Optimization of a municipal wastewater plant expansion: A real options approach

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Abstract. The municipal water and wastewater sector is considered to be the most capital intensive industrial sector. Naturally, any methodology that has the potential to improve capital allocation decision making, has the potential to make a positive financial contribution to this sector. Most managers are aware of the power of calculating the Net Present Value (NPV) of an investment decision using Discounted Cash Flows (DCF). The problem with DCF based NPV analysis is that the inherent value of future project options is not modeled. In this study, we consider a small resort-based municipality faced with the question of how big to make their new wastewater treatment facility to meet the expanding demand of 10% growth in the number of new residential connections to the wastewater treatment infrastructure. Since a significant number of new dwellings are second “weekend” homes, the planners felt strongly that growth rates were tied to the strength of the market index. Here we set the model framework for considering optimal plant size based on correlation assumptions of municipal growth to the market index. The model takes on the form of an Asian option. The results show that the greater the (assumed) correlation, the smaller the required plant size. Penalty costs associated with not building a large enough plant are hedged in the market. This paper sets the basis for future analysis of staged plant expansion analysis.

Keywords: real options; capital budgeting; public infrastructure; Asian option; incomplete markets

Introduction

The municipal water and wastewater industrial sector is considered to be one of the most capital intensive industrial sectors and unfortunately the American Society of Civil Engineers (2005) rated the condition of the drinking water and wastewater infrastructure systems as poor, citing specifically a lack of investment in capital assets over a prolonged period of time. Clearly, methods based on sound financial principles that enhance capital asset allocation strategies can add significant value to municipal decision makers. It is recognized that projects face future uncertainties. The ability of project managers to react to these uncertainties at a future time adds intrinsic value to the project, and this value is not captured by standard discounted cash flow (DCF) / net present value (NPV) methods. To adequately account for the uncertainty and its impact on the project value, financial engineering methods applied in the financial markets can be utilized in “real” capital investment projects. Trigeorgis (1996) provides a thorough introduction and review of real option theory and how it can be utilized to enhance an entity’s strategy in resource allocation.

While capital asset and project valuation using real options has seen a significant research focus over the last 15 years (see, for example Jacoby and Laughton (1992), Ingersoll and Ross (1992), Emhiellen and Alalouze (2003), and van Putten and MacMillan (2004)), real options theory has seen limited application in the municipal infrastructure
sector. Of note, Schubert and Barenbaum (2007) discuss how public managers can employ real options technique to
to better value their capital budgeting opportunities and improve the efficacy of capital budgeting decisions.

Other studies, Ho and Liu (2002), Garvin and Cheah (2004), consider the application of real options to value
public infrastructure projects under private management arrangements. Arboleda and Abraham (2006) propose a
method using real option analysis to evaluate capital investments in public infrastructure projects managed by
private operators. The proposed methodology develops a valuation based on deterioration curves of infrastructure
and the associated value of flexibility to invest at optimal states within the model.

In this study, real option valuation is used to determine the optimal size for a wastewater plant expansion required
for a small municipality undergoing significant residential growth. Specifically, the community is located in a resort
area and has experienced increases in the growth rates of approximately 10% over the last 15 years. Since a significant
number of new dwellings are second “weekend” homes, the planners felt strongly that growth rates were tied to
the strength of the market index. Assumptions based on the strength of the correlation will be examined.

Model development

The optimization problem consists of balancing the costs associated with plant construction versus penalty costs
associated with a shortfall in plant capacity. Note that revenues from new connections and operations have no
impact on the optimization.

Plant construction cost model

Gillot et al (1999) provide a detailed methodology to optimize a wastewater plant based on cost. For simplicity,
the capital cost to build / expand a plant in this study is assumed to have a fixed and variable component only, as
follows:

\[ C_{\text{plant}}(K) = \alpha_{\text{plant}} + \phi_{\text{plant}} K \]  

(1)

where \( C_{\text{plant}} \) is the present value of the capital cost to build the plant (including any salvage value component), \( K \)
is the design size of the new plant based on number of (new) customers, \( \alpha_{\text{plant}} \) is the fixed cost and \( \phi_{\text{plant}} \) is the
variable cost.

