Valuation of area development project investments as compound real option problems

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Abstract. Area development projects are large construction projects that aim for designated areas to be constructed in a concentrated time frame. Starting from owning the land on which the project is to be built, the projects can be divided into three main stages: planning & zoning, construction, and post-construction. Each of the stages has its own cash in- and out-flows, and advancing from one stage to the next is optional for the involved parties – this makes area development investments compound real option problems. The projects involve large sunk investments, are often long-term, and the size of the cash-flows are often difficult to accurately estimate, this makes analysis of area development project profitability complex and challenging. This paper presents how ex-ante analysis of area development projects can be considered as compound real options and illustrates with a numerical example how the compound real-option valuation of a potential project can be performed by using the pay-off method for real option valuation.

Keywords: area development; compound real option valuation; pay-off method; decision support

Introduction

Area development investments are large long-term construction projects that aim for designated areas to be constructed in a concentrated time frame. In this paper we consider area development investments from the point of view of a developer (public or private) who has the possibility to make project level choices based on the profitability of different project combinations, that is a developer with project level real options. A prerequisite for being able to start an area development project is that a designated area is available for development. The alternatives project owners most often have are to purchase land, or to lease land for the project purposes. The profitability level of the possible project to-be-built on the land determines the acceptable cost for obtaining the use of the land. Once designated land is bought or leased, area development projects can be divided into three separate phases, according to the conducted activities: planning and zoning, construction, and post construction. The planning and zoning phase is the first part of the area development process that includes the investment of the acquired / leased land into the project, planning the area to be developed (e.g. architecture, municipal engineering & infrastructure plans etc.), and possible zoning / re-zoning the area. The construction phase can start when the zoning is ready and construction permits are valid. The phase includes the construction of the municipal engineering & infrastructure into the area (roads, pipelines etc.) and the construction of the planned buildings. The post-construction phase starts after the construction of the buildings are ready and rentable/sellable. The phase includes “owning” the buildings and maintenance of the municipal engineering and infrastructure constructs.

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For each of these three phases specific investment and revenue cash-flows can be identified, but the duration of each of the phases is difficult to estimate accurately, due to a number of factors. This causes that the size of the connected cash-flows also becomes difficult to estimate accurately. Indeed, large construction investments are notorious for cost overruns, see e.g. (Flyvbjerg et al., 2003; Flyvbjerg et al., 2002). Because of these estimation difficulties the methods used should be able to consider cash-flow estimation uncertainty. As there can be a lot of uncertainty involved in area development projects, and the projects can be naturally divided into optional stages, real option thinking and valuation are very likely to bring added value to the profitability analysis of area development investments. Research on real options with a direct connection to area development has mainly been concentrated in estimation of option value in land prices and in the use of real option valuation in the optimal timing of development projects; see e.g. (Ooi, 2006; Quigg, 1993; Titman, 1985; Williams, 1991; Yamaguchi, 2000). Issues having to do with real options in performing zoning are discussed, e.g., in (Capozza and Li, 1994; Capozza and Sick, 1994).

The paper by (Rocha, 2007) approaches the options in land use from the perspective of having the option to stage development of large area development projects (sequential development), in contrast to developing all-at-once. The paper shows that under high uncertainty, it may be beneficial to use the option to postpone after a first stage development, due to the possibility of changes in the market. The paper presents a case from the Rio de Janeiro that illustrates a situation where it is beneficial to postpone the second investment phase. The effect of staging the development on the risk of the project is that the downside is of the project is limited. Selecting optimal involvement strategies for municipalities in area development projects is discussed in (Collan, 2009). The paper presents strategic, or project-level real options available to the municipalities’ involvement are presented and discussed. The construct and the benefits of a decision support system for area development projects with a real option focus is discussed in (Collan, 2010). In area development projects, real options are available on two different levels: project-level real options are the “high-level” possibilities that the developer has to enter project stages or exit from the project and operational-level options are “lower-level” possibilities available within stages (building flexibility into the built premises etc.). In this paper we concentrate on the project-level real options and continue in the next section by discussing project-level real options available to developers in area development investments.

