A real options model for closing or not closing a production plant

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Abstract. The closing/not closing decisions for a production plant normally worries senior management as their decision will be scrutinized and criticized by many groups of influential actors. The shareholders will react negatively if they find out that share value will decrease (closing a profitable plant, closing a plant which may turn profitable, or not closing a plant which is not profitable, or which may turn unprofitable) and the trade unions, local and regional politicians, the press etc will always react negatively to a decision to close a plant almost regardless of the reasons. Real options models will support decision making in which senior managers search for the best way to act and the best time to act. The key elements of the closing/not closing decision may be known only partially and/or only in imprecise terms, which is why we show that meaningful support, can be given with a fuzzy real options model. The decision process and its consequences are worked out in terms of a real case in the forest products industry in which we have been able to show the models with realistic data.

Keywords: strategic decisions; fuzzy real options modelling; binomial model

Introduction

The forest industry, and especially the paper making companies, has experienced a radical change of market since the change of the millennium. Especially in Europe the stagnating growth in paper sales and the resulting overcapacity have led to decreasing paper prices, which have been hard to raise even to compensate for increasing costs. Other drivers to contribute to the misery of European paper producers have been steadily growing energy costs, growing costs of raw material and the Euro/USD exchange rate, which is unfavorable for an industry, which with some generalization is invoicing its customers in USD and pays its costs in Euro. The result has been a number of restructuring measures, such as closedowns of individual paper machines and production units. Additionally, a number of macroeconomic and other trends have changed the competitive and productive environment of papermaking. The current industrial logic of reacting to the cyclical demand and price dynamics with operational flexibility is losing edge because of shrinking profit margins. Simultaneously, new growth potential is found in the emerging markets of Asia, especially in China, which more and more attracts the capital invested in paper production. This imbalance between the current production capacity in Europe and the better expected return on capital invested in the emerging markets has set the paper makers in front of new challenges and uncertainties that are different from the ones found in the traditional paper company management paradigm. In a global business environment both challenges and uncertainties vary from market to market, and it is important to find new ways of managing them in the current dynamic business environment.

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World population and the paper consumption in different parts of the world in 2008 is shown in Figure 1; the problem for the paper producing companies is that the production capacity is concentrated in countries where the paper consumption/capita has reached saturation levels, not where the potential growth of consumption is highest. Another, related problem is that the paper production – mostly for historical reasons - is concentrated to countries where the costs for producing paper are high and growing. This combined with the less demand of saturated markets translates to growing problems with profitability.

![Population and paper consumption/capita worldwide](source: Finnish Forest Industries Federation, FFIF, 2009)

**Production, exports, jobs and investments**

The Finnish Forest Industries Federation continuously updates material on the forest industry on its website (http://www.forestindustries.fi, accessed 25 March 2010); from this material we can find the key observations summarised in Table 1.

<table>
<thead>
<tr>
<th>Table 1. Value of the production, exports and imports of the forest industry in 2006</th>
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<tr>
<td><strong>Production</strong></td>
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<td>Pulp</td>
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<td>Paper and paperboard</td>
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<td>Sawn wood</td>
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In 2007, the gross value of the forest industry’s production in Finland was about €23.7 billion, a third of which was accounted for by the wood products industry and two thirds by the pulp and paper industries. In 2007, the forest sector employed a total of 60,000, some 30,000 of whom worked for the paper industry and about 30,000 for the wood products industry. Finnish Forest Industries Federation member companies employed 50,000 people. In addition to their domestic functions, Finnish forest industry companies employed about 70,000 people abroad. The total investments of Finnish forest industry came to some €1.0 billion in 2008, €880 M of which were invested abroad.
World production of paper and paperboard totals some 370 million tonnes. Growth is the most rapid in Asia, thanks mainly to the quick expansion of industry in China. Asia already accounts for well over a third of total world paper and paperboard production. In North America, by contrast, production is contracting; a number of Canadian mills have had to shut down because of weak competitiveness.

Per capita consumption of paper and paperboard varies significantly from country to country and regionally. On average, one person uses about 58 kilos of paper a year; the extremes are 260 kilos for each US resident and some eight kilos for each African. Only around 40 kilos of paper per person is consumed in the populous area of Asia. This means that Asian consumption will continue to grow strongly in the coming years if developments there follow the precedent of the West. In Finland, per capita consumption of paper and paperboard is 205 kilos.

Rapid growth in Asian paper production in recent years has increased the region’s self-sufficiency, narrowing the export opportunities available to both Europeans and Americans. Additionally, Asian paper has started to enter Western markets – from China in particular. Global competition has intensified noticeably as the new entrants’ cost level is significantly lower than in competing Western countries. The European industry has been dismantling overcapacity by shutting down unprofitable mills. In total, over five percent of the production volume in Europe has been closed down in the last couple of years. Globally speaking, the products of the forest industry are primarily consumed in their production country, so it can be considered a domestic-market industry.

Globally speaking, the profitability of forest industry companies has been weak in recent years. Overcapacity has led to falling prices and this, coupled with rising production costs, is gnawing at the sector’s profitability.

The Finnish forest industry has earlier enjoyed a productivity lead over its competitors. The lead is primarily based on a high rate of investment and the application of the most advanced technologies. Investments and growth are now curtailed by the long distance separating Finland from the large, growing markets as well as the availability and price of raw materials. Additionally, the competitiveness of Finnish companies has suffered because costs here have risen at a faster rate than in competing countries.

Finnish energy policy has a major impact on the competitiveness of the forest industry. The availability and price of energy, emissions trading and whether wood raw material is produced for manufacturing or energy use will affect the future success of the forest industry. If sufficient energy is available, basic industry can invest in Finland. In decisions on how to use existing resources the challenges of changing markets become a reality when senior management has to decide how to allocate capital to production, logistics and marketing networks, and has to worry about the return on capital employed. The networks are interdependent as the demand for and the prices of fine paper products are defined by the efficiency of the customer production processes and how well suited they are to market demand; the production should be cost effective and adaptive to cyclic (and sometimes random) changes in market demand; the logistics and marketing networks should be able to react in a timely fashion to market fluctuations and to offer some buffers for the production processes. The conclusions made from the development of all these interdependent factors and processes are often simplified in a black-and-white fashion: continue operating production capacity or close it down.

Closing or not closing a production plant is often regarded as an isolated decision, without working out the possibilities and requirements of the interdependent networks.