Penalty cost model

In this study, we assume that the rate of wastewater connections follows a Geometric Brownian Motion (GBM)
process and that this process is correlated to the market index, which is also assumed to follow a GBM process.
The market index \( S_t \) is modeled as

\[ dS_t = \mu_S S_t dt + \sigma_S S_t dW_t^S \]  

(2)

Here, \( \mu_S \) and \( \sigma_S \) are constants representing the rate of growth and volatility of the index, respectively, and \( W_t \)
is a standard Brownian motion (or Wiener process) representing the fluctuations. Similarly, the wastewater connection
rate \( X_t \) (i.e. the number of connections per unit of time) is given by

\[ dX_t = \mu_X X_t dt + \sigma_X X_t dW_t^X \]  

(3)

\( \mu_X \) and \( \sigma_X \) assumed to be constant, and \( W_t^X \) is a second, correlated, Weiner process. The correlation between
the growth of the connection rate and the market index is captured by the correlation coefficient \( \rho \). It is often
convenient to decompose the correlated Brownian motions into two independent motions as follows:

\[ dW_t^X = \rho dW_t^S + \sqrt{1-\rho^2} dW_t^\perp \]  

(4)

where \( W_t^\perp \) is a Weiner process independent of \( W_t \). Under the risk-neutral measure, the risk-adjusted process
\[ d\tilde{W}_t = \frac{\mu_s - r}{\sigma_s} dt + dW_t \]  

is a standard Brownian motion. Here \( r \) is the risk-free rate. Assuming that the market price of risk associated with fluctuations uncorrelated to the market is zero (since they are not hedgeable), we do not risk-adjust the perpendicular component \( W_{\perp t} \). In this case, the connection rate becomes

\[ dX_t = \left( \mu_X - \frac{\rho \sigma_X}{\sigma_s} (\mu_s - r) \right) X_t dt + \sigma_X X_t \left( \rho d\tilde{W}_t + \sqrt{1 - \rho^2} d\tilde{W}_{\perp t} \right) \]

or

\[ dX_t = \tau X_t dt + \sigma_X X_t \left( \rho d\tilde{W}_t + \sqrt{1 - \rho^2} d\tilde{W}_{\perp t} \right) \]

Here \( \tau \equiv \mu_X - \frac{\rho \sigma_X}{\sigma_s} (\mu_s - r) \).

Defining \( N_t \) as the total number of connections to the plant, then

\[ N_t = N_0 + \int_0^t X_u du \]

\( N_0 \) denoting the number of existing customers whose wastewater will be treated by the new plant. If the number of connections exceeds plant capacity, the municipality will face extra costs associated with either wastewater removal and transport to an adjacent facility or face regulatory and environmental charges. The cost rate per each extra connection over capacity is \( PC_0 e^{r_{cpi} t} \), where \( r_{cpi} \) is the rate of inflation, \( PC_0 \) is cost rate per each extra connection over capacity at \( t = 0 \), and thus the penalty cost (rate) is

\[ PC_t = \left( (N_t - K) \cdot PC_0 e^{r_{cpi} t} \right)_+ \]

where \( K \) is the number of connections at capacity. Here we use the notation \( (\cdot)_+ \) to represent \( \max(\cdot, 0) \). Assuming the plant expansion will be completed by \( t_2 \), the present value of the penalty cost associated with the plant expansion is

\[ PC_{t_2, T}^{PV} = \int_{t_2}^T PC_0 e^{- (r - r_{cpi}) u} \left( (N_u - K)_+ \right) du \]

It is assumed that the size of the plant expansion does not impact the construction time. Clearly, penalty costs incurred before the plant expansion have no bearing on the optimization. The expected present value of the cost under the risk-neutral measure becomes

\[ \tilde{E}[PVC] = \tilde{E} \left[ PC_{t_2, T}^{PV} \right] + C_{\text{plant}} \]

now,

\[ \tilde{E} \left[ PC_{t_2, T}^{PV} \right] = \int_{t_2}^T PC_0 e^{- (r - r_{cpi}) u} \cdot \tilde{E} \left[ (N_u - K)_+ \right] du \]

\[ = \int_{t_2}^T PC_0 e^{- (r - r_{cpi}) u} \cdot \tilde{E} \left[ (N_0 + \int_0^u X_s ds - K)_+ \right] du. \]

