Optimizing one-day driving restriction using mathematical programming

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Abstract—A one-day driving restriction prohibits drivers from operating their vehicles on a given weekday in an attempt to reduce traffic congestion and air pollution. A restricted weekday is assigned based on vehicles’ license number, which is somewhat random. The one-day driving restriction has been popular in developing countries due to the low cost of implementation. However, it has not been effective in reducing air pollution in the long run. Many studies have pointed out that there is an incentive for the household to purchase an old used vehicle, which might have high emission potential, to circumvent the restriction. In addition, as shown in this paper, households drive more on non-restricted weekdays to compensate for lost driving time, creating a substitution effect which may lead to increased emissions. We propose a one-day driving restriction based on mathematically assigned household vehicle registrations rather than a randomly assigned license plate number. This is valuable in overcoming flaws of the current one-day driving restriction to increase opportunity cost of cheating as such, the new one-day driving restriction will increase the cost of buying the second vehicle. Furthermore, the mathematical programming model is utilized to assign the restricted weekday to each household, while also optimizing the distribution of high and low emission vehicles on the road to reduce air pollution effectively. The numerical simulation illustrates that the goal of driving restriction is achieved.

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Introduction

Driving restriction is a policy option which aims at reducing traffic congestion and air pollution caused by on-road air pollutants from motor vehicles. A one-day driving restriction prohibits drivers from driving their vehicles on a given weekday based on their vehicles’ license number. A growing number of cities around the world, especially in emerging economies, are implementing or planning to introduce the one-day driving restriction (e.g., Mexico City, Mexico; Santiago, Chile; São Paulo, Brazil; and Bogotá, Colombia) (Gallego et al., 2013; Carvallo and Lizana, 2011). Moreover, a driving restriction was implemented in Beijing in preparation for the 2008 Olympic Games (Lin et al., 2012). A different type of driving restrictions such as the implementation of a fee to enter a congested area has also been used in Stockholm, London, and Milan (Carnovale and Gibson, 2013). This paper is an effort to assess the effectiveness of one-day driving restriction programs.

One of the goals of such driving restrictions is to reduce air pollution from on-road vehicles. Small and Kazimi (1995) conducted a study aimed at measuring the cost associated with air pollution from motor vehicles. Pollutants such as methane (CH4), nitro-oxide (N2O) and Particulate Matter (PM) from vehicles are key sources of automobile pollution. When PM2.5 (i.e., particulate matter smaller than 2.5 µg/m³) particles are inhaled, some are retained along the respiratory tract, while others penetrate deeply into the lung where they can enter the bloodstream (US EPA, 2012). These particles aggravate the severity of chronic lung diseases and impair airway functions, which can lead to inflammation of lung tissue, aggravated asthma and decreased lung function (US EPA, 2012). Recent epidemiological studies have shown that high levels of PM are closely correlated with substantial adverse health effects such as acute respiratory infections and mortality in the short-term (Chen et al. 2000; Sastry 2002; Tham et al. 2009). Long-term exposure to the combustion-related PM and the SO2-related air pollution could lead to cardiopulmonary and lung cancer (Viswanathan et al. 2006). Dockery et al. (1992), Pope (2000) and Pope et al. (2002), quantified the effects of chronic exposure of PM, and conclude that exposure to PM2.5 has been consistently linked with increased mortality from cardiopulmonary diseases, lung cancer, numerous other respiratory illnesses, and associated morbidity.

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The current one-day driving restriction, however, has not been effective in reducing air pollutants even though 20% of vehicles were removed from the roads. Some of the previous studies, e.g., Eskeland and Feyzioglu (1997), Davis (2008), Lin, et al. (2012), and Gallego et al. (2013), were unable to find an indication that the one-day driving restriction improved air quality, especially in the long-run. Two reasons are suggested.