**Project level real options in area development investments**

The “high-level” real options that developers of area development projects have are for the purposes of this paper called project-level real options. A mapping of project-level real options is shown in figure 1, and listed and discussed in below the figure.

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**Diagram:**

**Fig. 1.** The three phases of area development projects with project level real options and with cash-flow information (-/+)

1. **Planning & Zoning phase**
   - Planning costs (-)
   - Zoning costs (-)
   - *INVESTMENT* Value of land into the process (-)
   - Selling (zoned) land (+)

2. **Construction phase**
   - Construction oversight(-)
   - *INVESTMENT* Building municipal engineering (-)
   - Value of zoned land (-)
   - Construction of buildings (-)
   - Sale of buildings (+)

3. **Post Construction Phase**
   - Land property tax (-)
   - Corporate income tax (-)
   - Maintenance of municipal engineering (-)
   - Rental income (+)
   - Building maintenance (-)
   - *INVESTMENT* Value of buildings (-)
   - Sale of buildings (+)

Arrows indicate project level real options connected to entering into a new project phase or to exiting the project.

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*Timing options to time the acquisition / leasing of the (un-zoned) land for the project – the developer has the power to decide when and if the land is procured; optimization of the timing may depend on the market situation.*
Option to continue to the construction phase is exercisable by investing the zoned land to the next phase. This is done if the expected profit reached from entering the construction phase is higher than the profit that can be realized from selling the land after zoning.

Option to sell the zoned land is used if the profit from the sale is higher than the expected value of continuing onto further stages of the project.

Option to continue to the post-construction phase, by not selling the constructed buildings and land. This means that the developer remains a landlord in (part or all of) the constructed buildings.

Option to sell the buildings and the underlying land. The developer should also think about the timing of the sale to optimize the sales income. The option should be exercised if the expected profit from continuing as property owner is smaller than the profit from selling.

Option to re-zone the land used in the project is available during the whole project, but is most usually used before construction or after the economic life of the project, in which cases the erected buildings are often demolished.

The above mapping project-level real options can be used in populating the decision space (the different decision alternatives) that is available to the decision-maker. By listing all (feasible) combinations for an area development project the investment decision problem becomes selecting the most profitable combination. This is not a trivial problem; because the above described problems in future cash-flow estimation inaccuracy (both timing and size), make the profitability analysis difficult. The combinations must also be ranked to yield the best alternative.

Using cash-flow scenarios is a widespread practice of modeling the uncertain future of projects. The scenario approach is also compatible with area development projects and is also used in planning the profitability of area development investments (Collan, 2010).

Pay-off method for real option valuation

The pay-off method (POM) is a real options-thinking based profitability analysis method that is suitable for cases, where input information is in the form of cash-flow scenarios (usually the case with area development projects) and when cash-flow estimation accuracy is not necessarily very high. The method is presented in (Collan et al., 2009b). The method is usable also in the valuation of compound real options (Collan et al., 2009a). These characteristics make the pay-off method a good fit with the profitability analysis of area development projects.

The pay-off method calculates a real option value for a project from the project pay-off distribution (NPV distribution) that can be constructed from the project cash-flow scenarios. The created NPV distribution is treated as a fuzzy number. To calculate the real option value by using the pay-off method the area weighted mean of the number (distribution) is calculated, such that distribution values below zero are counted as zero and multiplied by the ratio of area over the positive values over the whole area of the distribution.