The term \( \tilde{E} \left[ (N_0 + \int_0^u X_s ds - K)_+ \right] \) takes on the form of an Asian option’s payoff. Defining
\[ v(t, X_t, N_t) \equiv \mathbb{E}\left[ \left( N_t + \int_{t}^{T} X_s ds - K \right)_+ | Y_t \right], \]  

(13)

the solution to \( v(t,x,y) \), where \( X_t \) and \( N_t \) are replaced by the dummy variables \( x \) and \( y \), is given by the following partial differential equation (PDE)

\[
\frac{\partial v}{\partial t} + \bar{r} x \frac{\partial v}{\partial x} + x \frac{\partial v}{\partial y} + \frac{1}{2} \sigma^2 x^2 \frac{\partial^2 v}{\partial x^2} = 0
\]

(14)

The boundary conditions are determined as follows. For \( x = 0 \), \( v \) will only depend on the total connections, thus

\[ v(t,0,y) = (y - K)_+, \quad 0 \leq t \leq T \]

(15)

and similarly at the terminal time, \( t = T \), \( v \) is determined by

\[ v(T,x,y) = (y - K)_+ \]

(16)

As per Shreve (2004), there is no mathematical reason to restrict \( y \) to positive values and by allowing the computational domain to include values of \( y < 0 \), the boundary condition for \( y \to -\infty \) gives

\[ \lim_{y \to -\infty} v(t,x,y) = 0, \quad 0 \leq t \leq T \]

(17)

note that

\[ y_{t_2} = y_{t_1} + \int_{t_1}^{t_2} x_s ds, \]

and since \( x > 0 \) then \( y_{t_2} \geq y_{t_1} \) for \( t_2 > t_1 \). Define \( y_{t_1} \equiv y_{\text{max}} \) with \( y_{\text{max}} > K \), then \( y_{t_2} \) will always be in the money and

\[
v(t,x,y_{\text{max}}) = \mathbb{E}\left[ y_{\text{u}} - K | X_t = x, y_t = y_{\text{max}} \right]
\]

(18)

\[
= \mathbb{E}\left[ y_{\text{u}} | X_t = x, y_t = y_{\text{max}} \right] - K
\]

\[
= y_{\text{max}} - K + \int_{t}^{u} \mathbb{E}\left[ X_s | X_t = x \right] ds.
\]

From equation (6),

\[
\mathbb{E}\left[ X_u | X_t = x \right] = xe^{\bar{r}(u-t)}
\]

(19)

and so equation (18) gives

\[
v(t,x,y_{\text{max}}) = y_{\text{max}} - K + \frac{x}{\bar{r}} \left( e^{\bar{r}(u-t)} - 1 \right).
\]

(20)

Finally, for large \( x \) values, set \( x = x_{\text{max}} \), a constant. Thus

\[
v(t,x_{\text{max}},y) = \mathbb{E}\left[ \left( y + \int_{t}^{u} X_s ds - K \right)_+ | X_t = x_{\text{max}}, y_t = y \right]
\]

\[
\sim \left( y + (u-t)x_{\text{max}} - K \right)_+.
\]

(21)

The finite difference method was used to solve equation (14).
Hedging strategy

Clearly for values of $\rho$ different from zero, the optimal new plant size will be reduced and “extra” penalty costs associated with building a smaller plant can be hedged. In this section, the hedging strategy is developed.

The expected present value of the penalty cost at time $t$, including the (known) incurred penalty to $t$, as

$$
G_t = \mathbb{E}\left[PC_{t,t}^{PV} \mathbb{P} \mathbb{R}\right] + \int_0^T PC_0 e^{-(r-\rho)t} \cdot (N_t - K) \cdot du
$$

then

$$
dG_t = PC_0 \left[ \left( e^{-(r-\rho)t} \cdot (N_t - K) - v \right) + \int_t^T e^{-(r-\rho)u} \frac{\partial v}{\partial t} du \right]
$$

$$
+ TX_1 \left[ \int_t^T e^{-(r-\rho)u} \frac{\partial v}{\partial X} du + X_1 \left[ \int_t^T e^{-(r-\rho)u} \frac{\partial^2 v}{\partial X^2} du + \frac{\sigma^2}{2} X_1^2 \int_t^T e^{-(r-\rho)u} \frac{\partial^2 v}{\partial X^2} du \right] \right] dt
$$

$$
+ \rho \sigma_X X_1 \left[ \int_t^T e^{-(r-\rho)u} \frac{\partial v}{\partial X} du dW + \sqrt{1-\rho^2} \sigma_X X_1 \int_t^T e^{-(r-\rho)u} \frac{\partial v}{\partial X} du dW \right].
$$