First, the one-day driving restriction has unique imperfections which allow households to find alternative ways to dodge regulations. The presence of loopholes makes it difficult for the one-day driving restriction to achieve policy goals which lead to pollution leakage. A common type of dodging behavior is the purchase of an additional vehicle (Lavinson and Shetty, 1992; Eskeland and Feyzioglu, 1997; Davis, 2008; Wang et al., 2013). Under the current system, an individual can circumvent the one-day restriction by purchasing another vehicle with a different license plate number. Purchasing an additional vehicle attenuates the effect of the one-day driving restriction. According to Davis (2008), vehicle registration increased after the implementation of the one-day driving restriction in Mexico City. In addition, drivers set on avoiding the restriction are inclined to purchase cheap and older vehicles that may emit more harmful air pollutants (Davis, 2008). According to Gallego, et al. (2013), in Mexico City, the one-day driving restriction reduced certain pollutants when it first went into effect, but eventually, air pollution became worse; such change could be attributed to the purchase of older and high emitting vehicles.

Second, as we will show in the theory section below, drivers increase their driving on unrestricted weekdays to attend to their routine responsibilities (e.g., groceries, chores) since they cannot drive on the restricted weekday. This creates a substitution effect that results in the emission of more pollutants.

The objective of this study is to design a new driving restriction that inhibits the present flaws reducing policy effectiveness. The proposed restriction uses a mathematical programming model that assigns the restricted workday based on vehicle registration rather than license plate number. This not only discourages the purchase of a second vehicle to avoid driving restrictions because any such vehicle would have to be registered at a different address, it also reduces air pollution by optimizing the distribution of high and low emission vehicles on the road. The numerical simulation illustrates that the goal of the driving restriction is achieved effectively. It might increase the administration cost due to running a vast mathematical programming model, but in the long run, the expected reduction in computational costs would allow the program to be within the financial means of local government.

Literature review

The iconic driving restriction, Hoy no Circula in Mexico City, Mexico, started in 1989, the program prohibits vehicles from driving one day per week based on the last digit of a vehicle’s license plate. For example, a plate where the last digit ends in 0 or 1 cannot drive on Monday. This aims at reducing traffic congestions and air pollution from vehicle emissions. Tovar (1995) shows that pollution has worsened under the program because many drivers get around the restriction by purchasing a second car. Eskeland and Feyzioglu (1997) investigated the changes in car ownership and gasoline demand after the implementation of the driving restriction in Mexico City, and found that gasoline demand increased by about 7%. Davis (2008) reported that air quality in Mexico City did not improve and gasoline consumption increased. Gallego et al. (2013) determined that air pollutants such as carbon monoxide and sulfur dioxide were reduced in the short run, but increased in the long run leading to lower air quality. Both, Davis (2008) and Gallego, et al. (2013) concluded that these results were associated with residents purchasing additional vehicles. Many of such purchases were made in used car sale markets, which contains older and less efficient vehicles.

In an attempt to reduce traffic congestion and air pollution during the 2008 Olympic Games, China implemented a temporary driving restriction in Tianjin and Beijing in 2007-2008 (Lin et al., 2012). This restriction, based on cars with odd or even license plate numbers being driven on alternating days, removed 50% of the car from the roads. After the Olympic Games, a less strict restrictive program was adopted in Beijing. Viard and Fu (2015) investigated the driving restrictions in Beijing and found that particulate matter was reduced by 18% during the every-other-day restriction, and by 21% in the latter program. Lin et al. (2012) found that during the 2008 Olympics, the every-other-day driving restriction in China was related with at least a 38% reduction in PM concentration.

One of the key issues with such restrictions is evasion. Survey data from Wang et al. (2013) show that 50% of Chinese families planned to purchase an additional vehicle to circumvent the driving restriction. Furthermore, roughly 48% of Chinese drivers actually purchased the second vehicle, borrowed license plates or created alternative ways to bypass the driving restriction (Wang et al., 2013), which is consistent with findings in Chen (2012) where 58% of drivers violated the driving restriction.
Other studies for São Paulo and Bogotá (Lin et al., 2012), Santiago (Gallego et al. 2013), San Jose - Costa Rica (Osakwe, 2010) show that the driving restriction might have a positive impact on air quality. Gallego, et al. (2013) also determined that the driving restriction program in Santiago had a positive impact on air quality improvement prior to households adjusting their stock of vehicles. Osakwe (2010) found a reduction in gasoline sales in Costa Rica, although the impact on air quality and traffic congestions was unclear. Lin et al. (2012) found no significant impact on the level of PM$_{10}$ and sulfur dioxide level in Bogotá.