Profitability analysis has usually had an important role as the threshold phase and the key process when a decision should be made on closing or not closing a production plant. Economic feasibility is of course an important consideration but – as pointed out – more issues are at stake. There is also the question of what kind of profitability analysis should be used and what results we can get by using different methods. Senior management worries – and should worry – about making the best possible decisions on the close/not close situations as their decisions will be scrutinized and questioned regardless of what that decision is going to be. The shareholders will react negatively if they find out that share value will decrease (closing a profitable plant, closing a plant which may turn profitable, or not closing a plant which is not profitable, or which may turn unprofitable) and the trade unions, local and regional politicians, the press etc will always react negatively to a decision to close a plant almost regardless of the reasons.

The idea of optimality of decisions comes from normative decision theory. The decisions made at various levels of uncertainty can be modelled so that the ranking of various alternatives can be readily achieved, either with certainty or with well-understood and non-conflicting measures of uncertainty. However, the real life complexity, both in a static and dynamic sense, makes the optimal decisions hard to find many times. What is often helpful is to relax the decision model from the optimality criteria and to use sufficiency criteria instead. Modern profitability plans are usually built with methods that originate in neoclassical finance theory. These models are by nature normative
and may support decisions that in the long run may be proved to be optimal but may not be too helpful for real life decisions in a real industry setting as conditions tend to be not so well structured as shown in theory and – above all – they are not repetitive (a production plant is closed and this cannot be repeated under new conditions to get experimental data).

In practice and in general terms, for profitability planning a good enough solution is many times both efficient, in the sense of smooth management processes, and effective, in the sense of finding the best way to act, as compared to theoretically optimal outcomes. Moreover, the availability of precise data for a theoretically adequate profitability analysis is often limited and subject to individual preferences and expert opinions. Especially, when cash flow estimates are worked out with one number and a risk-adjusted discount factor, various uncertain and dynamic features may be lost. The case for good enough solutions is made in fuzzy set theory (Carlsson and Fullér, 2003; Carlsson et al., 2005): at some point there will be a trade-off between precision and relevance, in the sense that increased precision can be gained only through loss of relevance and increased relevance only through the loss of precision.

In a practical sense, many theoretically optimal profitability models are restricted to a set of assumptions that hinder their practical application in many real world situations. Let us consider the traditional Net Present Value (NPV) model - the assumption is that both the microeconomic productivity measures (cash flows) and the macroeconomic financial factors (discount factors) can be readily estimated several years ahead, and that the outcome of the project, such as a paper machine with an expected economic lifetime of 20-25 years, is tradable in the market of production assets without friction. In other words, the model has features that are unrealistic in a real world situation. The idea of the NPV is based on a fixed coupon bond that generates a fixed stream of cash flows during a pre-defined lifetime. For real investments with long economic lifetimes that are subject to intense competition, technological deterioration and radically changing context factors (currency exchange rates, energy costs, raw material costs, etc.) the NPV gives rather a simplistic picture of real life profitability. In reality, the decision makers have to face a complex set of interdependencies that change dynamically and are uncertain, and uncertain in their uncertainty.

Having now set the scene, the problem we will address is the decision to close – or not to close – a production plant in the forest products industry sector. The plant we will use as a context is producing fine paper products, it is rather aged, the paper machines were built a while ago, the raw material is not available close by, energy costs are reasonable but are increasing in the near future, key markets are close by and other markets (with better sales prices) will require improvements in the logistics network. The intuitive conclusion is, of course, that we have a sunset case and senior management should make a simple decision and close the plant. On the other hand we have the trade unions, which are strong, and we have pension funds commitments until 2013, which are very strict, and we have long-term energy contracts, which are expensive to get out of. Finally, by closing the plant we will invite competitors to fight us in markets we have served for more than 50 years and which we cannot serve from other plants at any reasonable cost. We will show in section 2 that real options models will support decision making in which senior managers search for the best way to act and the best time to act. The key elements of the closing/not closing decision may be known only partially and/or only in imprecise terms, which are why we show that meaningful support, can be given with a fuzzy real options model (cf. Carlsson and Fullér, 2001a; Collan et al., 2003; Alcaraz Garcia, 2006). The real world case is introduced in section 3 where we show the dilemma(s) senior management had to deal with and the (low) level of precision in the data to be used for making a decision. In section 4 we will show the models we worked with and the results we were able to get with fuzzy real options models. Finally, section 5 summarizes some discussion points and offers some conclusions.

Fuzzy real option valuation

In traditional investment planning investment decisions are usually taken to be now-or-never, which the firm can either enter into right now or abandon forever. The decision on to close/not close a production plant has been understood to be a similar now-or-never decision for two reasons: (i) to close a plant is a hard decision and senior management can make it only when the facts are irrefutable; (ii) there is no future evaluation of what-if scenarios after the plant is closed. Nevertheless, as we will show, it could make sense to work a bit with what-if scenarios as closing the plant will cut off all future options for the plant.
Making hard decisions is the macho thing and new CEOs often want to earn their first spurs by closing production plants; they are quite often rewarded by the shareholders who think that decisive action is the mark of a manager who is going to build good shareholder value. Nevertheless, the exact outcomes in terms of shareholder value of the decision are uncertain as a consequence of changing markets, changes in raw material and energy costs, changes in the technology roadmap, changes in the economic climate, etc. In order to support and motivate tough decisions a number of valuation methods have been developed and the standard approach is to use NPV or other discounted cash flow (DCF) methods with a number of assumptions about the future development of key cost and profitability drivers.

Only very few decisions are of the type now-or-never – often it is possible to postpone, modify or split up a complex decision in strategic components, which can generate important learning effects and therefore essentially reduce uncertainty. If we close a plant we lose all alternative development paths which could be possible under changing conditions; on the other hand, senior management may have a difficult time with shareholders if they continue operating a production plant in conditions which cut into its profitability as their actions are evaluated and judged every quarter.

In these cases we can utilize the idea of real options. The new rule, derived from option pricing theory, is that we should only close the plant now if the net present value of this action is high enough to compensate for giving up the value of the option to wait. Because the value of the option to wait vanishes right after we irreversibly decide to close the plant, this loss in value is actually the opportunity cost of our decision. The value of a real option is computed by (cf. Carlsson et al, 2005; Collan, 2004).

\[
ROV = S_0 e^{-\delta} N(d_1) - X e^{-rT} N(d_2),
\]

where,

\[
d_1 = \frac{\ln(S_0 / X) + (r - \delta + \sigma^2 / 2)T}{\sigma \sqrt{T}},
\]

\[
d_2 = d_1 - \sigma \sqrt{T} = \frac{\ln(S_0 / X) + (r - \delta - \sigma^2 / 2)T}{\sigma \sqrt{T}}.
\]

Here, \(S_0\) denotes the present value of the expected cash flows, \(X\) stands for the nominal value of the fixed costs, \(r\) is the annualized continuously compounded rate on a safe asset, \(\delta\) is the value lost over the duration of the option, \(\sigma\) denotes the uncertainty of the expected cash flows, and \(T\) is the time to maturity of the option (in years). Furthermore, the function \(N(d)\) gives the probability that a random draw from a standard normal distribution will be less than \(d\), i.e.

\[
N(d) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{d} e^{-x^2 / 2} dx.
\]

Facing a deferrable decision, the main question that a company needs to answer is the following: How long should we postpone the decision – up to \(T\) time periods – before (if at all) making it? From the idea of real option valuation we can develop the following natural decision rule for an optimal decision strategy (Benaroch and Kaufman, 2000).