The hedge portfolio will consist of the market index and a money market account, whose value is denoted as $M_t$, such that the value of the portfolio is

$$
\Pi_t = a_t S_t + M_t
$$

and (in order for the hedge to be self-financing)

$$
d\Pi_t = a_t r S_t dt + r M_t dt + a_t \sigma_S S_t dW.
$$

Here $a_t$ represents the number of shares invested in the market index. Since we aim to hedge away all tradable risks, the appropriate hedge requires equating the $dW$ coefficients resulting in

$$
a_t = PC_0 \frac{\rho \sigma_X X_1}{\sigma_S S_t} \int_t^T e^{-(r-\rho)u} \frac{\partial v}{\partial X} du.
$$

Assuming continuous trading, the volatility of this strategy will be $PC_0 \sqrt{1-\rho^2} \sigma_X X_1 \int_t^T e^{-(r-\rho)u} \frac{\partial v}{\partial X} du$ – the last term in equation (23), which cannot be hedged away.

Standard engineering approach

The standard engineering approach is to look at historical growth rates and use this rate to determine the future plant size to meet the demand at some future time, say, $\tau$. The calculation for this approach is straightforward,

$$
K_{eng} = N_0 + \int_0^\tau X_t e^{rt} dt
$$

It will be shown that building a “conservatively” sized plant, i.e. by applying equation (27) with $\tau = T$, will lead to sub-optimal sizing for the case where no hedge is applied, i.e. $\rho = 0$. 
Results

As mentioned above, the municipality in this study is located in a resort area and has seen its growth rate increasing at approximately 10% (i.e. $\mu_X = 0.1$) over a 15 year time span. The volatility in the growth rate was approximately 0.16 during this period. For the last year, the connection rate, $N_{in}$, was 81. Current plant capacity is estimated to have the ability to accommodate another approximately 200 new connections. The penalty cost rate was estimated to be $5000 per connection not served per year. It is assumed that if not enough capacity is installed, extra sewage entering the treatment facility will need to be hauled to other locations, thus bearing significant yearly costs associated with under design. Construction for the wastewater capacity expansion project is estimated to be 3 years. For simplicity of analysis, the plant is assumed to have a 20 year lifespan. The analysis presented here assumes a 23-year time horizon (3 years for construction plus 20 years for the life of the plant). Penalty costs after 23 years are ignored – it is assumed that future plant construction / expansion will negate future penalty costs. Feasibility studies conducted by consulting engineers provided estimates of the capital cost for plant expansion (and salvage) and for the type and size of plant considered here, $\alpha_{plant}$ was estimated to be $3,500,000 and $\phi_{plant}$ to be $860 per connection served (c.f. equation (1)).

In this study, the risk-free rate, $r$, was taken as 5% and the inflation rate, $r_{cpi}$, as 3%. Market index parameters were determined to be $\mu_S = 0.08$ and $\sigma_S = 0.1$. As previously noted, due to the sparseness of the data, the correlation between the growth rates of the market index and connection rates, $\rho$, was not determinable with statistical significance, but values in the 0.1 to 0.5 range were observed using a number of standard methods. Figure 1 plots the present value of the total risk-neutral expected cost versus new plant size for varying $\rho$. The optimal plant size, i.e. $\min \left\{ C_{plant} + PC_{t,Tmin} \right\}$, is also indicated in the figure. As expected, the higher the correlation of the connection rate growth to market growth, the smaller the optimal plant size.

