the same NPV as the original set of cash-flows. Annuity stream function of the cash-flows associated with a production or inventory system can be used in the classical framework for deriving a total profit or cost function. In NPV analysis, all cash flows related to activities are valued by their time of occurrence using as one common discount rate \( \alpha \), which represents the opportunity cost of the next best alternative for the firm. Beullens and Janssens (2010a) introduced NPV to identify the ‘supplier’s reward’ in term of profit function for the vendor in the classic buyer-vendor models. Beullens and Janssens (2010a) presented NPV to identify the gap between the classic function and the NPV-derived profit functions in the infinite production rate model. There are several researches and relevant articles for introducing the same approach such as Hadley (1964); Trippi and Lewin (1974); Thompson (1975); Grubbstrom and Thorstenson (1986); Grubbstrom (1998, 2007); Rummel (1985); Klein Haneveld and Teunter (1986); Hofmann (1998).

Vendor managed inventory (VMI) has been defined as a collaborative strategy between a buyer and a supplier to optimize the availability of products at minimal cost to both companies. The supplier takes responsibility for the operational management of the inventory within a mutually agreed framework of performance targets which are constantly, monitored and updated to create an environment of continuous improvement in Jame and Rich (1997). In a VMI system, the operations of the vendor and buyer can be integrated through information sharing by using the information technologies such as electronic data interchange (EDI) or internet-based protocols. The vendor can use this information to plan production, schedule deliveries, and manage inventory levels at the buyer.

As a consequence, system cost can likely be reduced while capacity utilization will be increased. These benefits of VMI have been widely recognized in different industries, especially in the retail industry. There are several VMI strategies from previous researches, Bernstein et al. (2006) presented VMI\(^+\) and VMI\(^-\) whereby the supplier determines lot-size and a number of shipments simultaneously from his own profit function. They show how vendor-managed inventory (VMI) partnerships create echelon operational autonomy (EOA) and compare the
resulting coordinating pricing schemes with those required in a traditional decentralized setting (without EOA). Gumus et al. (2008), presented consignment stock whereby the buyer only pays the supplier in whole sale price at the moment a sale is made to the final customer. Darwish and Odah (2010), presented a model that allows the supplier to determine lot-size whereby buyer pays for the goods upon delivery, but the supplier still pays the fixed and variable delivery charges. Many successful retailers such as Wal-Mart and Kmart were the pioneer firms to adopt VMI system. The popularity of VMI has lead to the claim that it is the vital policy in the future and this concept will revolutionize the tactical decision in the distribution channel see in Andel (1996); Burrke (1996); and Cottrill (1997). For further discussion on the VMI system, we refer the reader to Xu et al. (2001), Waller et al. (1999) and Yao et al. (2005).

In this paper, we propose coordination buyer-supplier with VMI strategies from NPV perspective. The objective is to find the total profit functions when both actors accepted coordinative decision to optimize inventory policies in the supply chain system. This study is separated into two scenarios: the first scenario, buyer supplier without coordination is studied with the classical model whereby each party act independently. The second scenario, coordination buyer supplier is considered with VMI strategies proposed here with four strategies consisting of consignment inventory (CI), VMI$, VMI$, and vendor managed replenishment (VMR). In addition, NPV framework has been applied to both scenarios to find the optimal lot-size and analyze the profit functions.

The rest of this paper is organized according to the following structure. In the next section we propose the assumptions and notation. A section 2, without coordinated buyer-supplier model is proposed and alternative model with NPV approach is applied. Section 3, the model development derived from NPV approach is applied in coordination buyer-supplier model with VMI strategies. Section 4, Numerical experiments and discussions are presented and the conclusions and further research are given in section 5.