**Theorem 1.** Let us assume that we have a deferrable decision opportunity \(P\) of length \(L\) years with expected cash flows \(\{c_{f_0}, c_{f_1}, \ldots, c_{f_L}\}\), where \(c_{fi}\) is the cash inflows that the plant is expected to generate at year \(i\), \(i = 0, 1, \ldots, L\). We note that \(c_{fi}\) is nothing else but the anticipated net income (revenue – costs) of decision \(P\) at year \(i\), which we can readily obtain from the \(i\)-th row of the NPV table of the plant operations. In these circumstances, if the maximum deferral time is \(T\), we shall make the decision, i.e. exercise the option at time \(t'\), \(0 < t' < T\), for which the value of the option, \(ROV_t'\), is positive and attends its maximum value; namely,

\[
ROV_t' = \max_{i=0,1,\ldots,T} \max_{i=0,1,\ldots,T} V_i e^{-\delta} N(d_1) - X e^{-rT} N(d_2) > 0,
\]

where

\[
V_i = PV(c_{f_0}, c_{f_1}, \ldots, c_{f_i}; \beta_p) - PV(c_{f_0}, c_{f_1}, \ldots, c_{f_{i-1}}; \beta_p) = PV(c_{f_1}, \ldots, c_{f_i}; \beta_p)
\]
is the present value of the aggregate cash flows generated by the decision, which we postpone \( t \) years before undertaking. Hence,

\[
V_t = \sum_{i=0}^{L-1} \frac{cf_i}{(1 + \beta_p)^i} - \sum_{i=0}^{L-1} \frac{cf_i}{(1 + \beta_p)^i} = \sum_{i=0}^{L} \frac{cf_i}{(1 + \beta_p)^i},
\]

where \( \beta_p \) stands for the risk-adjusted discount rate of the decision.

**Note 1.** Obviously, in the case we obtain or learn some new information about the decision alternatives, their associated NPV table and the cash flows \( cf_i \) which may change. Thus, we have to reapply this decision rule every time when new information arrives during the deferral period to see how the optimal decision strategy might change in light of the new information.

**Note 2.** If we make the decision now without waiting, then

\[
V_0 = PV(cf_0, cf_1, \ldots, cf_L; \beta_p) = \sum_{i=0}^{L} \frac{cf_i}{(1 + \beta_p)^i},
\]

and since we can formally write

\[
d_1, d_2 \rightarrow_{r \rightarrow x} n, \quad N(d_1), N(d_2) \rightarrow_{r \rightarrow 1},
\]

we obtain

\[
ROV_0 = V_0 - X = \sum_{i=0}^{L} \frac{cf_i}{(1 + \beta_p)^i} - X.
\]

That is, this decision rule also incorporates the net present valuation of the assumed cash flows. For more details, we refer the interested reader to Carlsson and Fullér (2001a).

\( \square \)

**Possibility distributions**

A fuzzy number \( A \) is a fuzzy set of the real line with a normal, (fuzzy) convex and continuous membership function of bounded support. The family of all fuzzy numbers will be denoted by \( F \). For any fuzzy number \( A \in F \) its \( \gamma \)-level (“gamma-level”) set is defined by \( [A]_{\gamma} = \{ x \in R \mid A(x) \geq \gamma \} \) if \( 0 < \gamma \leq 1 \), and \( [A]_{\gamma} = \text{cl supp}(A) = \text{cl} \{ x \in R \mid A(x) > \gamma \} \) (the closure of the support of \( A \)) if \( \gamma = 0 \). If \( A \in F \) is a fuzzy number then we shall use the notation \( [A]_{\gamma} = [a_\gamma, b_\gamma] \) for the \( \gamma \)-level sets of \( A \), \( \gamma \in [0,1] \). Fuzzy numbers can also be considered as possibility distributions (Dubois and Prade, 1988). If \( A \in F \) is a fuzzy number, and \( x \in R \) a real number then \( A(x) \) can be interpreted as the degree of possibility of the statement “\( x \) is in \( A \)”.

**Definition 1.** A fuzzy set \( A \in F \) is called a trapezoidal fuzzy number with core \([a, b]\), left width \( \alpha \) and right width \( \beta \), if its membership function has the following form

\[
A(t) = \begin{cases} 
1 - \frac{a - t}{\alpha} & \text{if } a - \alpha \leq t < a, \\
1 & \text{if } a \leq t \leq b, \\
1 - \frac{t - b}{\beta} & \text{if } b < t \leq b + \beta, \\
0 & \text{if } t \notin [a - \alpha, b + \beta],
\end{cases}
\]

and we use the notation \( A = (a, b, \alpha, \beta) \). It can easily be shown that \( [A]_{\gamma} = [a - (1 - \gamma)\alpha, b + (1 - \gamma)\beta] \), \( \forall \gamma \in [0,1] \), and that the support of \( A \) is \( \text{sup}(A) = (a - \alpha, b + \beta) \). A trapezoidal fuzzy number with core \([a, b]\) can be seen as a context-dependent description of the property “the value of a real variable is approximately in \([a, b]\)”, where \( \alpha \) and \( \beta \) define the context. If \( A(t) = 1 \) then \( t \) belongs to \( A \) with degree of membership one (i.e. now \( a \leq t \leq b \)); if \( A(t) = 0 \) then \( t \) belongs to \( A \) with degree of membership zero (i.e. now \( t \leq a - \alpha \).
or \( t \geq b + \beta \), thus \( t \) is considered to be “too far” from \([a, b]\); and if \( 0 < A(t) < 1 \) then \( t \) belongs to \( A \) with an intermediate degree of membership (i.e. now \( t \) is “close enough” to \([a, b]\)). Generally, in a possibilistic environment \( A(t) \), \( t \in F \) can be interpreted as the degree of possibility of the statement “\( t \) is approximately in \([a, b]\)”. Let \([A] = [a_1(\gamma), a_2(\gamma)]\) and \([B] = [b_1(\gamma), b_2(\gamma)]\) be two fuzzy numbers, and let \( \lambda \in R \) be a real number. Using the sup-min extension principle (Carlsson and Fullér, 2003) we can verify the following rules for addition and scalar multiplication of fuzzy numbers

\[
[A + B] = [a_1(\gamma) + b_1(\gamma), a_2(\gamma) + b_2(\gamma)] \quad \forall \gamma \in [0, 1].
\]