![Fig. 2. An example triangular NPV distribution from where the real option value can be calculated. Values between a-α and 0 are counted as zero](image)
Definition 1. The pay-off method calculates the real option value from the project NPV distribution as follows:

\[
ROV = \frac{\int_0^\infty A(x) \, dx \times E(A_x)}{\int_0^\infty A(x) \, dx}
\]

Where \( A \) stands for the (fuzzy) NPV, \( E(A_x) \) denotes the (fuzzy) mean value of the positive side of the NPV and \( \int_0^\infty A(x) \, dx \) computes the area below the whole NPV distribution \( A \), and \( \int_0^\infty A(x) \, dx \) computes the area below the positive part of \( A \). For the purposes of area development project profitability analysis with the pay-off method, we suggest that triangular or trapezoidal pay-off distributions are used, because of their simplicity. Derivation of the fuzzy mean for the positive side of different types of distributions in this context is presented in (Collan et al., 2009b) the definition of fuzzy mean is presented in (Carlsson and Fullér, 2001).

Definition 2. The fuzzy mean for different cases of the triangular distribution case is:

\[
E(A_x) = \begin{cases} 
2a + \beta - \alpha & \text{if } a - \alpha > 0 \\
(\alpha - a)^3 + a + \beta - \alpha & \text{if } a - \alpha < 0 < a \\
(\alpha + \beta)^3 & \text{if } a < 0 \\
0 & \text{if } a + \beta
\end{cases}
\]

In the next section we will illustrate with a numerical case how project-level real option combinations of area development projects can be valued as compound options with the pay-off method, and describe how, by being able to value them the optimal (most profitable) combination may be selected.

Case: Valuing an area development project as a compound real option

As we have seen above, the selection of the most profitable way of conducting area development project depends on our ability to be able to correctly value different combinations of project-level real options, in other words, on our ability to value compound real options. In this case we show how we can value one possible combination of the different available project-level real options available for an area development project.

This case illustrates a situation where the developer exercises the real option to enter the area development project planning and zoning phase (\( n \) in figure 1.) and accepts the uncertain costs and duration of the planning and zoning phase. The duration of the phase is between three and five time units (tu) and the total costs range between 1500 and 2500 monetary units (mu). The best guess scenario is four tu duration and a cost of 1700 mu. (See figure 3.). Then the developer exercises the option to enter the construction phase (\( o \) in figure 1.) with the connected construction costs and durations ranging roughly between 2 tu and 4 tu (see figure 3). Continued by the exercise of the option to exit the project by selling the constructed buildings (\( r \) in figure 1.). The stages \( o \) and \( r \) are partially overlapping as some sales income starts already coming in while the construction is still underway. Developers often receive cash-inflow from sales already even before starting the construction. It is trivial to accommodate such cash-flow information with timing to the project cash flow scenarios.

The distribution of the cash-flows and durations for the three project-level option combination \( 1-2-3 \) is visible in figure 3. The upper bound of the distribution is determined by the combination of the optimistic scenarios for the combination and the lower bound by the pessimistic scenarios. The best guess scenario is obviously the combination of the three best guess (or most likely) scenarios. We can see that there is a lot of uncertainty in each of the three stages and that the uncertainty is different in each case; this means that using the same process for all three stages would result in results that possibly would grossly misrepresent the situation.
Fig. 3. The three cash-flow scenarios for the project level real options combination 1, 2, 3. The options and 2 and 3 are temporally overlapping (sales of the first completed buildings take place before the construction of the whole project is finished)

For the purposes of this illustration we use a discount rate of 5% (near risk-free) for the cost-cash flows as making the investment is in the hands of the developer. For the revenues (market governed) the discount rate used is 15%. It is possible to estimate and easy to implement the discount rates separately for each stage of the project. The estimation uncertainty connected to the size of the cash-flows is covered by using the three scenarios. Present value (PV) of the aggregate yearly cash-flows, for the combination under scrutiny, are visible in figure 3 and a graphical presentation of the cumulative PV of the combination is visible in figure 4.

Using the net present values (NPV) of the three compound real option scenarios we construct a simple NPV distribution, i.e. a pay-off distribution, for the compound option. We do this by first by observing that the best guess scenario is the most likely one (as it is a composition of the best guess / most likely scenarios) and assign it full membership in the set of possible outcomes. Secondly, we decide that the optimistic and pessimistic scenarios are the upper and lower bounds of the distribution - we make a simplifying assumption and consider values higher than the optimistic scenario and lower than the pessimistic scenario so unlikely that they need not be taken into consideration. We understand that this may, or may not be, very accurate, but we consider this simplification such that it does not jeopardize the reliability of the results that we feel, will remain at a “good enough” level.