Assumptions and notation

The supply chain model studied in this paper contains two independent parties which consist of single supplier (vendor), single buyer. In this paper the buyer purchases a single product from the supplier and both parties have perfect information about the other player’s characteristics. The assumptions of the model are summarized: the final demand rate ($D$) is deterministic and constant; the supplier produces at (in) finite rate ($R$) where ($R > y$); sales price per product ($p(y)$); ($w$) denote contract purchase price where ($p > w$); ($c$) denotes production cost where ($w > c$); ($h_i$) is holding cost per product per year and ($s_i$) is fixed order processing cost for actor; the optimal lot-size is $Q_j$ whereby is the $j \in \{b, s\}, (j = b$ for buyer, $j = s$ for supplier) and integer multiplication $m$ ($Q_b = m Q_b, m \geq 1$); shortages and backlogs are not allowed; lead time is zero; the internal annual holding cost rate $c_i$ for actor $i \in \{b, s\}$; and also there are no capacity constraints.

Whereby is the $j \in \{b, s\}, (j = b$ for buyer, $j = s$ for supplier) and shortages and backlogs are not allowed; lead time is zero; the internal annual holding cost rate $c_i$ for actor $i \in \{b, s\}$; and also there are no capacity constraints. The flow of the lot-sizes follows the standard assumptions. The buyer receives items in batches of $Q_b$ at equidistant points in time with cycle time $T = Q_b / D$. The supplier's inventory position over time follows the classical saw-tooth function, as given in Figure 1. The supplier produces in batches of $Q_s = m Q_b$, where $m \in \{1, 2, 3, \ldots \}$, and ships the first batch $Q_b$ as soon as it is available, i.e. $DT / R$ time after the start of the production run, and delivers all subsequent $m - 1$ batches at equidistant points in time corresponding to the buyer's cycle time $T$. The supplier's cycle time, the time between the start of two subsequent production runs, is $m T$.

Without-coordinated buyer-supplier

In this model, classical inventory model is considered as without-coordinated case. We assume that buyer and supplier manage inventory independently. Each actor manages its own replenishments or production, respectively. Both actors must decide to maximize own profit. The result is then compared to the coordinated system where the two parties cooperate to make decision to manage inventory centrally. The inventory model from buyer-supplier is shown in Fig. 1.
**Buyer’s profit function:**
The classical model from Monahan (1984) is applied to find the buyer’s profit function. The elements of the buyer’s profit are as follows: the revenue from selling the product to final customer and the buyer’s annual cost can be expressed as the sum of annual fixed cost of ordering from the supplier, the holding cost from stock level at buyer’s site. Then, his optimal lot-size and the buyer’s profit function are shown in Eq. (1) and Eq. (2) as:

\[ Q_b = \frac{2S_p D}{h_b} \]  
\[ \Pi_b = (p - w)D - s_b \frac{D}{Q_b} - h_b \frac{Q_b}{2} \]

**Supplier’s profit function:**
In this case, the model from Joglekar (1988) who was considered supplier’s profit function for the case of (in) finite production rate \( R \) and \( m \geq 1 \) and integer is considered. The buyer has been decided the optimal lot-size \( Q_b \) and transfer order quantity to supplier, then supplier accepted the buyer’s lot-size and will define the production lot-size and integer multiple \( m \) of \( Q_s = mQ_b \). The profit function for supplier is given:

\[ \Pi_s = (w - c)D - \left( \frac{S_s}{m} + S_p \right) \frac{D}{Q_b} - h_s \left[ (m - 1) - (m - 2) \frac{D}{R} \right] \frac{Q_b}{2} \]

\[ m^* = \frac{s_s h_b}{s_b h_s} \]

Let \( m^* \) be the optimal integer value, where \( [m^*] \) if \( \Pi_s([m^*]) \geq \Pi_s([m^*]), \text{and } [m^*] \) otherwise.