In the following we shall define the possibilistic mean value and variance of fuzzy numbers (Carlsson and Fullér, 2001a). Let \( A \in F \) be a fuzzy number with \([A] = [a_1(\gamma), a_2(\gamma)], \gamma \geq \gamma_0 \in [0, 1] \). Then, the (crisp) possibilistic mean (or expected) value of \( A \) is defined as

\[
E(A) = \int_0^1 (a_1(\gamma) + a_2(\gamma)) \gamma \, d\gamma = \frac{1}{2} \int_0^1 \gamma (a_1(\gamma) + a_2(\gamma)) \, d\gamma.
\]

That is, \( E(A) \) is the level-weighted average of the arithmetic means of all \( \gamma \)-level sets, where the weight of the arithmetic mean of \( a_1(\gamma) \) and \( a_2(\gamma) \) is just \( \gamma \). It can easily be proven that \( E: F \to R \) is a linear function (with respect to addition and scalar multiplication defined by the sup-min extension principle). Furthermore, the (possibilistic) variance of \( A \in F \) is defined by

\[
\sigma^2(A) = \int_0^1 \left( \frac{1}{2} \left[ a_1(\gamma) + a_2(\gamma) \right]^2 - a_1(\gamma)^2 \right) \, d\gamma = \frac{1}{2} \int_0^1 \left[ a_1(\gamma) + a_2(\gamma) \right]^2 \, d\gamma.
\]

That is, the possibilistic variance of \( A \) is computed as the expected value of the squared deviations between the arithmetic mean and the endpoints of the level sets of \( A \).

It is easy to verify that if \( A = (a, b, \alpha, \beta) \) is a trapezoidal fuzzy number then

\[
E(A) = \int_0^1 (a - (1 - \gamma)\alpha + b + (1 - \gamma)\beta) \gamma \, d\gamma = \frac{a + b}{2} + \frac{\beta - \alpha}{6},
\]

and

\[
\sigma^2(A) = \int_0^1 \left[ (b + (1 - \gamma)\beta - a + (1 - \gamma)\alpha)^2 \right] \gamma \, d\gamma = \frac{(b - a)^2}{4} + \frac{(b - a)(\alpha + \beta)}{6} + \frac{(\alpha + \beta)^2}{24}.
\]

The reasons for using fuzzy numbers are, of course, not self-evident. The imprecision we encounter when judging or estimating future cash flows is not in many cases stochastic in nature, and the use of probability theory gives us a misleading level of precision and a notion that consequences somehow are repetitive. This is not the case; the uncertainty is genuine as we simply do not know exact levels of future cash flows. Without introducing fuzzy numbers it would not be possible to formulate this genuine uncertainty. Fuzzy numbers incorporate subjective judgments and statistical uncertainties which may give managers a better understanding of the problems involved in assessing future cash flows.

**Fuzzy real option optimization**

We pointed out that the closing/not closing decisions for production plants have consequences that are significant and will have an impact on the market and competitive positions of the company, which are good reasons to apply fuzzy real option valuation. More specifically, we have a decision rule that we should only close a production plant immediately if the net present value of this action is high enough to compensate for giving up the value of the option to wait; this is the opportunity cost of the decision to close the plant. Our next step is to determine the time \( t' \) to exercise the fuzzy real option.
Let us now assume that the expected cash flows of the close/not close decision cannot be characterized with single numbers (which should be the case in serious decision making). With the help of possibility theory we can estimate the expected incoming cash flows at each year of the project by using a trapezoidal possibility distribution of the form

\[ \overline{c_i} = (s_i^L, s_i^R, \alpha_i, \beta_i), \quad i = 0,1,\ldots,L, \]

that is, the most possible values of the expected incoming cash flows lie in the interval \([s_i^L, s_i^R]\) (which is the core of the trapezoidal fuzzy number describing the cash flows at year \(i\) of the investment); \((s_i^R + \beta_i)\) is the upward potential and \((s_i^L - \alpha_i)\) is the downward potential for the expected cash flows of the investment at year \(i\), \((i = 0, 1, \ldots, L)\). In a similar manner we can estimate the expected costs by using a trapezoidal possibility distribution of the form

\[ \overline{X} = (x_i^L, x_i^R, \alpha', \beta'), \]

i.e. the most possible values of the expected costs lie in the interval \([x_i^L, x_i^R]\); \((x_i^R + \beta')\) is the upward potential and \((x_i^L - \alpha')\) is the downward potential for the expected fixed costs. By using possibility distributions we can extend the pure probabilistic decision rule for an optimal decision strategy to a possibilistic context.

**Theorem 2.** Let \(P\) be a deferrable decision opportunity with incoming cash flows and costs that are characterized by the trapezoidal possibility distributions given above. Furthermore, let us assume that the maximum deferral time of the decision is \(T\), and the required rate of return on this project is \(\beta_P\). In these circumstances, we shall make the decision (exercise the real option) at time \(t'\), \(0 < t' < T\), for which the value of the option, \(C_t\), is positive and reaches its maximum value. That is,

\[ \text{FROV}_t = \max_{t=0,1,\ldots,T} \text{FROV}_t = \max_{t=0,1,\ldots,T} \overline{V} e^{-\delta t} N(d_1^{(t)}) - \overline{X} e^{-\delta t} N(d_2^{(t)}) > 0, \]

where

\[ d_1^{(t)} = \frac{\ln(E(V_t)/E(X_t)) + (r - \delta - \sigma^2/2) t}{\sigma \sqrt{t}}, \]

\[ d_2^{(t)} = \frac{\ln(E(V_t)/E(X_t)) + (r - \delta + \sigma^2/2) t}{\sigma \sqrt{t}}. \]

Here, \(E\) denotes the possibilistic mean value operator defined in the previous section, and \(\sigma = \sigma(V_t)/E(V_t)\) is the annualized possibilistic variance of the aggregate expected cash flows relative to its possibilistic mean (and therefore represented as a percentage value). Furthermore,

\[ \overline{V}_i = \text{PV}(\overline{c_f_0}, \overline{c_f_1}, \ldots, \overline{c_f_L}; \beta_P) - \text{PV}(\overline{c_f_0}, \overline{c_f_1}, \ldots, \overline{c_f_L}; \beta_P) = \text{PV}(\overline{c_f}, \ldots, \overline{c_f_L}; \beta_P) = \sum_{i=0}^{L} \overline{c_f}, \]

computes the present value of the aggregate (fuzzy) cash flows of the project if this has been postponed \(t\) years before being undertaken. Let

\[ \overline{V}_t = (v_i^L, v_i^R, \alpha', \beta') = \sum_{i=0}^{L} (s_i^L, s_i^R, \alpha_i, \beta_i). \]