General model

The general model closely follows Lin et al. (2011) and Zhang et al. (2017) without changing most of the mathematical notations and then the new policy model is introduced. Let $v_{id}$ denote the daily vehicle miles traveled, where $i = \text{household } (1, \cdots, N)$, $d = \text{day of the week (Monday-Friday)}$. The model uses only one subscript, $i$, to describe vehicle type information (passenger vehicle, light truck, medium/heavy truck, and others), make-year information, and emission potential without loss of generality. It is possible to avoid adding a subscript when one assumes that each household owns one vehicle, and each household makes a driving decision on a weekly basis. The weekly vehicle miles are given by a sum of daily driving such that $v_i = \sum_d v_{id}$. The benefit from driving is a function of $v_i$ and given by $B_i(v_i) = B_i(\sum_d v_{id})$, where $\frac{\partial B_i}{\partial v_{id}} > 0$ and $\frac{\partial^2 B_i}{\partial v_{id}^2} < 0$ (marginal benefit of driving is diminishing).

Cost of driving consists of two parts. First part is the cost of gas which is linearly related to the daily vehicle miles, i.e.,

$$c^g_i v_{id},$$

where $c^g_i$ is the gas cost in dollars per mile. It has the subscript $i$ because it depends on the mileage of the vehicle such that $c^g_i = \frac{p^g}{\text{mpg}}$, where $p^g$ is gas price per gallon and mpg is mile per gallon of the vehicle. Second part of the driving cost is disutility from the driving (e.g. depreciation of a car, drivers’ fatigue, and increase in car accident probability). A cost function is given by $C_i(v_i) = \sum_d [c^g_i v_{id} + c_i(v_{id})] + F_i$, where $F_i$ is the fixed cost. The expression inside the bracket, $c^g_i v_{id} + c_i(v_{id})$, is the daily driving cost. The cost function has the properties that $\frac{\partial c_i}{\partial v_{id}} = c^g_i + c'_i > 0$ and $\frac{\partial^2 c_i}{\partial v_{id}^2} = c''_i > 0$ (increasing marginal cost of driving).

Emission of pollutants on the particular day of the week is the function of vehicle miles and type of vehicle and is written as $E_d = \sum_i e_i v_{id}$, where $e_i$ is the emissions factor which is different over households, vehicles types, and vehicle make-year. Damages from the pollution in a day is written as $D_d = D(E_d)$, where $D$ evaluate environmental and public health damage ($\frac{\partial D_d}{\partial E_d} > 0$).

Household decision (private optimal)

Individual household is maximizing the net benefit of driving such that,

$$\max_{v_{id}} w_i^{UR} = B_i\left(\sum_d v_{id}\right) - \sum_d \left(c^g_i v_{id} + c_i(v_{id})\right) - F_i,$$

where $B$ is the benefit from driving and $C$ is cost of driving. Superscript $UR$ represents the net benefit from driving without the driving restriction. First-order condition (FOC) is:

$$\frac{\partial w_i^{UR}}{\partial v_{id}} = \frac{\partial B_i}{\partial v_{id}} - c^g_i - \frac{\partial c_i}{\partial v_{id}} = 0.$$

Equation (2) implies that each household $i$ will indicate its driving mileage to set marginal private benefit ($\frac{\partial B_i}{\partial v_{id}}$) equal to marginal private cost ($c^g_i + \frac{\partial c_i}{\partial v_{id}}$).
Social planner’s problem

The social planner’s problem (equation 3) is to choose the vehicle miles and day of the week for each household in order to maximize total (social) welfare, including environmental and public health.

\[
\max_{v_{id}} W = \sum_i w_i = \sum_i B_i \left( \sum_d v_{id} \right) - \sum_i \sum_d \left[ (c_i^g v_{id} + c_i(v_{id})) - F_i \right] - \sum_d D(v_{id})
\]  
(3)

where the FOC is:

\[
\frac{\partial W}{\partial v_{id}} = \frac{\partial B_i}{\partial v_{id}} - c_i^g - \frac{\partial c_i}{\partial v_{id}} - \frac{\partial D}{\partial v_{id}} = 0.
\]  
(4)

Comparing equations (2) and (4), we know that households’ vehicle miles are higher than social optimum because of (negative externality). To fix the externality, a fee or daily tax can be imposed at \( t_d = \frac{\partial D}{\partial v_{id}} \) per vehicle mile; however, it is not practical.