**Joint profit function:**
The joint profit function would be obtained by maximize the supply chain profit function in Eq. (5) which is given in the sum of Eq. (2) and Eq. (3) as:

\[ \Pi_{sc} = (p - c)D - (S_b + S_p + \frac{S_s}{m}) \frac{D}{Q_b} - h_s \left[ (m - 1) - (m - 2) \frac{D}{R} \right] \frac{Q_b}{2} - h_b \frac{Q_b}{2} \]

The joint profit from buyer and supplier, say the optimal \((Q_b^{sc} \text{ and } m^{sc})\), was derived previous based supply chain profit function:

\[ Q_b^{sc} = \sqrt{\frac{2(S_p + \frac{S_s}{m} + S_b)D}{h_s(m - 1) - h_s(m - 2) \frac{D}{R} + h_c}} \]

\[ m_l^{sc} = \sqrt{\frac{s_s(w - c) + 2S_s \frac{D}{R}}{c(S_b + S_p)(1 - \frac{D}{R})}} \]

The optimal \( m^{sc} \), as always, should be rounded up to 1 if it is smaller, and otherwise rounded to nearest higher or lower integer, whichever gives the highest profit function value.
Fig. 1. Buyer-supplier inventory model

Fig. 2. Buyer-supplier cash-flows over time
An alternative model

In this section the Net Present Value (NPV) approach is used to model the inventory systems and also can be used as financial measurement to capture the impact of interactions of various cash-flows from inventory models such as effects of timing of cash-flows by including in holding cost of inventories and the opportunity cost of the capital:

\[
NPV = E\left\{ \sum_{k=1}^{\infty} C e^{\alpha_i kT} \right\}
\]

In addition we define the Annuity Stream (AS) to derive NPV as \(AS = \alpha_i \{NPV\}\).

The annuity stream function of the cash-flows associated with a production or inventory system can be used for deriving a total profit or cost functions. These approximations are obtained by Maclaurin expansion of the exponential terms in \(\alpha_i kT\), where \(k \in \{1, D/R, m\}\), and then the linear approximation of the expansion in \(\alpha_i (i \in \{b, s\})\).

**Buyer’s profit function:**

In order to derive NPV results, the cash-flow in the model need to be specified. The following assumptions are made (see also Figure 2). The buyer sells the product to final customers at a constant rate \(pD\), inventory starting at time \(L\) when the first batch arrives. The buyer incurs a set-up cost \(S_b\) upon the arrival of every batch \(Q_b\), and pays the supplier \(wDT\). The buyer’s annuity stream \(AS_b\) function is thus:

\[
AS_b = [pD - (S_b + wDT)] \sum_{i=0}^{\infty} \alpha e^{-\alpha T} e^{-\alpha L}
\]

Maclaurin expansion of the exponential term with respect to \(\alpha_b T\) gives:

\[
AS_b = pD - (S_b + wDT) \left( \frac{1}{T} + \frac{\alpha_b}{2} + O\left(\frac{\alpha_b^2 T}{12}\right) \right)
\]

The linear approximation can be written as:

\[
\overline{AS}_b = \left( (p - w)D - S_b \frac{D}{Q_b} - \frac{\alpha S_b}{2} - \alpha w \frac{Q_b}{2} \right) e^{-\alpha L}
\]

The first term in Eq.(11) represents the marginal profits on sales, the second the set-up costs, the third the average interest cost of set-up payments, and the fourth the holding costs. The holding cost for the buyer is valued at \(w\), the money invested in keeping products in inventory, and corresponds to the traditional viewpoint of holding costs.

**Supplier’s profit function:**

In this assumption at the start of every production run, a set-up cost \(S_s\) is incurred, and a variable production cost at rate \(cD\) starts, and stops \(mDT/R\) time later in the supplier’s cycle. Every time the supplier ships a batch to the buyer, he incurs a set-up cost \(S_p\) and receives the income \(wDT\) from the buyer. The supplier’s annuity stream profit function is thus:

\[
AS_s = [-S_s \frac{\alpha e^{\alpha DT/R}}{1 - e^{-\alpha mT}} - cR \frac{1 - e^{-\alpha DT/R}}{1 - e^{-\alpha mT}} e^{\alpha DT/R} - (S_p - wDT) \frac{\alpha}{1 - e^{-\alpha T}}] e^{-\alpha T}
\]