Thirdly we assume that the shape of the pay-off distribution is triangular. These three steps allow us to construct the pay-off distribution that is visible in figure 5. From the distribution we can see that expectation is for the most part positive, i.e., the majority (82%) of the distribution is on the positive side of the zero. This is good information as it tells us something about the likelihood of the profitability of the investment.
Fig. 5. Resulting pay-off distribution for the analyzed real option combination with the resulting real option value indicated

From the created pay-off distribution we calculate the value of the real option combination \( \hat{V} \) by using the pay-off method and get 3057 mu as the expected value of the analyzed real option combination. Similar analysis can be performed for all feasible project-level real option combinations and the one with the highest profitability (optimal) selected. Three facsimile pay-off distributions are shown in figure 6., for the purposes of illustrating what a situation of comparing different compound real options could graphically look like. It is easy to see that the method used above allows for different forms of distributions that tell a lot about the uncertainty connected to the three different combinations, e.g. large downside is clearly visible and the division between expected positive and sub-zero outcomes is clear. We can then compare these distributions, e.g. with the cost of obtaining the rights to start a project by acquiring land.

Looking at the distributions rather than only looking at a single number value gives a more complete picture of the risks involved; the middle alternative in figure 6. has the highest “most likely” NPV, however it also has the lowest upside and the largest downside. If we only look at the single most-likely NPV value we may be mislead. We feel that showing the whole distribution is better decision support, because it more clearly shows the “whole picture”. The pay-off method can then be used in comparing the distributions in a way that takes the project upside and the downside correctly in consideration, and treats starting the project as a real option.

Summary and conclusions

We have presented area development projects as large long-term construction projects and discussed the complexity and difficulties in assessing the profitability of area development projects as investments. Difficulties arise from the inaccuracy in estimating the size and the timing of cash-flows connected to area development projects and the fact that area development projects are actually divisible into three phases that are optional to enter. It is also possible to exit the area development investments by selling assets at each phase. We understand the project-level choices (to continue or to exit) as project-level real options and have presented them shortly. We have further observed that any involvement in area development projects is actually a combination of the project-level real options. As these real options are sequential choices, the valuation of the combinations is compound real option valuation.

We observed that area development data is often in the form of project cash-flow scenarios; to value the area development projects a suitable method must be able to handle the available data, to be able to cope with estimation inaccuracy, and to be able to accommodate compound real options. For the valuation we have presented the pay-off method that is compatible with the above-mentioned requirements. With the pay-off method it is possible to value the different feasible area-level real option combinations and after the valuation the most profitable, i.e. optimal, combination can be selected. The valuation of an area development project-level real option combination, a possible “involvement strategy” was illustrated with a numerical case example. It was shown that the pay-off method is simple to use and the analysis can be performed rather easily with a commonly available spreadsheet program.
without any simulation or complicated mathematical formulations from the available project cash-flow scenario data. A pay-off distribution was generated from the cumulative three-stage compound option NPV scenario data and used as an input to the pay-off method. By comparing the distributions from the different compound options it is possible to easily and intuitively understand the value of these different involvement strategies. When many resulting distributions seem to be similar the compound real option value from the pay-off method can be used to distinguish the best, most profitable alternative. The decision-support received from the using the pay-off method and the connected distributions is very promising, as the results are easy to generate and simple to interpret. Also any changes to the cash-flow scenarios are directly reflected in the results and inversely the results can be traced back to singular cash-flows. The author was a project manager in an industry run development project that mapped involvement strategies for area development projects and built a decision support system for the task of evaluation and profitability analysis of these projects.

![Graph showing pay-off distributions from different project-level real option combinations.](image)

**Fig. 6.** Results from three different project-level real option combinations (facsimile)

**References**