Driving regulation – one-day restriction

A one-day driving restriction is one that requires that \( v_{id}^* = 0 \) for a household \( i \) for a day, \( d^* \). When faced with a driving restriction, the individual household’s optimization problem is:

\[
\max_{v_{id}} w_i^R = B_i \left( \sum_d v_{id} \right) - \sum_d \left( c_i^g v_{id} + c_i(v_{id}) \right) - F_i \text{ s.t. } v_{id} = 0 \text{ when } d = d^*,
\]  
(5)

where superscript \( R \) indicates the restricted model, and \( d^* \) is the day of applied restriction; \( d^* \) is assigned here based on license plate’s number given by local government/authorities (e.g. plates ending in 1, 2 are restricted on Monday; 3, 4 on Tuesday, etc.). Lagrangian function is:

\[
L = B_i \left( \sum_d v_{id} \right) - \sum_d \left( c_i^g v_{id} + c_i(v_{id}) \right) - F_i - \lambda_{id^*} v_{id^*},
\]  
(6)

where \( \lambda_{id^*} \) is a multiplier, and FOC are:

\[
\frac{\partial w_i}{\partial v_{id}} = \frac{\partial B_i}{\partial v_{id}} - c_i^g - \frac{\partial c_i}{\partial v_{id}} = 0 \ \forall \ d \notin d^*  
\]  
(7.1)

\[
\frac{\partial w_i}{\partial v_{id}} = \frac{\partial B_i}{\partial v_{id}} - c_i^g - \frac{\partial c_i}{\partial v_{id}} - \lambda_{id} = 0 \text{ and } v_{id}^* = 0 \ \forall \ d \in d^*  
\]  
(7.2)

Because the benefit of driving is the function of the sum of daily driving mileage, the benefit of driving during unrestricted days is higher under the driving restriction than in the absence of regulation, i.e., \( \frac{\partial^2 B_i(v_{id})}{\partial v_{id}^2} > 0 \). Regulation evasion happens when \( w_i^R - w_i^R \) is greater than the (opportunity) cost of buying the second vehicle.

While the one-day driving restriction removes 20% of vehicles from the road, emission of pollutants may not be reduced by 20% because:

- households own vehicles which have different emission factors, and they are not optimally spread over the weekdays. This is because the day of restriction applied (\( d^* \)) is determined based on the license plate number not vehicle type.
- \( \frac{\partial^2 B_i(v_{id})}{\partial v_{id}^2} > 0 \), meaning that, daily driving mileage is increased whereas total driving mileage is reduced.
- drivers have the incentive to buy used vehicles that are old and have high emissions. Thus less than 20% of vehicles are off the roads.
New driving restriction based on registration

A driving restriction is one that requires that \( v_{i,d^*} = 0 \) for some household \( i^* \) based on vehicle registration information such as emission factors, which is collected when the vehicle is inspected and registered. Focused on minimizing daily emissions, regulators will decide \( i^* \) and \( d^* \) based on the emission level of each vehicle. Like the one-day driving restriction, 20% of vehicles are expected to be off the roads. Individual household’s optimization problem is unchanged as they will be assigned the specific restricted day.

Rather than randomly assigning individual plates, the optimization model in the new restriction program will decide which household is restricted on which day. In order to avoid intertemporal substitution on the same day, the new system will be effective from 12:00 a.m. to 11:59 p.m. The household which cannot drive on a specific day is the decision variable. Let \( h_{id} \) stands for a binary variable, 0 or 1, to indicate this decision, (i.e. when \( h_{id} = 0 \) household \( i \) cannot drive on \( d \)), the social planner’s optimization problem is given by:

\[
\max_{h_{id}} W = \sum_i B_{i} \left( \sum_{d} h_{id} v_{id} \right) - \sum_i \sum_{d} \left[ (c_{i}^\beta h_{id} v_{id} + c_{i} (h_{id} v_{id})) - F_{i} \right] - \sum_{d} D (h_{id} v_{id})
\]

\[
\text{s.t. } \sum_{i} e_{i} h_{id} v_{id} \leq \bar{e}_{d}, \quad \sum_{d} h_{id} = 4, \quad \text{and } h_{id} \text{ binary}
\]

The constraint, \( \sum_{d} h_{id} = 4 \) assures that each household can drive four days a week. \( \bar{e}_{d} \) is a predetermined daily emission target (e.g. 25% emission reduction from baseline). Once the regulator decides \( h_{id}^* \), each household will maximize the benefit (which is identical to equation 7).