The linear approximation can be written as:

\[
\overline{AS}_s = \left( (w - c)D - \left( S_p + \frac{S_s}{m} \left( 1 + \frac{D}{R} \right) \right) \frac{D}{Q_b} - \alpha_s S_p + \frac{S_s}{2} - \alpha_s c (m - 1) \frac{Q_b}{2} \right) e^{-\alpha L}
\]

\[+ \alpha_s c (m - 2) \frac{D Q_b}{R^2} + \alpha_s (w - c) \frac{Q_b}{2} \right] e^{-\alpha L}
\]
The first term in Eq. (13) represents marginal profits on sales, the second the set-up costs, the third the average interest costs of set-up payments, the other three terms in $Q_b$ are the inventory holding costs and that the last inventory holding cost term in $(w-c)$ actually implies additional profits. This term is the 'supplier's reward', identified in Beullens and Janssens (2010a).

**Joint profit function:**

The joint profit function can be defined as the sum of buyer's and supplier's profit functions. The linear approximation of the annuity stream profit function is thus:

$$\overline{AS}_c = [(p-c)D - (S_b + S_p + \frac{S_s}{m}(1 - \frac{D}{R})) \frac{D}{Q_b} - \frac{S_b + S_p + S_s}{2} - ac(m-1) \frac{Q_b}{2} + ac(m-2) \frac{D}{R} \frac{Q_b}{2}]e^{-at}$$

The supplier’s reward term has the effect in the last term in Eq. (14), which can be interpreted as the holding cost term for the buyer’s inventory, is now valuing this inventory at production cost $c$ rather than at $w$. The joint total profits, as well as the joint optimal policy, are independent of the transfer price $w$. Note that the buyer’s and supplier’s profit functions, as well as their joint profit function, can be interpreted as annuity stream functions calculated for time $L$, and then transferred to the annuity stream at time $0$ by multiplication with ‘delay’ term $e^{at}$. This delay term has no impact on optimal policies, and can thus be ignored. In addition, when comparing profit function values with the classic models, since $L$ is arbitrary, this delay term should also be ignored.

The supplier’s optimal lot-size $Q_{bc}^*$ given by Eq. (15) and the optimal value for $m$ is shown in Eq. (16):

$$Q_{bc}^* = \sqrt{\frac{2(S_b + S_p + \frac{S_s}{m}(1 - \frac{D}{R}))D}{\alpha_c [m(m-1) - (m-2) \frac{D}{R}] - \alpha_s (w-c)}}$$

where $h_s \equiv \alpha_c$, and $h_p \equiv \alpha_w$.

Using the approach presented in Munson and Rosenblatt (2001), the optimal integer value for $m$ based on his profit function:

$$m^* = \left\lfloor 1 + \frac{S_s (1 + \frac{\alpha_s D}{R}) (2 \frac{D}{R} h_s - \alpha_s w)}{S_p h_s (1 - \frac{D}{R}) \frac{2}{2}} \right\rfloor$$

**Proposed coordination buyer-supplier model with VMI**

In general, VMI is one of inventory policies where the supplier is authorized to manage inventory for retailer by making decision for replenishment orders on behalf of buyers and can decide when and how to deliver product to the buyer. In this section coordination buyer-supplier model with VMI is proposed. Four different VMI strategies derived from NPV approach are developed and compared in term of the optimal policy and profit function which included of VMI”, VMI”, Consignment inventory (CI), and Vendor managed inventory (VMR). In order to derive NPV results, the cash-flows in the model need to be specified at the moment when the money are paid-in, or paid-out from buyer and supplier and then annuity stream $AS_i^t$ function is applied to derive profit function where $(t \in \{-, +, c, r\})$, and $(i \in \{b, s\})$. The proposed models are presented as follows:
Vendor managed inventory (VMI-)

VMI- is the strategy whereby the supplier determines lot-size $Q_b$ and $m$ simultaneously from his own profit function to optimize for the replenishment decisions while the buyer still pays all costs upon every delivery. The buyer and supplier’s profit functions from are defined as follows.