Then, using the formulas for arithmetic operations on trapezoidal fuzzy numbers we have

\[ \text{FROV}_t = (v_i^L, v_i^R, \alpha', \beta') e^{-\delta t} N(d_1^{(t)}) - (x_i^L, x_i^R, \alpha', \beta') e^{-\delta t} N(d_2^{(t)}) = (v_i^L e^{-\delta t} N(d_1^{(t)}) - x_i^L e^{-\delta t} N(d_2^{(t)}), v_i^R e^{-\delta t} N(d_1^{(t)}) - x_i^R e^{-\delta t} N(d_2^{(t)}), v_i^L e^{-\delta t} N(d_1^{(t)}) + \alpha' e^{-\delta t} N(d_2^{(t)}), v_i^R e^{-\delta t} N(d_1^{(t)}) + \beta' e^{-\delta t} N(d_2^{(t)}), v_i^L e^{-\delta t} N(d_1^{(t)}) + \alpha' e^{-\delta t} N(d_2^{(t)}), v_i^R e^{-\delta t} N(d_1^{(t)}) + \beta' e^{-\delta t} N(d_2^{(t)})) \]

when we find the maximizing element from the set

\[ \{\text{FROV}_0, \text{FROV}_1, \ldots, \text{FROV}_T\} \]

we can work out the time \(t'\) to exercise the fuzzy real option. This is, however, a task that involves some ambiguity, because it involves the ordering of trapezoidal fuzzy numbers.
There are a number of studies of the ordering of trapezoidal fuzzy numbers (cf. Carlsson and Fullér, 2001b; Dubois and Prade, 1988 for details). Basically, we can simply apply some value function to order fuzzy real option values of trapezoidal forms \( FROV_t = (c^L_t, c^R_t, \alpha'_t, \beta'_t) \), \( t = 0, 1, \ldots, T \). Let
\[
\nu(FROV_t) = \frac{c^L_t + c^R_t}{2} + r_A \left( \frac{\beta'_t - \alpha'_t}{6} \right),
\]
where \( r_A \geq 0 \) denotes the degree of the manager’s risk aversion. If \( r_A = 1 \) then the manager compares trapezoidal fuzzy numbers by comparing their pure possibilistic means. Furthermore, in the case \( r_A = 0 \), the manager is risk neutral and compares fuzzy real option values by comparing the centre of their cores, i.e. he does not care about their upward or downward potentials.

\[\Box\]

### Binomial model

For practical purposes and when working with senior management the binomial version of the real options model is easier to use and easier to explain in terms of the available data, i.e. we sometimes have very limited data available which makes it hard to carry out the parameter estimates we need for the possibility distributions. For our case the basic binomial setting is presented as a setting of two lattices, the underlying asset lattice and the option valuation lattice; for adding insight we can also include a decision rule lattice. In Figure 2 the weights \( u \) and \( d \) describe the geometric movement (Brownian motion) of the cash flows \( V \) over time, \( q \) stands for a movement up and \( 1-q \) movement down, respectively. The value of the underlying asset develops in time according to probabilities attached to movement \( q \) and \( 1-q \), and weights \( u \) and \( d \), as described in the figure.

![Asset value](image)

**Fig. 2.** Underlying asset lattice of two periods

The input values for the lattice are approximated with the following set of formulae (Cox et al, 1979):
\[
u = e^{\sigma \sqrt{\Delta t}} \quad \text{Movement up}
\]
\[d = e^{-\sigma \sqrt{\Delta t}} \quad \text{Movement down}
\]
\[
q = \frac{1}{2} + \frac{1}{2} \left( \frac{\alpha - \frac{1}{2} \sigma^2}{\sigma} \right) \sqrt{t} \quad \text{Probability of movement up}
\]

The option valuation lattice is composed of the intrinsic values \( I \) of the latest time to invest retrieved as the maximum of present value and zero, the option values \( O \) generated as the maximum of the intrinsic or option values of the next period (and their probabilities \( q \) and \( 1-q \)) discounted, and the present value \( S - F \) of the period in question.
This formulation describes two binomial lattices that capture the present values of movements up and down from the present values of the previous states of time and the incremental values \( I \) directly contributing to option value \( O \). The relation of geometric movements up and down is captured by the ratio \( d = 1/u \). The binomial model is a discrete time model and its accuracy improves as the number of time steps increases.

In summary, the benefit of using fuzzy numbers and the fuzzy real options model – both in the Black-Scholes (Black and Scholes, 1973) and in the binomial (Cox et al., 1979) version of the real options model - is that we can represent genuine uncertainty in the estimates of future costs and cash flows and take these factors into consideration when we make the decision to either close the plant now or to postpone the decision by \( t \) years (or some other reasonable unit of time). The simpler, classical representation does not adequately show the uncertainty.

**The production plant and future scenarios**

The production plant we are going to describe is a real case, the numbers we show are realistic (but modified for reasons of confidentiality) and the decision process is as close to the real process as we can make it. We worked the case with the fuzzy real options model in order to help senior management decide if the plant should (i) be closed as soon as possible, (ii) not closed, or (iii) closed at some later point of time (and then at what point of time). The background for the decisions can be found in the following general development of the profitability of the Finnish forest products companies in Figure 4 (the Finnish Forest Industries Federation, 2009):

![Fig. 4. Profit before taxes and ROCE, Finnish forest products companies](image-url)
The analysis carried out for the production plant started from a comparison of the present production and production lines with four new production scenarios with different production line set-ups (cf. Figure 5). In the analysis each production scenario is analysed with respect to one sales scenario assuming a match between performed sales analysis and consequent resource allocation on production. Since there is considerable uncertainty involved in both sales quantities and sales prices the resource allocation decision is contingent to a number of production options that the management has to consider, but which we have simplified here in order to get to the core of the case. There were a number of conditions, which were more or less predefined. The first one was that no capital could/should be invested as the plant was regarded as a sunset plant. The second condition was that we should in fact consider five scenarios: the current production set up with only maintenance of current resources and four options to switch to set-ups that save costs and have an effect on production capacity used. The third condition is that the plant together with another unit has to carry considerable administrative costs of the sales organization in the country. The fourth condition is that there is a pension scheme that needs to be financed until 2013. The fifth condition is the energy contract of the unit which is running until 2013. These specific conditions have consequences on the cost structure and the risks that various scenarios involve.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Optimistic sales volume</th>
<th>Sales volume as today</th>
<th>Pessimistic sales volume</th>
<th>Joker</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
<td>200000</td>
<td>150000</td>
<td>125000</td>
<td>105000</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>200000</td>
<td>150000</td>
<td>125000</td>
<td>105000</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>200000</td>
<td>150000</td>
<td>125000</td>
<td>105000</td>
</tr>
<tr>
<td>Scenario 4</td>
<td>200000</td>
<td>150000</td>
<td>125000</td>
<td>105000</td>
</tr>
</tbody>
</table>

Fig. 5. Production plant scenario

Each scenario assumes a match between sales and production, which is a simplification; in reality there are significant, stochastic variations in sales, which cannot be matched by the production. Since no capital investment is assumed there will be no costs in switching between the scenarios (which is another simplification). The possibilities to switch in the future were worked out as (real) options for senior management; the opportunity to switch to another scenario is a call option. The option values are based on the estimates of future cash flows, which are the basis for the upward/downward potentials.