The new structure is superior in two aspects: low and high emission vehicles are spread optimally over weekdays and the cost of dodging the regulation is increased. Thus there is a reduction of additional registration for additional vehicles, leading to close to 20% of vehicles being kept off the roads.

Numerical simulation

In this section, numerical simulation is undertaken. We compare the results from the one-day driving restrictions based on license plate number and vehicle registration.

Calibration

Let the benefit function of driving be expressed by

\[
B_i(v_i) = v_i^{\alpha_i} = \left( \sum_{d} v_{id} \right)^{\alpha_i},
\]

where \( \alpha_i \) is a benefit function parameter (0 < \( \alpha_i < 1 \)). The shape of the benefit function might be different over households. In favor of simplicity, all households are assumed to have a constant benefit function parameter, (i.e., \( \alpha_i = \alpha \)).

Cost of driving is given by:

\[
C_i(v_i) = \sum_{d} (c_{i}^\beta v_{id} + b v_{id}^k) + F_i,
\]

where \( b \) and \( k \) are cost function parameters, and \( b > 0 \) and \( k_i > 1 \). Likewise, assume that all the household have a constant cost parameter, \( k_i = k \). The individual household is maximizing the net benefit of driving. If \( \alpha_i = 0.85 \), \( b = 0.2 \), and \( c_{i}^\beta = 0.2 \), then \( v_{id}^* \approx 9.8 \) miles per day and \( v_{id}^* \approx 48.8 \) miles per week. This assumes that the price of gasoline is $4 per gallon and the mileage per gallon of vehicle is 20 miles, then \( c_{i}^\beta = 0.2 \). where a household’s \( c_{i}^\beta \) differs based on the different mileage of the vehicle owned.
Mobile combustion CH₄ and N₂O emission factors are a function of vehicle type and make year. Emission factor, $e_{1,t}$, is calculated based on the data which are compiled from emission factors for greenhouse gas inventories at US EPA (2014). As shown in Figure 1, newer vehicles emit a smaller amount of CH₄ and N₂O. For example, if a household owns a 2008 passenger car, it will emit 0.017 grams of CH₄ and 0.004 grams of N₂O per mile.

We selected 1000 random households from 309,164 household observations from Household Travel Survey (NHTS) dataset. This dataset was gathered from the US Department of Transportation (2009) and contains significant information such as vehicle year and type. Each household is assumed to own one vehicle and each vehicle has different information regarding mileage, make year, and vehicle type. Figure 2 presents the mile per gallon (MPG) distribution of the 1000 sample household randomly selected from the dataset and the mileage distribution of the full sample in the dataset for comparison. MPG sample average is 20.1, median 19.6, and standard deviation 5.8. Roughly half of the households own passenger cars, and 28% of household have Vans or SUVs, while 20% of households drove pick-up trucks.

Figure 1. Emission Factors, Source: EPA (2014)

Figure 2. PDF Approximation of Vehicle Mileage (mile per gallon)
Notes:
1. Sample – 1000 random sample from VEHV2PUB dataset. Full – all of the observations in VEHV2PUB dataset.
2. The horizontal axis represents the age of vehicle
3. High mileage vehicles include hybrid cars.
Make-year distribution is also represented in Figure 3. Sample average of make year is 2000 (9 years old at the time the survey was conducted).

![Figure 3. PDF Approximation of Vehicle Make-year](image)

Notes:
1. Sample – 1000 random sample from VEHV2PUB dataset. Full – all of the observations in VEHV2PUB dataset.
2. The horizontal axis represents the age of vehicle
3. A peak exists at the rightmost of the distribution due to the collectible vehicles from the late 1960s and early 1970s.