**Buyer’s profit function:**
In order to derive the profit function the cash-flow in the model needs to be specified. In this strategy the buyer sells the product to customers at a constant rate $pD$, inventory starting at time $L$ when the first batch arrives. The buyer incurs the order processing cost $S_b$ upon the arrival of every batch $Q_b$, and pays the supplier $wDT$. The buyer’s annuity stream $A S_b^b$ function is thus:

$$A S_b^b = A S_b^b = \left[pD - (S_b + wDT) \sum_{t=0}^{\infty} a e^{-at}\right]$$

The linear approximation is:

$$\overline{A S_b^b} = \overline{A S_b^b} = \left[(p - w)D - s_b \frac{D}{Q_b} - \frac{a_b s_b}{2} - \frac{a_b w Q_b}{2}\right] e^{-a b L}$$

The first term in Eq.(18) represents the marginal profits on sales, the second term is the set-up costs, the third the average interest cost of set-up payments, and the fourth the holding costs. The holding cost for the buyer is valued at $w$, the money invested in keeping products in inventory.

**Supplier’s profit function:**
Supplier incurred a set-up cost $S_s$ every production run, and a variable production cost at rate $cD$, which will be started and stopped at $mDT/R$ time later in the supplier’s cycle. Every time the supplier delivers a batch to the buyer, he incurs a set-up cost $S_p$ and receives the income $wDT$ from the buyer. The supplier’s annuity stream profit function is thus:

The linear approximation is:

$$\overline{A S_s^s} = \overline{A S_s^s} = \left[(w - c)D - \left(s_p + \frac{s_s}{m} \frac{D}{Q_b}\right) - \frac{a_s c}{2} \left[(m - 1) - (m - 2) \frac{D}{R} \frac{Q_b}{2}\right] - \frac{a_s}{2} (1 + \frac{a_D}{mR}) + a_s (w - c) \frac{Q_b}{2}\right] e^{-a s L}$$

The supplier’s profit function from VMI’ are composed of the marginal profits on product sales from supplier to buyer shown at the first term, the second term is the set-up costs, the third term is the inventory holding costs, and the forth terms is the average interest costs of set-up payments. The set-up cost in $S_s$ is function of production $R$, and that the last inventory holding cost term in $(w - c)$ actually implies the supplier’s reward.

Vendor managed inventory (VMI+)

VMI’ is also known as Consignment with VMI. The supplier determines $Q_b$ and $m$ simultaneously from his own profit function. He is paying for all delivery-related costs for buyer such as order processing cost, and will receives payments $w$ per item from the buyer at the rate which the buyer sells products to customers.

**Buyer’s profit function:**
The cash-flow is constructed to derive NPV results. In this strategy the buyer will receive revenue from sells the product to customers at a constant rate $pD$, and pays for transfer price to supplier at $w$ but no payment for the order processing cost $S_b$ which this cost is transferred to supplier. The buyer’s annuity stream function is thus:

$$A S_b^b = A S_b^b = \left[pD - (S_b + wDT) \sum_{t=0}^{\infty} a e^{-at}\right]$$

The linear approximation is:

$$\overline{A S_b^b} = \overline{A S_b^b} = \left[(p - w)D - s_b \frac{D}{Q_b} - \frac{a_b s_b}{2} - \frac{a_b w Q_b}{2}\right] e^{-a b L}$$

The first term in Eq.(18) represents the marginal profits on sales, the second term is the set-up costs, the third term is the inventory holding costs, and the fourth the holding costs. The holding cost for the buyer is valued at $w$, the money invested in keeping products in inventory.
The marginal profits achieved on products sales from buyer to customer at rate \( p \) and then pay to supplier for transfer price at rate \( w \). In this strategy, all of delivery-related costs for buyer will be paid by supplier.