To test the effectiveness of the delta-hedging strategy, simulations were run with $\rho = 1.0$ and $\rho = 0.5$. For $\rho = 1.0$, the optimal plant size was determined to be approximately 3100, whereas for $\rho = 0.5$, it was found to be 5800 and for $\rho = 0.0, 11,600$. Applying equation (27), the engineering estimate for the plant size would be $K = K_{eng} = 7069$. For the results presented here, 1000 sample paths were simulated, and the delta hedge was re-adjusted once per week. A sample of 10 paths of X and N are presented in Figure 2 for $\rho = 0.5$ and $K = 5800$. The corresponding delta hedge, value of the money market and the penalty cost paths are given in Figure 3. It should be noted that the “money market”, when negative, represents the cost associated with paying for the hedge and the penalty. As expected, the money market value somewhat mirrors the delta hedge until penalty costs begin to be incurred.
Histograms of the negative accumulated present value of the penalty costs (these amounts are equal to $-e^{\rho T}PV_{C_t}$) and the present values of the final money market values for $K = 3100$ (for $\rho = 1.0$) and for $K = 5800$ (for $\rho = 0.5$) are plotted in Figure 4. As can be seen, both the penalty costs for the smaller sized plant, $K = 3100$, are substantially higher than those for the larger plant, $K = 5800$, resulting in a higher negative final money market value. However, the higher penalty cost is offset by the lower cost of constructing a smaller plant. Figure 5 presents a histogram of the accumulated negative present value of the penalty costs for the case of no hedge and the plant built to the engineering size estimate of $K_{eng} = 7069$. 
The total present value of the “savings” for applying the hedging strategy compared to building the plant to the engineering estimate of $K_{\text{eng}} = 7069$ is given by

$$\text{Savings} = C_{\text{plant}}(7069) - C_{\text{plant}}(K) + M_t e^{-rT} + PC_{t_{2},T}^\text{PV}(7069) - PC_{t_{2},T}^\text{PV}(K)$$

and the associated histograms are given in Figure 6 for $\rho = 1.0$ and $\rho = 0.5$. For both cases, the municipality has effectively hedged its penalty cost risk in the market, such that if the savings associated with building a smaller plant are considered, the municipality’s savings are significant, approximately $14.5$ million for $\rho = 1.0$ and $10.4$ million for $\rho = 0.5$. The 95% confidence intervals for the two cases are $3.2$ million to $99.2$ million for $\rho = 1.0$ and $1.0$ million to $79.5$ million for $\rho = 0.5$. 
Fig. 6. Histogram of the present value of Total Savings

For the case of not hedging, or, equivalently, for the case of setting $\rho = 0$, the optimal plant size is 11,600 — significantly higher than the extrapolated engineering estimated value of 7069. A histogram of the savings associated with building the plant to full scale based on the Asian option approach with $\rho = 0$ is given in Figure 7. The average savings relative to the engineering estimate is $4.5$ million, with the 95% confidence interval stretching from a loss of $3.8$ million, which is the difference in construction cost between building a plant with a capacity of 7069 connections and 11,600 connections, to a savings of $68.8$ million. Clearly, if the municipality were not to hedge the penalty, the plant should be built to a size of 11,600 connections.

Fig. 7. Histogram of the present value of Total Savings with no hedge

Discussion and conclusions

It is not likely that many municipalities will opt to build smaller wastewater treatment plants and try to hedge away their potential penalty costs in the stock market. However, the approach developed here highlights how private investment would value the cost of a plant expansion. Capital investment theory is based on the assumption that investors will only invest in a project if its expected payoffs, for a given risk level, are better than what could be
achieved in the market. In situations where private investors are burdened with the financial costs of a design-build-operate system, it may be important to ensure penalty costs associated with not meeting treatment are adequately adjusted at the outset.

On the one hand, the methodology developed here highlights the potential cost of extrapolating growth without properly accounting for the penalty cost dynamics in the case where no hedging is considered. A savings of approximately $4.5 million is expected if the plant were to be built based on the Asian option approach versus the standard engineering extrapolation method. On the other hand, this framework could be used by municipal planners to improve their capital expenditures, and provides an alternative to the current practice of extrapolating the growth and building for extreme events forecasted far into the future. Shorter term hedging strategies may be applicable with local real estate market indices to ward off massive capital requirements in the shorter term. Real hedges can be introduced through development / real estate taxes, user fees, or even strategic agreements with neighbouring municipalities with respect to load and fees sharing. Finally, the model developed here can now be easily expanded to consider staged investment of the facilities.

References