Table 1 includes the basic statistics and values of parameters of numerical simulation with household information.

<table>
<thead>
<tr>
<th>$a_i$</th>
<th>Benefit parameter</th>
<th>0.85</th>
<th>Number of households</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>Cost parameter</td>
<td>0.2</td>
<td>Average MPG</td>
<td>20.14</td>
</tr>
<tr>
<td>$P_g$</td>
<td>Gas price</td>
<td>$4/gallon</td>
<td>Percentage of passenger cars</td>
<td>51%</td>
</tr>
<tr>
<td>$k_i$</td>
<td>Cost Parameter</td>
<td>1.20</td>
<td>Average age of vehicles</td>
<td>9</td>
</tr>
<tr>
<td>$r$</td>
<td>Damage parameter</td>
<td>1.11</td>
<td>% of vehicles younger than 5 years old</td>
<td>25.8%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>% of vehicles older than 15 years old</td>
<td>12.5%</td>
</tr>
</tbody>
</table>
**Buying another vehicle**

Buying and driving the second vehicle happens when the (opportunity) cost of buying and driving another car is lower than the welfare loss from the driving restriction. Denote the difference of net benefit between driving with and without a driving restriction as $\Delta w_i = w_i^{UR} - w_i^R$. We assume that the household has an incentive to buy the second vehicle when $\Delta w_i > \bar{u}$, where $\bar{u}$ is the threshold that satisfies $\int_{\bar{u}}^{\infty} f(\Delta w_i) d\Delta w_i = 1 - \alpha$. Where $f(\Delta w_i)$ is the probability distribution of $\Delta w_i$ under the one-day driving restriction, and $\alpha$ is the noncompliance rate, (i.e. probability of buying the second vehicle). We set up the noncompliance rate to be 7.5% (i.e. 7.5% of household would buy the second car under the current one-day driving restriction). In other words, in this simulation, a total of 75 used vehicles would be added after implementing the one-day driving restriction. Eskeland and Feyzioğlu (1997) show that people in Mexico City bought a second car with a different license plate to avoid the one-day driving restriction, increasing the number of total cars registered. Gallego et al. (2013) found similar results in Santiago. For proper comparison between simulated scenarios, households are assumed to buy the same model and aged vehicle they currently own; this avoids a random selection of second vehicles that could add unnecessary noise between models.

**Numerical simulation results**

The numerical simulation model is run using the General Algebraic Modeling System (GAMS). Table 2 contains the results of five simulations. Model (1) is the private optimum where no driving restriction is implemented. Weekly driving mileage to maximize the household welfare is given by 48.2 miles with 525.7 units of combined emission of CH$_4$ and N$_2$O. Note that daily driving mileage is 9.6 miles.

Under the one-day driving restriction without regulation dodging, Model (3), household reduces weekly diving mileage to 43.6 miles (9.4% less than the private optimum) and emits 477.3 units of air pollutants (9.2% less than the private optimum). Although the one-day driving restriction removes 20% of vehicles from the roads, the emission is not decreased by a similar magnitude. It is because a household drives more, i.e., daily driving mileage is 10.9 miles (13.3% more than private optimum) due to the substitution effect. Also, high emission vehicles and low emission vehicles are mixed on the road, i.e., not optimally spread over the weekdays.

When households are able to dodge regulations by buying a second vehicle, Model (4), the one-day driving restriction removes only 18.6% of vehicles from the roads. The emission is 482.9 units which is 8.1% less than the private optimum. The threshold, $\bar{u}$, is estimated around 0.83 units (Figure 4). Figure 4 has the probability distribution of welfare change, $\Delta w_i$, under the one-day driving restriction and dodging restriction cutoff point, $\bar{u}$. The household who loses more than $\bar{u}$, is likely to purchase the second vehicle. As constructed, the area under the distribution above $\bar{u}$ is 7.5%. The effect of the driving restriction would be attenuated when the probability of dodging the regulation increases.