**Supplier’s profit function:**
Supplier is incurred a set-up cost \( S_s \) for every production run, and also pay a variable production cost at rate \( cD \) starts, which will be start and stops \( mDT/R \) time later in the supplier’s cycle. Every time the supplier delivery a batch to the buyer, he incurs a set-up cost \( S_p \) and set-up cost \( S_b \) at buyer’s site and will receives the income \( wDT \) from the buyer. The supplier’s annuity stream profit function is thus:

The linear approximation is:

\[
\frac{AS_s^+}{e^{-\alpha_b L}} = (p - w)D e^{-\alpha_b L} = \frac{AS_s^+}{e^{-\alpha_b L}}
\]

\( AS_s^+ \) is composed of the marginal profits on product sales from supplier to buyer at the first term in Eq. (21), the second term is the set-up costs from buyer and supplier sites. In the third is the inventory holding costs. The first term in the second line of Eq.(21) tells that holding costs are valued at rate \( c \) in line with the traditional derivation and the second terms is the average interest costs of set-up payments. Note that the effect of the transfer price on the capital holding costs has been removed and the supplier reward not incurred from this strategy.

**Consignment inventory (CI)**

CI is the strategy whereby the buyer chooses \( Q_b \) to maximize the buyer's profit function. The supplier accepts this lot-size, and finds the integer multiplier \( m \) that maximises the supplier's profit function. In this approach supplier will receive sourcing \( w \) per item from the buyer at the rate which the products have been sold to customers. The profit functions are defined as follows.

**Buyer's profit function:**
In this strategy the buyer sells the product to customers at a constant rate \( pD \), inventory starting at time \( L \) when the first batch arrives. The buyer incurs a set-up cost \( S_b \) upon the arrival of every batch \( Q_b \), and pays the product price to the supplier at \( wDT \). The buyer's annuity stream function is thus:

The linear approximation is:

\[
\frac{AS_b^{ce}}{e^{-\alpha_b L}} = [(p - w)D - s_b \frac{D}{Q_b} - \alpha_b \frac{Q_b}{2} - \alpha_b \frac{s_b}{2}] e^{-\alpha_b L}
\]

The first term in Eq. (22) represents the marginal profits on sales, the second term is the set-up costs, the third term is the average interest cost of set-up payments, and the fourth term is the holding costs.

**Supplier’s profit function:**
Supplier is incurred a set-up cost \( S_s \) every production run and a variable production cost at rate \( cD \) starts, and stops \( mDT/R \) time later in the supplier’s cycle. Every time the supplier delivery a batch to the buyer, he incurs a set-up cost \( S_p \) and receives the income at the rate \( w \) where the product has been sold to customer. The supplier’s annuity stream profit function is thus:

The linear approximation is:
The supplier’s profit function from CI is composed of the marginal profits on product sales from supplier to buyer in the first term of Eq. (23), the second term is the set-up costs, the third term are composed of the inventory holding costs, and the fourth terms in the second line tells that holding costs are valued at rate \( c \) in line with the traditional derivation and the fifth terms is the average interest costs of set-up payments. In this strategy the transfer price on the capital holding costs does not affect to the profit function and the supplier rewards have become zero.

**Vendor managed replenishment (VMR)**

This strategy the supplier determines \( Q_b \) and \( m \) simultaneously from his own profit function. The buyer pays for the goods upon delivery, but the supplier is carrying the delivery costs.

**Buyer’s profit function:**
In this strategy the buyer achieve income from sells the product to customers at a constant rate \( pD \), and pays for product price to supplier at rate \( w \). The buyer’s annuity stream function is thus:

\[
\overline{AS^r_b} = [(p - w)D - \alpha_b w \frac{Q_b}{2}] e^{-\alpha_b L}
\]  

The first term in Eq. (24) represents the marginal profits on product sales at constant rate \( pD \), and pays the product price to the supplier at \( wDT \), the second term is holding costs. Note that the holding cost for the buyer is valued at \( w \), the money invested in keeping products in inventory.