In discussions with senior management they (reluctantly) adopted the view that options can exist and that there is a not-to-decide-today possibility for the close/not close decision. We should remember that the close/not close decisions also have a few strains of politics and that the timing of the decision matters, postponing a decision after the groundwork has been done may turn out to be difficult.

The motives to include options into the decision process were reasoned through with the following logic:

- New information changes the decision situation (Good or Bad News in Figure 6)
- Consequentially, new information has a value and it increases the flexibility of the management decisions
- The value of the new information can be analysed to enable the management to make better informed decisions

Fig. 6. Committing now vs. having options
In the discussions we were able to show that companies fail to invest in valuable projects because the options embedded in a project are overlooked and left out of the profitability analysis. The real options approach shows the importance of timing, as the real option value is the opportunity cost of the decision to wait in contrast with the decision to act immediately. We also worked out the use of decision trees as a way to work with the binomial form of the real options model as shown in Figure 6. We were then able to give the following practical description of how the option value is formed:

\[
\text{Option value} = \text{Discounted cash flow} \times \left( \frac{\text{Value of uncertainty (usually standard deviation)}}{\text{Investment}} \right) - \text{Risk free interest}
\]

If we compare this sketch of the actual work with the decision to close/not close the production plant with the theoretical models we introduced in the previous sections, we cannot avoid the conclusion that things are much simplified. There are two reasons for this: (1) the data available is scarce and imprecise as the scenarios are more or less ad hoc constructs; (2) senior management will distrust results of an analysis they cannot evaluate and verify with numbers they recognize or can verify as “about right”. Difficult decisions are not easier to make if the alternatives and their consequences are hard to understand.

**Closing/not closing the plant: Decisions**

The capital investment in a new paper machine – the type of project normally analysed with the real options models - is a project of several hundred M€. As a capital investment such a project is a long-time venture of 10-15 years of operational lifetime, which means that the productivity and profitability of the machine should be worked out over this period of time. Productivity is largely defined by the technological deterioration rate. Generally, the longer the plant stays in the technological race for productivity, the longer it is able to compete profitably. The conventional wisdom in the paper making industry is to build a paper machine with the most advanced features of technology development, so that high profits can be retrieved during the early years to pay back the capital invested.

The story is a bit different when we are nearing the end of the economic lifetime of a paper mill (with 1 or 2 paper machines). Closing a paper mill is usually understood as a decision at the end of the operational lifetime of the real asset. In the aging unit considered here the two paper machines were producing three paper qualities with different price and quality characteristics. The newer machine M2 had a production capacity of 150 000 tons of paper per year; the older M1 produced about 50 000 tons. The three products were: (1) an old product with declining, shrinking prices, (2) a product at the middle-cycle of its lifetime, and (3) a new innovative product with large valued added potential.

As background information a scenario analysis had been made with market and price forecasts, competitor analyses and the assessment of paper machine efficiency. Our analysis was based on the assumptions of this analysis with five alternative scenarios to be used as a basis for the profitability analysis (cf. Figure 5). After a preliminary screening (a simplifying operation to save time) two of the scenarios, one requiring sales growth and another with unchanged sales volume were chosen for a closer profitability assessment. The first one, Scenario 1 (sales volume 200 000 ton) included two sub options; 1A with the current production set up and 1B with a product specialization for the two paper machines. The 1B would offer possibilities for a closedown of a paper coating unit, which will result in savings of over 700 000 €. Scenario 1A was chosen for the analysis illustrated here. Scenario 2 starts from an assumption of a smaller sales volume (150 000 ton) and allows a closedown of the smaller M1, with savings of over 3.5 M€.

In addition to operational costs a number of additional cost items needed to be considered by the management. There is a pension scheme agreement, which would cause extra costs for the company if M1 is closed down. Additionally, the long-term energy contracts would cause extra cost if the company wants to close them before the end term.

The scenarios are summarised here as production and product set up options, and are modelled as options to switch a production set up. They differ from typical options - such as options to expand or postpone - in that they do not include major capital commitments; they differ from the option to abandon, as the opportunity cost is not calculated to the abandonment, but to the continuation of the current operations.
Binomial analysis

Cash flow estimates for the binomial analysis were estimated for each of the scenarios from the sales scenarios of the three products and by considering changes in the fixed costs caused by the production scenarios. Each of the products had their own price forecast that was utilised as a trend factor. For the estimation of the cash flow volatility there were two alternative methods of analysis. Starting from the volatility of sales price estimates one can retrieve the volatility of cash flow estimates by Monte Carlo simulation or by applying expert opinions directly to the added value estimates. In order to illustrate the latter method the volatility is here calculated from added value estimates ($AVE$) (with fuzzy estimates: $a: AVE \times -10\%$, $b: AVE \times 10\%$, $\alpha: AVE \times 10\%$, $\beta: AVE \times 10\%$) (cf. Fig. 7).

(Fuzzy) interval assumptions

\begin{align*}
(b+\beta) & = 20\% \\
b & = 10\% \\
a & = -10\% \\
\alpha & = -20\% \\
\beta & = -20\% \\
Volatility measure & = 10.3\% 
\end{align*}

Volatility measure $= 10.3\%$

\begin{align*}
(b+\beta) & = 20\% \\
b & = 10\% \\
a & = -10\% \\
\alpha & = -20\% \\
\beta & = -20\% \\
Volatility measure & = 10.3\% 
\end{align*}

(Fuzzy) interval assumptions

\begin{align*}
(b+\beta) & = 20\% \\
b & = 10\% \\
a & = -10\% \\
\alpha & = -20\% \\
\beta & = -20\% \\
Volatility measure & = 10.3\% 
\end{align*}

Volatility measure $= 10.3\%$

The annual cash flows in the option valuation were calculated as the cash flow of postponing the switch of production subtracted with the cash flows of switching now. The resulting cash flow statement of switching immediately is shown below (Figure 8). The cash flows were transformed from nominal to risk-adjusted in order to allow risk-neutral valuation.