![Figure 4. Probability Distribution of $\Delta w_i$ under One-day Driving Restriction and Dodging Restriction Cutoff Point](image-url)
Note: horizontal axis represents the loss of welfare between driving with and without the one-day driving restriction. $\bar{u}$ is the threshold for the household to buy the second vehicle, i.e., the household who loses more than $\bar{u}$ would buy the second vehicle. Model (5) is the optimized one-day driving restriction based on the vehicle registration. The emission is reduced to 394.3 which is 25% less than the emission under the private optimum. Note that 25% reduction in emission is the predetermined policy goal of the driving restriction and it is achieved. Individual household daily driving increases due to substitution effect (9.8 miles instead of 9.6 miles on average). No one purchases the second vehicle under this one-day driving restriction because the cost of buying the second vehicle is much higher.

### Table 2. Simulation results

<table>
<thead>
<tr>
<th></th>
<th>Private Optimum (1)</th>
<th>Social Optimum (2)</th>
<th>One-day Driving Restriction (3)</th>
<th>One-day with Regulation Dodging (4)</th>
<th>Household Driving Restriction (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>% of vehicles out of roads</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weekly driving mileage</td>
<td>HH Average: (StDev)</td>
<td>48.15 (20.36)</td>
<td>24.94 (17.57)</td>
<td>43.62 (18.11)</td>
<td>44.65 (20.03)</td>
</tr>
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<td></td>
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<td></td>
<td>39.12 (17.72)</td>
</tr>
<tr>
<td>Daily driving mileage</td>
<td>HH Average: (StDev)</td>
<td>9.63 (4.07)</td>
<td>4.99 (3.51)</td>
<td>8.72 (5.95)</td>
<td>8.93 (5.62)</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>7.83 (5.57)</td>
</tr>
<tr>
<td>Daily emission</td>
<td>Total emission: (% change):</td>
<td>525.67 (−67%)</td>
<td>174.83 (−9.2%)</td>
<td>477.27 (−9.2%)</td>
<td>485.94 (−7.6%)</td>
</tr>
<tr>
<td></td>
<td>HH Average: (StDev)</td>
<td>0.53 (0.36)</td>
<td>0.18 (0.10)</td>
<td>0.48 (0.37)</td>
<td>0.49 (0.37)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.39 (0.28)</td>
</tr>
<tr>
<td>Net benefit of HH from driving</td>
<td>HH Average: (StDev)</td>
<td>6.24 (2.57)</td>
<td>5.00 (2.81)</td>
<td>5.75 (2.33)</td>
<td>5.86 (2.54)</td>
</tr>
</tbody>
</table>

- Dodging of Regulation happens when $\Delta w_i > \bar{u}$, where $\int_0^\bar{u} f(\Delta w_i) d\Delta w_i = 92.5\%$. Assume that household buys the same vehicle s/he owns currently;
- 25% emission reduction goal;
- Sum of CH$_4$ and N$_2$O; $d 18.6\% = \frac{200}{1000+75} \times 100$.

### Conclusion

The one-day driving restriction programs have been popular in developing countries to reduce traffic congestion and air pollution. Studies, however, fail to find clear evidence that the one-day driving restriction improves air quality. Three reasons are discussed, i) Households drive vehicles which have different emission factors and vehicles are not optimally spread over the weekdays. This is because the day of driving restriction applied is determined based on the license plate’s numbers, not the vehicle types. ii) Daily driving mileage is increased due to the substitution effect although total driving mileage is reduced. iii) Drivers have the incentive to buy used vehicles that might have high emission potential. Thus less than 20% of vehicles are off the roads and more air pollutants are emitted. The new driving restriction we propose uses the numerical optimization model based on the vehicle registration, rather than license plate’s number. This new program increases the opportunity cost of buying a second vehicle. Also, new and old vehicles are optimally mixed over the weekdays. As numerical simulation suggests, the original one-day driving restriction fails to achieve the policy goal effectively while the new structure achieves the goal.

The proposed new driving restriction might face challenges in its application to the real world for two reasons; i) the difficulty to estimate the benefit function and ii) administration costs would be high under the new driving restriction due to maintaining optimization model and the consistent enforcement of driving restriction. Perhaps the benefit function would not matter much as the study focuses on emissions; the optimization model will work as long as the benefit function is concave. In the future, increase in administration cost might be marginal due to advances in computer technology. Enforcement of regulation could be completed in creative ways, such as colored registration stickers on the vehicles.
References