**Supplier’s profit function:**
Every production run, supplier incurs a set-up cost \( S_s \) every production run, and a variable production cost at rate \( cD \) starts, and stops \( mDT / R \) time later in the supplier’s cycle. Every time the supplier delivery a batch to the buyer, he incurs a set-up cost \( S_p \) and receives the income \( wDT \) from the buyer. The supplier’s annuity stream profit function is thus:

\[
\overline{AS^r_s} = \left[ (w - c)y - \left( s_b + s_p + \frac{s_s}{m}\right)D - \alpha_s c \left( m - 1 \right) - \left( m - 2 \right) \frac{D}{R} \right] \frac{Q_b}{2} \left( s_b + s_p + s_s \left( 1 + \frac{\alpha D}{m R} \right) \right) e^{-\alpha_s L}
\]  

The supplier’s profit function from VMR is composed of the marginal profits on sales from supplier to buyer in the first term, the second term is the set-up costs, the third term is the inventory holding costs at the average of inventory level, and the forth terms is the average interest costs of set-up payments. This strategy the transfer price on the capital holding costs does affect to the profit function which the supplier’s reward has been incurred in the last inventory holding cost term at \( (w - c) \).
Joint optimal policy:
In this strategy when buyer and supplier agree to adopt the discount rate \( \alpha \), the annuity stream function for the joint system, \( AS_J \), is given by:

\[
AS_J = \left[ (p - c)y - \left( s_b + s_p + \frac{s_s}{m} \right) \frac{D}{Q_b} - \alpha c((m - 1) - (m - 2) \frac{D}{R}) \frac{Q_b}{2} 
- \alpha c -\frac{Q_b}{2} - \alpha \frac{s_b + s_p + s_s(1 + \frac{2D}{mR})}{2} \right] e^{-\alpha L}
\] (26)

Note that this function is independent of the strategy chosen and of the transfer price \( w \). The individual profit functions for buyer and supplier, however, are dependent on cash-flow structure of the particular strategy and on the transfer price \( w \), and are given by their annuity stream functions presented in the previous section.

Numerical experiments and discussions

Four instances have been used in this paper consisted of instance 1 has been proposed by Goyal (1976), and other instances are derived from these by changing parameters such as demand rate \( D \), the purchase price \( p \) and fixed ordering cost \( S_b \) and the relative importance of set-up costs \( S_s \). The input parameters are listed in Table 1.

| Table 1. Problem instance characteristics |
|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| Instance | \( D \) | \( p \) | \( w \) | \( c \) | \( s_b \) | \( s_s \) | \( R \) |
| 1 | 12000 | 3 | 1.5 | 1.2 | 100 | 400 | 38400 |
| 2 | 20000 | 4 | 2.9 | 1.5 | 120 | 400 | 38400 |
| 3 | 25000 | 30 | 25 | 20 | 150 | 700 | 38400 |
| 4 | 30000 | 40 | 34 | 25 | 200 | 700 | 38400 |

For each instance \( \alpha = 0.2 \), \( s_p = 0 \)

| Table 2. Optimal order quantities, profit in individual and joint the buyer-supplier supply chain |
|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| Instance | Model | \( Q_b^* \) | \( Q_s^* \) | \( m^* \) | \( Q_c^* \) | \( m^o \) | Buyer Profit | Supplier Profit | Total Profit | Buyer Profit | Supplier Profit | Total Profit |
| 1 | Classic | 2828 | 5656 | 2 | 2818 | 3 | 17151 | 2412 | 19563 | 17151 | 2462 | 19613 |
| | NPV | 2828 | 5657 | 2 | 3953 | 2 | 17141 | 2394 | 19535 | 17090 | 2549 | 19640* |
| 2 | Classic | 2876 | 8628 | 3 | 2745 | 4 | 20331 | 26435 | 46766 | 20330 | 26465 | 46795 |
| | NPV | 2876 | 8630 | 3 | 3791 | 3 | 20319 | 26686 | 47005 | 20256 | 26858 | 47113* |
| 3 | Classic | 1224 | 2448 | 2 | 1049 | 5 | 118876 | 115403 | 234280 | 118802 | 117370 | 236172 |
| | NPV | 1224 | 2449 | 2 | 1125 | 5 | 118861 | 114998 | 233860 | 118839 | 117351 | 236190* |
| 4 | Classic | 1328 | 2656 | 2 | 1084 | 6 | 170967 | 258773 | 429740 | 170779 | 261690 | 432469 |
| | NPV | 1328 | 2657 | 2 | 1182 | 6 | 170947 | 258685 | 429632 | 170885 | 262049 | 432934* |
Table 3. Optimal order quantities, profit in coordinated buyer-supplier with VMI strategies derived from NPV