![Fig. 7. Added value estimates, trapezoidal fuzzy interval estimates and retrieved volatilities (STDEV)](image)

![Fig. 8. Incremental cash flows and NPV with no delay in the switch to Scenario 1A](image)

The switch immediately to Scenario 1A seems to be profitable. In the following option value calculation the binomial process results are applied in the row “EBDIT, from binomial EBDIT lattice”. The calculation shows that when given volatilities are applied to all the products and the retrieved Added Value lattices are applied to EBDIT, the resulting EBDIT lattice returns cash flow estimates for the option to switch, adding 24 million of managerial flexibility (cf. Figure 9).
Fig. 9. Incremental cash flows, the NPV and the Option value when the switch to Scenario 1A is delayed by 1 year

The binomial process is applied to the Added Value Estimates (AVEs). The binomial process up and down parameters, $u$ and $d$, are retrieved from the volatility ($\sigma$) and time increment ($dt$). The binomial process is illustrated in Figure 10 and 11.

<table>
<thead>
<tr>
<th>Volatility of Product 1</th>
<th>$u_{up} = e^{\sigma \sqrt{dt}} = e^{10%}, dt = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatility of Product 2</td>
<td>$down = 1/u$</td>
</tr>
<tr>
<td>Volatility of Product 3</td>
<td></td>
</tr>
</tbody>
</table>

Binomial lattice: Product 1 process with volatility of 10.274023382816%

<table>
<thead>
<tr>
<th>Binomial value added process</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

Fig. 10. Binomial value added process, and following steps

EBDIT process = Sales revenue – Fixed Costs

Sales revenue process

Added value * Sales

EBDIT

Fig. 10. Binomial value added process, and following steps
Binomial process, example

### EBDIT

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1535949</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>8</td>
<td>811524</td>
<td>2540158</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>164086</td>
<td>1721123</td>
<td>4087253</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>-414548</td>
<td>989150</td>
<td>3119647</td>
<td>4700974</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>-931692</td>
<td>334984</td>
<td>2254411</td>
<td>3670258</td>
<td>5289020</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**EBDIT of year 1 extrapolated to the future with a trend factor**

**Calculation rolls down from up (to the future)**

**Calculation rolls up from down (from the future)**

1,535,949 1,537,138 1,538,327 1,539,518 1,540,709

**713701 = Max((2254411-1540709),0)**

### EBDIT

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1,948444</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1,035,607</td>
<td>2,345,683</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>346,506</td>
<td>1,335,483</td>
<td>2,785,291</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>497,294</td>
<td>1,700,238</td>
<td>3,257,473</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>713,701</td>
<td>2,129,549</td>
<td>3,748,311</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**For NPV calculation with options cash flow of 1948444 is used instead of 1535949**

\[ 497294 = 0 \times (1 - 0.696781795) + 713701 \times 0.696781795 \]

**Risk-adjusted probability:** \( 0.696781795 = (e^{rf} - \text{down})/(\text{up-down}) \)

Fig. 11. Binomial process, final node assessment (up) and node-wise comparison (down)
Fuzzy interval analysis

The fuzzy interval analysis allows management to make scenario based estimates of upward potential and downward risk separately. The volatility of cash flows is defined from a possibility distribution and can readily be manipulated if the potential and risk profiles of the project change. Assuming that the volatilities of the three product-wise AVEs were different from the ones presented in Figure 7 to reflect a higher potential of Product 3 and a lower potential of Product 1, the following volatilities could be retrieved (Figure 12). Note that the expected value with products 1 and 3 now differs from the AVEs.

![Fuzzy Added Value intervals and volatilities](image)

**Fig. 12.** Fuzzy Added Value intervals and volatilities

The fuzzy cash flow based profitability assessment allows a more profound analysis of the sources of a scenario value. In real option analysis such an asymmetric risk/potential assessment is realized by the fuzzy ROV (cf. section 2). Added values can now be presented as fuzzy added value intervals instead of single (crisp) numbers. The intervals are then run through the whole cash flow table with fuzzy arithmetic operators. The fuzzy intervals described in this way are called trapezoidal fuzzy numbers (cf. Fig. 13).

![Fuzzy interval assessment, applying interval assumptions to Added Value](image)

**Fig. 13.** Fuzzy interval assessment, applying interval assumptions to Added Value
In case of the risk-neutral valuation the discount factor is a single number. In our analysis the discounting is done with the fuzzy EBDIT based cash flow estimates by discounting each component of the fuzzy number separately. The expected value (EV) and the standard deviation (St.Dev) are defined as follows (Figure 14, cf. also section 2):

<table>
<thead>
<tr>
<th>Risk-neutral valuation parameter</th>
<th>0.955</th>
<th>0.911</th>
<th>0.870</th>
</tr>
</thead>
<tbody>
<tr>
<td>EBDIT, risk neutral</td>
<td>805875</td>
<td>1458518</td>
<td>1751484</td>
</tr>
<tr>
<td>EBDIT, risk neutral, Support up</td>
<td>2040102</td>
<td>2799376</td>
<td>3127935</td>
</tr>
<tr>
<td>EBDIT, risk neutral, Core up</td>
<td>1422989</td>
<td>2128947</td>
<td>2439710</td>
</tr>
<tr>
<td>EBDIT, risk neutral, Core down</td>
<td>188761</td>
<td>788088</td>
<td>1063258</td>
</tr>
<tr>
<td>EBDIT, risk neutral, Support down</td>
<td>-428352</td>
<td>117659</td>
<td>375032</td>
</tr>
<tr>
<td>EBDIT, risk neutral, Fuzzy EV</td>
<td>805875</td>
<td>1458518</td>
<td>1751484</td>
</tr>
<tr>
<td>EBDIT, risk neutral, St. Dev.</td>
<td>634024</td>
<td>688801</td>
<td>707085</td>
</tr>
<tr>
<td>EBDIT, risk neutral, St. Dev. %</td>
<td>78.7%</td>
<td>47.2%</td>
<td>40.4%</td>
</tr>
</tbody>
</table>

Fig. 14. Fuzzy interval assessment, discounting a fuzzy number

As a result from the analysis a NPV calculation now supplies results of the NPV and fuzzy ROV as fuzzy numbers. Also flexibility is shown as a fuzzy number.

<table>
<thead>
<tr>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present value at delay</td>
<td>7,174,624</td>
<td>6,494,629</td>
<td></td>
</tr>
<tr>
<td>Present value at delay, Support up</td>
<td>9,834,912</td>
<td>14,886,532</td>
<td></td>
</tr>
<tr>
<td>Present value at delay, Core up</td>
<td>7,552,125</td>
<td>11,824,291</td>
<td></td>
</tr>
<tr>
<td>Present value at delay, Core down</td>
<td>2,986,552</td>
<td>5,699,809</td>
<td></td>
</tr>
<tr>
<td>Present value at delay, Support down</td>
<td>703,765</td>
<td>2,637,568</td>
<td></td>
</tr>
<tr>
<td>Present value at delay, Fuzzy EV</td>
<td>6,410,732</td>
<td>10,293,171</td>
<td></td>
</tr>
<tr>
<td>Present value at delay, St. Dev.</td>
<td>2,345,340</td>
<td>3,146,154</td>
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</tr>
<tr>
<td>Present value at delay, St. Dev. %</td>
<td>36.6%</td>
<td>36.6%</td>
<td></td>
</tr>
</tbody>
</table>

NPV at present year, 2005

<table>
<thead>
<tr>
<th>Flexibility</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delay value without flexibility</td>
<td>-1,283,804</td>
<td>7,174,624</td>
<td>5,890,820</td>
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</tr>
<tr>
<td>Delay value with flexibility, Support Up</td>
<td>3,667,612</td>
<td>9,834,912</td>
<td>13,502,524</td>
<td></td>
</tr>
<tr>
<td>Delay value with flexibility, Core Up</td>
<td>3,172,855</td>
<td>7,552,125</td>
<td>10,724,981</td>
<td></td>
</tr>
<tr>
<td>Delay value with flexibility, Core Down</td>
<td>2,183,343</td>
<td>2,986,552</td>
<td>5,169,895</td>
<td></td>
</tr>
<tr>
<td>Delay value with flexibility, Support Down</td>
<td>1,688,587</td>
<td>703,765</td>
<td>2,392,352</td>
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<tr>
<td>Delay value with flexibility, Fuzzy EV</td>
<td>2,925,477</td>
<td>6,410,732</td>
<td>9,336,209</td>
<td></td>
</tr>
<tr>
<td>Delay value with flexibility, St. Dev.</td>
<td>508,314</td>
<td>2,345,340</td>
<td>2,853,654</td>
<td></td>
</tr>
<tr>
<td>Delay value with flexibility, St. Dev. %</td>
<td>17.4%</td>
<td>36.6%</td>
<td>30.6%</td>
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</tr>
</tbody>
</table>

Delay

Fig. 15. Fuzzy interval assessment, NPV and Fuzzy Real Option Value (FROV)

Analysis results

The following table summarizes the results from binomial process and cash flow interval analysis (the analogous analysis of the switch to Scenario 2 has not been shown).
NPV with option to switch

<table>
<thead>
<tr>
<th>Time of action</th>
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<th>2006</th>
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<td>22 000</td>
<td>19 500</td>
<td>14 800</td>
<td>18 300</td>
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<td>Difference to NPV</td>
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<td>25 800</td>
<td>14 800</td>
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<td>7 600</td>
<td>11 100</td>
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<tr>
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<td>7 000</td>
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<td>2 800</td>
<td>5 100</td>
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Fig. 16. Results comparison

Fig. 17. Results from the binomial option valuation
The analysis shows that there are viable alternatives to the ones that result in closing the paper mill and that there are several options for continuing with the current operations. The uncertainties in the Added Value processes, which we have modeled in two different ways, show significantly different results when, on the one hand, both risk and potential are aggregated to one single (crisp) number in the binomial process and, on the other hand, there is a fuzzy number that allows the treatment of the downside and the upside differently. In this case study management is faced with poor profitability and needs to assess alternative routes for the final stages of the plant with almost no real residual value. The specific costs of closedown (the pension scheme and the energy contracts) are a large opportunity cost for an immediate closedown (the actual cost is still confidential).

The developed model allows for screening alternative paths of action as options. The binomial assessment, based on the assumptions of the real asset tradability, overestimates the real option value, and gives the management flexibilities that actually are not there. On the other hand, the fuzzy cash flow interval approach allows an interactive treatment of the uncertainties on the (annual) cash flow level and in that sense gives the management powerful decision support. With the close/not close decision, the fuzzy cash flow interval method offers both rigor and relevance as we get a normative profitability analysis with readily available uncertainty and sensitivity assessments. Here we showed one scenario analysis in detail and sketched a comparison with a second analysis. For the real case we worked out all scenario alternatives and found out that it makes sense to postpone closing the paper mill with several years. The paper mill was closed on January 31st, 2007 at significant cost.
Discussion and conclusions

In decisions on how to use existing resources the challenges of changing markets become a reality when senior management has to decide how to allocate capital to production, logistics and marketing networks, and has to worry about the return on capital employed. The networks are interdependent as the demand for and the prices of forest industry products are defined by the efficiency of the customer production processes and how well suited they are to market demand; the production should be cost effective and adaptive to cyclic (and sometimes random) changes in market demand; the logistics and marketing networks should be able to react in a timely fashion to market fluctuations and to offer some buffers for the production processes. Closing or not closing a production plant is often regarded as an isolated decision, without working out the possibilities and requirements of the interdependent networks.

The problem we have addressed is the decision to close – or not to close – a production plant in the forest products industry sector. The plant was producing fine paper products, it was rather aged, the paper machines were built a while ago, the raw material is not available close by, energy costs are reasonable but are increasing in the near future, key markets are close by and other markets (with better sales prices) will require improvements in the logistics network. The intuitive conclusion was, of course, that we have a sunset case and senior management should make a simple, macho decision and close the plant. On the other hand we have the trade unions, which are strong, and we have pension funds commitments until 2013, which are very strict, and we have long-term energy contracts, which are expensive to get out of. Finally, by closing the plant we will invite competitors to fight us in markets we have served for more than 50 years and which we cannot serve from other plants at any reasonable cost. We showed that real options models would support decision making in which senior managers search for the best way to act and the best time to act.

The key elements of the closing/not closing decision may be known only partially and/or only in imprecise terms; then meaningful support can be given with a fuzzy real options model. We found the benefit of using fuzzy numbers and the fuzzy real options model – both in the Black-Scholes and in the binomial version of the real options model – to be that we can represent genuine uncertainty in the estimates of future costs and cash flows and use these factors when we make the decision to either close the plant now or to postpone the decision by \( t \) years (or some other reasonable unit of time). We used a real world case to show the dilemma(s) senior management had to deal with and the (low) level of precision in the data to be used for making a decision. We worked the case with fuzzy real options models and were able to find ways to work out the consequences of closing or not closing the plant. Then, management made their own decision.

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