<table>
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<tr>
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<th>Model</th>
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<th>Supplier lot size $Q_s$</th>
<th>$m^*$</th>
<th>Buyer's Profit</th>
<th>Supplier's Profit</th>
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Analysis and discussion

From Table 2 the optimal lot-sizes and profit functions obtained from the classical model without NPV, and classical model with NPV are compared. In the first scenario, the classical inventory model used as the based case which from this can be classified into four situations. Firstly, individual buyer-supplier without NPV, the second situation individual buyer-supplier with NPV, the third coordination buyer-supplier for joint optimal without NPV, and fourth situation the coordination buyer-supplier for joint optimal with NPV. The optimal lot-size and profit function from four situations are reported in Table 1 are simply based on the NPV framework. The comparison shows, of course, that the optimal solution derived from coordination buyer-supplier for joint optimal with NPV-framework give the better solution than coordination buyer-supplier for joint optimal without NPV, whereas the total profit functions increased from 19613 to 19640 for instance 1 and from 467951 to 47113 for instance 2. The total profit functions also increased from instance 3 and 4 at which from 236172 to 136190 and from 432469 to 432934, respectively.

In coordinated optimisation scenario (VMI approaches), all instance show that the VMI+ strategy derived from NPV framework given the higher profit for supply chain more than other strategies. The experimental results have shown that the optimal solution is given from a coordinated of VMI+ derived from NPV framework in Table 3. The benefit from coordinated optimisation shown that the profit functions have significantly increased from VMI+ for buyers see in (column 6), and total profit for supply chain see in (column 8). For instance 1, the maximum total profit derived from VMI+ and VMR at 19640. For instance 2, 3, and 4, VMI+ gives the maximum total profit at 47117, 236593, and 433337, respectively.

Conclusions and further research

This paper presented the model for coordination single-buyer and single-supplier with VMI derived from NPV approach. Unlike earlier work, this paper focused on implicit costs which associated with opportunity cost of capital on managing inventory. Deriving profit function from NPV approach the mathematical modelling has been developed from cash-flows in order to derive the optimal lot-size for buyer $Q_b$ and supplier $Q_s$ and maximise profit for supply chain system as the performance measure. The NPV approach does make clearly distinction between physical inventory and opportunity costs, since the two are not related. The later does not depend on physical stocks at all,
but on the amount and timing of the investment. With the present contribution can be concluded that the NPV is useful financial method to derive the inventory model and investigated the optimal lot-size and profit function. This paper shows that the positive financial effect from NPV framework is given the larger lot-size for supplier which this gives supplier to achieve the higher profit in supply chain and her profit function. However, those of VMI strategies included of VMI−, CI, and VMR models cannot guarantee a good channel coordination, but VMI+ can guarantee a good coordination. This model guarantees that the profit function for both players remains the same without coordination and increases as a result of coordination.

For further research, the new model in which one supplier faces two or more buyers should be focused. The model in which the shortage is in the form of lost sale for the buyer should be also investigated and the conditions in which the VMI system will work better with respect to the traditional mode should be identified. It is also suggested to consider and analyze the problem presented in this paper in the three-level mode.

References


